HW MATH227/2

1. Consider the matrices

$$A = \begin{bmatrix} 3 & 0 \\ -1 & 2 \\ 1 & 1 \end{bmatrix}, \quad B = \begin{bmatrix} 1 & 5 & 2 \\ -1 & 0 & 1 \\ 3 & 2 & 4 \end{bmatrix}, \quad C = \begin{bmatrix} 6 & 1 & 3 \\ -1 & 1 & 2 \\ 4 & 1 & 3 \end{bmatrix}$$

compute the following (where possible).
(a) tr(B-3C) (b)-3(B+2C) (c) tr(A).

2. Let

$$A = \begin{bmatrix} 3 & -2 & 7 \\ 6 & 5 & 4 \\ 0 & 4 & 9 \end{bmatrix}, \quad B = \begin{bmatrix} 6 & -2 & 4 \\ 0 & 1 & 3 \\ 7 & 7 & 5 \end{bmatrix},$$

Use the method of Example 7 (page 29) to find
(a) the first row of AB (b) the third row of AB (c) the second column of AB.

3. In each part find matrices A, $\mathbf{x}$, and $\mathbf{b}$ that express the given system of linear equations as a single matrix equation $A\mathbf{x} = \mathbf{b}$.

$$(a) \quad \begin{array}{rrrcr} 2x_1 & -3x_2 & +5x_3 & = & 7 \\ 9x_1 & -x_2 & +x_3 & = & -1 \\ x_1 & +5x_2 & +4x_3 & = & 0 \end{array}$$

$$(b) \quad \begin{array}{rrrrcr} 4x_1 & & -3x_3 & +x_4 & = & 1 \\ 5x_1 & +x_2 & & -8x_4 & = & 3 \\ 2x_1 & -5x_2 & +9x_3 & -x_4 & = & 0 \\ & 3x_2 & -x_3 & +7x_4 & = & 2 \end{array}$$

4. If A and B are partioned into submatrices, for example,

$$A = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline A_{21} & A_{22} \end{array}\right], \quad B = \left[\begin{array}{c|c} B_{11} & B_{12} \\ \hline B_{21} & B_{22} \end{array}\right]$$

then AB can be expressed as

$$AB = \left[\begin{array}{c|c} A_{11}B_{11} + A_{12}B_{21} & A_{11}B_{12} + A_{12}B_{22} \\ \hline A_{21}B_{11} + A_{22}B_{21} & A_{21}B_{12} + A_{22}B_{22} \end{array}\right]$$

provided the sizes of the submatrices of A and B are such that the indicated operations can be performed. This method of multiplying partioned matrices is called **block multiplication**. In each part compute by block multiplication. Check your results by multiplying directly.

$$(a)\quad A = \left[\begin{array}{rr|rr} -1 & 2 & 1 & 4 \\ 0 & -3 & 4 & 2 \\ \hline 1 & 5 & 6 & 1 \end{array}\right] \qquad B = \left[\begin{array}{rr|r} 2 & 1 & 4 \\ -3 & 5 & 2 \\ \hline 7 & -1 & 5 \\ 0 & 3 & -3 \end{array}\right]$$

$$(b)\quad A = \left[\begin{array}{rrr|r} -1 & 2 & 1 & 5 \\ \hline 0 & -3 & 4 & 2 \\ 1 & 5 & 6 & 1 \end{array}\right] \qquad B = \left[\begin{array}{rr|r} 2 & 1 & 4 \\ -3 & 5 & 2 \\ 7 & -1 & 5 \\ \hline 0 & 3 & -3 \end{array}\right]$$

5. Use Theorem 1.4.5 to compute the inverse of the following matrices.

$$A = \begin{bmatrix} 3 & 1 \\ 5 & 2 \end{bmatrix}, \quad B\begin{bmatrix} 2 & -3 \\ 4 & 4 \end{bmatrix}, \quad C\begin{bmatrix} 6 & 4 \\ -2 & -1 \end{bmatrix}, \quad D\begin{bmatrix} 2 & 0 \\ 0 & 3 \end{bmatrix}$$

6. In each part use the given information to find A.

$$(a)\quad A^{-1} = \begin{bmatrix} 2 & -1 \\ 3 & 5 \end{bmatrix}$$

$$(b)\quad (I + 2A)^{-1} = \begin{bmatrix} -1 & 2 \\ 4 & 5 \end{bmatrix}$$

7. Let A be the matrix

$$\begin{bmatrix} 3 & 1 \\ 2 & 1 \end{bmatrix}$$

In each part find $p(A)$.
(a) $p(x) = x - 2$ (b) $p(x) = 2x^2 - x + 1$ (c) $p(x) = x^3 - 2x + 4$.

8. Let A be the matrix

$$\begin{bmatrix} 2 & 0 \\ 4 & 1 \end{bmatrix}$$

Compute A^3, A^{-3}, and $A^2 - 2A + I$.