

HW MATH227/12

1. Determine $\text{rank}(A)$ and $\text{rank}(A^T)$

$$A = \begin{bmatrix} 1 & 2 & 4 & 0 \\ -3 & 1 & 5 & 2 \\ -2 & 3 & 9 & 2 \end{bmatrix}$$

2. Find the rank and nullity of the matrix

(a) $A = \begin{bmatrix} 1 & -1 & 3 \\ 5 & -4 & -4 \\ 7 & -6 & 2 \end{bmatrix}$

(b) $A = \begin{bmatrix} 2 & 0 & -1 \\ 4 & 0 & -2 \\ 0 & 0 & 0 \end{bmatrix}$

(c) $A = \begin{bmatrix} 1 & 4 & 5 & 2 \\ 2 & 1 & 3 & 0 \\ -1 & 3 & 2 & 2 \end{bmatrix}$

(d) $A = \begin{bmatrix} 1 & 4 & 5 & 6 & 9 \\ 3 & -2 & 1 & 4 & -1 \\ -1 & 0 & -1 & -2 & -1 \\ 2 & 3 & 5 & 7 & 8 \end{bmatrix}$

(e) $A = \begin{bmatrix} 1 & -3 & 2 & 2 & 1 \\ 0 & 3 & 6 & 0 & -3 \\ 2 & -3 & -2 & 4 & 4 \\ 3 & -6 & 0 & 6 & 5 \\ -2 & 9 & 2 & -4 & -5 \end{bmatrix}$

3. In each part, find the largest possible value for the rank of A and the smallest possible value for the nullity of A.

- (a) A is 4×4
(b) A is 3×5
(c) A is 5×3 .

4. What conditions must be satisfied by b_1 , b_2 , b_3 , b_4 and b_5 for the overdetermined

linear system

$$\begin{aligned}x_1 - 3x_2 &= b_1 \\x_1 - 2x_2 &= b_2 \\x_1 + x_2 &= b_3 \\x_1 - 4x_2 &= b_4 \\x_1 + 5x_2 &= b_5\end{aligned}$$

to be consistent.

5. Are there values of r and s for which the rank of

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & r-2 & 2 \\ 0 & s-1 & r+2 \\ 0 & 0 & 3 \end{bmatrix}$$

is one or two? if so, find those values.