HW MATH227/12 Solutions

1. $rank(A) = 2$ and $rank(A^T) = 2$

2.
(a) rank=2, nullity=1.
(b) rank=1, nullity=2.
(c) rank=2, nullity=2.
(d) rank=2, nullity=3.
(e) rank=3, nullity=2.

3.
(a) The largest possible value for the rank of 4×4 matrix is 4, and the smallest possible value for the nullity of 4×4 matrix is 0.
(b) The largest possible value for the rank of 3×5 matrix is 3, and the smallest possible value for the nullity of 3×5 matrix is 2.
(c) The largest possible value for the rank of 5×3 matrix is 3, and the smallest possible value for the nullity of 5×3 matrix is 0.

4. Using Gauss-Jordan elimination, the augmented matrix is equivalent to

$$\begin{bmatrix} 1 & 0 & 3b_2 - 2b_1 \\ 0 & 1 & b_2 - b_1 \\ 0 & 0 & b_3 - 4b_2 + 3b_1 \\ 0 & 0 & b_4 + b_2 - 2b_1 \\ 0 & 0 & b_5 - 8b_2 + 7b_1 \end{bmatrix}$$

then the system is consistent if and only if b_1, b_2, b_3, b_4, and b_5 satisfy

$$\begin{aligned} b_3 - 4b_2 + 3b_1 &= 0 \\ b_4 + b_2 - 2b_1 &= 0 \\ b_5 - 8b_2 + 7b_1 &= 0. \end{aligned}$$

On solving this homogeneous linear system we get $b_1 = r$, $b_2 = s$, $b_3 = -3r + 4s$, $b_4 = 2r - s$, $b_5 = -7r + 8s$ where r and s are arbitrary.

5. The rank can not be 1 because there are two independent column vectors for any value of r and s. The rank is 2 for values of $r = 2$ and $s = 1$ so that the second column of the matrix is a zero vector.