HW MATH227/10 Solutions

1.

(a) The set of vectors are linearly independent since there is no $(\alpha,\beta,\gamma) \neq (0,0,0)$ such that
$\alpha(2-x+4x^2)+\beta(3+6x+2x^2)+\gamma(2+10x-4x^2)=0.$
(b) The set of vectors are linearly independent since there is no $(\alpha,\beta,\gamma) \neq (0,0,0)$ such that
$\alpha(3+x+x^2)+\beta(2-x+5x^2)+\gamma(4-3x^2)=0.$
(c) The set of vectors are linearly independent since there is no $(\alpha,\beta) \neq (0,0)$ such that
$\alpha(6-x^2)+\beta(1+x+4x^2)=0.$
(d) The set of vectors
$1+3x+3x^2,\ x+4x^2,\ 5+6x+3x^2,\ 7+2x-x^2$
in $\mathbf{P}_2$ are linearly dependent since
$-17(\mathbf{1+3x+3x^2})+5(\mathbf{x+4x^2})+9(\mathbf{5+6x+3x^2})-4(\mathbf{7+2x-x^2})=0.$

2.

(a) $\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3$ form a linearly independent set, so they do not lie on the same plane.
(b) $\mathbf{v}_1,\ \mathbf{v}_2,\ \mathbf{v}_3$ form a linearly dependent set, so they lie on the same plane.

3.

For $\lambda=1$ or $\lambda=-\frac{1}{2}$ the vectors form a linearly dependent set in $\mathbf{R}^3$ since
$\mathbf{v}_1+\mathbf{v}_2+\mathbf{v}_3=0$ when $\lambda=1$ or $\lambda=-\frac{1}{2}$.

4.

(a) The vectors $(2,1)$, $(3,0)$ span $\mathbf{R}^2$ and are linearly independent, therefore this set of vectors is a basis for $\mathbf{R}^2$.
(b) The vectors $(4,1)$, $(-7,-8)$ span $\mathbf{R}^2$ and are linearly independent, therefore this set of vectors is a basis for $\mathbf{R}^2$.
(c) $(0,0)$, $(1,3)$ are not linearly independent, therefore this set of vectors is not a basis for $\mathbf{R}^2$.
(d) $(3,9)$, $(-4,-12)$ are not linearly independent, therefore this set of vectors is not a basis for $\mathbf{R}^2$.

5.

(a) $(\mathbf{w})_S = (3, -7)$ which means $\mathbf{w} = 3\mathbf{u}_1 + (-7)\mathbf{u}_2$.
(b) $(\mathbf{w})_S = (\frac{5}{28}, \frac{3}{14})$ which means $\mathbf{w} = \frac{5}{28}\mathbf{u}_1 + \frac{3}{14}\mathbf{u}_2$.
(c) $(\mathbf{w})_S = (a, \frac{b-a}{2})$ which means $\mathbf{w} = a\mathbf{u}_1 + \frac{b-a}{2}\mathbf{u}_2$.

6.

(a) A basis for the set of all vectors of the form $(a, b, c, 0)$ is

$$\begin{bmatrix} a \\ 0 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ b \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} 0 \\ 0 \\ c \\ 0 \end{bmatrix}$$

therefore the dimension of this subspace is 3.
(b) A basis for the set of all vectors of the form (a, b, c, d), where $d = a + b$ and $c = a - b$ is

$$\begin{bmatrix} a \\ 0 \\ a \\ a \end{bmatrix}, \begin{bmatrix} 0 \\ b \\ -b \\ b \end{bmatrix}$$

therefore the dimension of this subspace is 2.
(c) A basis for the set of all vectors of the form (a, b, c, d), where $a = b = c = d$ is

$$\begin{bmatrix} a \\ a \\ a \\ a \end{bmatrix}$$

therefore the dimension of this subspace is 1.