HW MATH227/6 Solutions

1. (a) The components of the vector $\mathbf{v} - \mathbf{w}$ is $(-2, 1, -4)$.
(b) The components of the vector $6\mathbf{u} + 2\mathbf{v}$ is $(-10, 6, -4)$.
(c) The components of the vector $-\mathbf{v} + \mathbf{w}$ is $(-7, 1, 10)$.

2. (a) The norm of the vector $\mathbf{u} = (4, -3)$ is $||\mathbf{u}|| = \sqrt{4^2 + (-3)^2} = 5$.
(b) The norm of the vector $\mathbf{u} = (2, 2, 2)$ is $||\mathbf{u}|| = 2\sqrt{3}$.
(c) The norm of the vector $\mathbf{u} = (-7, 2, -1)$ is $||\mathbf{u}|| = 3\sqrt{6}$.

3. (a) $\mathbf{u}.\mathbf{v} = 0$, for the given vectors we have $||u|| = \sqrt{53}$ and $||v|| = \sqrt{13}$ so that

$$cos\ \theta = \frac{\mathbf{u}.\mathbf{v}}{||u||||v||} = 0$$

thus, $\theta = 90$.
(b) $\mathbf{u}.\mathbf{v} = -3$, for the given vectors we have $||u|| = \sqrt{6}$ and $||v|| = \sqrt{6}$ so that

$$cos\ \theta = \frac{\mathbf{u}.\mathbf{v}}{||u||||v||} = \frac{-3}{\sqrt{6}\sqrt{6}} = -\frac{1}{2}$$

thus, $\theta = 120$.

4. (a) $(0, 0, 5)$ is a point in the plane and $\mathbf{n} = (-3, 7, 2)$ is a normal vector so that $-3(x - 0) + 7(y - 0) + 2(z - 5) = 0$ is a point-normal form.
(b) $(x - 0) + 0(y - 0) - 4(z - 0) = 0$.

5. (a) The line of intersection consists of all the points (x, y, z) that satisfy the two equations in the system

$$\begin{aligned} -3x + 2y + z &= -5 \\ 7x + 3y - 2z &= -2 \end{aligned}$$

Solving the system gives

$$\begin{cases} x &= \frac{11}{23} + \frac{7}{23}t \\ y &= -\frac{41}{23} - \frac{1}{23}t \\ z &= t \end{cases}$$

(b) The line of intersection consists of all the points (x, y, z) that satisfy the two equations in the system

$$5x - 7y + 2z = 0$$
$$y = 0$$

Solving the system gives

$$\begin{cases} x &= -\frac{2}{5}t \\ y &= 0 \\ z &= t \end{cases}$$

6. $\mathbf{u}$ and $\mathbf{v}$ are orthogonal when

(a) $k = -3$

(b) $k = -2$ and $k = -3$

7.

$$\begin{cases} x_1 &= 1 \\ x_2 &= -1 \\ x_3 &= 2 \end{cases}$$