

## HW MATH227/3

1. Which of the following are elementary matrices?

$$(a) \begin{bmatrix} 1 & 0 \\ -5 & 1 \end{bmatrix}, \quad (b) \begin{bmatrix} -5 & 1 \\ 1 & 0 \end{bmatrix}, \quad (c) \begin{bmatrix} 1 & 0 \\ 0 & \sqrt{3} \end{bmatrix}, \quad (d) \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix},$$

$$(e) \begin{bmatrix} 1 & 1 & 0 \\ 0 & 0 & 1 \\ 0 & 0 & 0 \end{bmatrix}, \quad (f) \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 9 \\ 0 & 0 & 1 \end{bmatrix}, \quad (g) \begin{bmatrix} 2 & 0 & 0 & 2 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

2. Consider the matrix

$$A = \begin{bmatrix} 1 & 0 \\ -5 & 2 \end{bmatrix}$$

- (a) Find elementary matrices  $E_1$  and  $E_2$  such that  $E_2 E_1 A = I$ .
- (b) Write  $A^{-1}$  as a product of two elementary matrices.
- (c) Write  $A$  as a product of two elementary matrices.

3. In each part perform the stated row operation on

$$A = \begin{bmatrix} 2 & -1 & 0 \\ 4 & 5 & -3 \\ 1 & -4 & 7 \end{bmatrix}$$

by multiplying  $A$  on the left by a suitable elementary matrix. Check your answer in each case by performing the row operation directly on  $A$ .

- (a) Interchange the first and third rows.
- (b) Multiply the second row by  $\frac{1}{3}$ .
- (c) Add twice the second row to the first row.

*of equation*

4. Solve the system  $\lambda$  by inverting the coefficient matrix and using Theorem 1.6.2

$$\begin{aligned} x_1 + x_2 &= 2 \\ 5x_1 + 6x_2 &= 9 \end{aligned}$$

5. Solve the system by inverting the coefficient matrix and using Theorem 1.6.2

$$\begin{array}{rcl} x_1 + 3x_2 + x_3 & = & 4 \\ 2x_1 + 2x_2 + x_3 & = & -1 \\ 2x_1 + 3x_2 + x_3 & = & 3 \end{array}$$

6. Solve the following matrix equation for  $X$

$$\begin{bmatrix} 1 & -1 & 1 \\ 2 & 3 & 0 \\ 0 & 2 & -1 \end{bmatrix} X = \begin{bmatrix} 2 & -1 & 5 & 7 & 8 \\ 4 & 0 & -3 & 0 & 1 \\ 3 & 5 & -7 & 2 & 1 \end{bmatrix}$$

7. Determine whether the matrix is invertible; if so, find the inverse by inspection

$$(a) \begin{bmatrix} 2 & 0 \\ 0 & -5 \end{bmatrix}, \quad (b) \begin{bmatrix} 4 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 5 \end{bmatrix}, \quad (c) \begin{bmatrix} -1 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & \frac{1}{3} \end{bmatrix}$$

8. Find all values of  $a$ ,  $b$ , and  $c$  for which  $A$  is symmetric.

$$A = \begin{bmatrix} 2 & a - 2b + 2c & 2a + b + c \\ 3 & 5 & a + c \\ 0 & -2 & 7 \end{bmatrix}$$