HW MATH227/5

1. Verify that $det(kA) = k^n det(A)$ for

$$(a) \quad A = \begin{bmatrix} -1 & 2 \\ 3 & 4 \end{bmatrix}; \; k = 2$$

$$(b) \quad A = \begin{bmatrix} 2 & -1 & 3 \\ 3 & 2 & 1 \\ 1 & 4 & 5 \end{bmatrix}; \; k = -2$$

2. Verify that $det(AB) = det(A)det(B)$ for

$$A = \begin{bmatrix} 2 & 1 & 0 \\ 3 & 4 & 0 \\ 0 & 0 & 2 \end{bmatrix} \text{ and } B = \begin{bmatrix} 1 & -1 & 3 \\ 7 & 1 & 2 \\ 5 & 0 & 1 \end{bmatrix}$$

3. For which value(s) of k does A fail to be invertible?

$$(a) \quad \begin{bmatrix} k-3 & -2 \\ -2 & k-2 \end{bmatrix}$$

$$(b) \quad \begin{bmatrix} 1 & 2 & 4 \\ 3 & 1 & 6 \\ k & 3 & 2 \end{bmatrix}$$

4. Express the following linear systems in the form $(\lambda I - A)x = 0$.

$$(a) \quad \begin{aligned} x_1 + 2x_2 &= \lambda x_1 \\ 2x_1 + x_2 &= \lambda x_2 \end{aligned}$$

$$(b) \quad \begin{aligned} 2x_1 + 3x_2 &= \lambda x_1 \\ 4x_1 + 3x_2 &= \lambda x_2 \end{aligned}$$

$$(c) \quad \begin{aligned} 3x_1 + x_2 &= \lambda x_1 \\ -5x_1 - 3x_2 &= \lambda x_2 \end{aligned}$$

5. Let

$$A = \begin{bmatrix} 1 & -2 & 3 \\ 6 & 7 & -1 \\ -3 & 1 & 4 \end{bmatrix}$$

(a) Find all the minors of A. (b) Find all the cofactors.

6. Evaluate $det(A)$ by cofactor expansion along a row or a column of your choice.

$$A = \begin{bmatrix} 3 & 3 & 1 \\ 1 & 0 & -4 \\ 1 & -3 & 5 \end{bmatrix}$$

7. Solve by cramer's rule

$$\begin{array}{rrrcl} 4x & +5y & & = & 2 \\ 11x & +y & +2z & = & 3 \\ x & +5y & +2z & = & 1 \end{array}$$

8. If $A = \left[\begin{array}{c|c} A_{11} & A_{12} \\ \hline 0 & A_{22} \end{array}\right]$ is an upper triangular block matrix, where A_{11} and A_{22} are square matrices, then $det(A) = det(A_{11})det(A_{22})$. Use this result to evaluate $det(A)$ for

$$\left[\begin{array}{cc|ccc} 2 & -1 & 2 & 5 & 6 \\ 4 & 3 & -1 & 3 & 4 \\ \hline 0 & 0 & 1 & 3 & 5 \\ 0 & 0 & -2 & 6 & 2 \\ 0 & 0 & 3 & 5 & 2 \end{array}\right]$$