

HW MATH227/11 Solutions

1.

(a) $\mathbf{b}$ is in the column space of A

$$\mathbf{b} = \begin{bmatrix} 1 \\ 4 \end{bmatrix} - \begin{bmatrix} 3 \\ -6 \end{bmatrix}$$

(b) $\mathbf{b}$ is not in the column space of A .

(c) $\mathbf{b}$ is in the column space of A

$$\mathbf{b} = \begin{bmatrix} 1 \\ 9 \\ 1 \end{bmatrix} - 3 \begin{bmatrix} -1 \\ 3 \\ 1 \end{bmatrix} + \begin{bmatrix} 1 \\ 1 \\ 1 \end{bmatrix}$$

(d) $\mathbf{b}$ is in the column space of A

$$\mathbf{b} = \begin{bmatrix} 1 \\ 1 \\ -1 \end{bmatrix} + \begin{bmatrix} -1 \\ 1 \\ -1 \end{bmatrix} + 2 \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

(e) $\mathbf{b}$ is in the column space of A

$$\mathbf{b} = -26 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + 13 \begin{bmatrix} 2 \\ 1 \\ 2 \\ 1 \end{bmatrix} - 7 \begin{bmatrix} 0 \\ 2 \\ 1 \\ 2 \end{bmatrix} + 4 \begin{bmatrix} 1 \\ 1 \\ 3 \\ 2 \end{bmatrix}$$

2.

$$(a) \mathbf{x} = r \begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 4 \\ -1 \\ 0 \\ 1 \end{bmatrix}.$$

$$(b) \mathbf{x} = \begin{bmatrix} -1 \\ 2 \\ 4 \\ -3 \end{bmatrix} + r \begin{bmatrix} -3 \\ 1 \\ 1 \\ 0 \end{bmatrix} + s \begin{bmatrix} 4 \\ -1 \\ 0 \\ 1 \end{bmatrix}.$$

3. The vector form of the general solution of the given linear system $A\mathbf{x} = \mathbf{b}$ is

$$(a) \mathbf{x} = \begin{bmatrix} 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} 3 \\ 1 \end{bmatrix}.$$

$$(b) \mathbf{x} = \begin{bmatrix} -2 \\ 7 \\ 0 \end{bmatrix} + t \begin{bmatrix} -1 \\ -1 \\ 1 \end{bmatrix}.$$

$$(c) \mathbf{x} = \begin{bmatrix} -1 \\ 0 \\ 0 \\ 0 \end{bmatrix} + r \begin{bmatrix} 2 \\ 1 \\ 0 \\ 0 \end{bmatrix} + s \begin{bmatrix} -1 \\ 0 \\ 1 \\ 0 \end{bmatrix} + t \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}.$$

4. A basis of the null space of A is:

$$(a) \begin{bmatrix} 16 \\ 19 \\ 1 \end{bmatrix} \quad (b) \begin{bmatrix} 1 \\ 0 \\ 2 \end{bmatrix}, \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix} \quad (c) \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} 2 \\ -4 \\ 0 \\ 7 \end{bmatrix} \quad (d) \begin{bmatrix} -1 \\ -1 \\ 1 \\ 0 \\ 0 \end{bmatrix}, \begin{bmatrix} -2 \\ -1 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -1 \\ -2 \\ 0 \\ 0 \\ 1 \end{bmatrix}$$

$$(e) \begin{bmatrix} -2 \\ 0 \\ 0 \\ 1 \\ 0 \end{bmatrix}, \begin{bmatrix} -16 \\ 2 \\ 5 \\ 0 \\ 12 \end{bmatrix}$$

5. A basis for the column space of A is:

$$(a) \begin{bmatrix} 1 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} -1 \\ -4 \\ -6 \end{bmatrix} \quad (b) \begin{bmatrix} 2 \\ 4 \\ 0 \end{bmatrix} \quad (c) \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}, \begin{bmatrix} 4 \\ 1 \\ 3 \end{bmatrix} \quad (d) \begin{bmatrix} 1 \\ 3 \\ -1 \\ 2 \end{bmatrix}, \begin{bmatrix} 4 \\ -2 \\ 0 \\ 3 \end{bmatrix}$$

$$(e) \begin{bmatrix} 1 \\ 0 \\ 2 \\ 3 \\ -2 \end{bmatrix}, \begin{bmatrix} -3 \\ 3 \\ -3 \\ -6 \\ 9 \end{bmatrix}, \begin{bmatrix} 2 \\ 6 \\ -2 \\ 0 \\ 2 \end{bmatrix}$$