HW MATH227/9

1. Determine whether the set of all triplets of real numbers (x, y, z) with the operations $(x, y, z) + (x', y', z') = (x + x', y + y', z + z')$ and $k(x, y, z) = (0, 0, 0)$ is a vector space.

2. Determine whether the set of all 2×2 matrices of the form

$$\begin{bmatrix} a & 1 \\ 1 & b \end{bmatrix}$$

with matrix addition and scalar multiplication is a vector space.

3. Determine which of the following are subspaces of $\mathbf{R}^3$
(a) All vectors of the form $(a, 0, 0)$.
(b) All vectors of the form $(a, 1, 1)$.
(c) All vectors of the form (a, b, c), where $b = a + c$.
(d) All vectors of the form (a, b, c), where $b = a + c + 1$.

4. Determine which of the following are subspaces of $\mathbf{M}_{nn}$
(a) All $n \times n$ matrices A such that $tr(A) = 0$.
(b) All $n \times n$ matrices A such that $A^T = -A$.
(c) All $n \times n$ matrices A such that the linear system $A\mathbf{x} = \mathbf{0}$ has only the trivial solution.
(d) All $n \times n$ matrices A such that $AB = BA$ for a fixed $n \times n$ matrix B.

5. Express the following as linear combinations of $\mathbf{p}_1 = 2 + x + 4x^2$, $\mathbf{p}_2 = 1 - x + 3x^2$, and $\mathbf{p}_3 = 3 + 2x + 5x^2$.
(a) $-9 - 7x - 15x^2$.
(b) $6 + 11x + 6x^2$.
(c) $7 + 8x + 9x^2$.

6. In each part determine whether the given vectors span R^3.
(a) $\mathbf{v}_1 = (2, 2, 2)$, $\mathbf{v}_2 = (0, 0, 3)$, $\mathbf{v}_3 = (0, 1, 1)$.
(b) $\mathbf{v}_1 = (2, -1, 3)$, $\mathbf{v}_2 = (4, 1, 2)$, $\mathbf{v}_3 = (8, -1, 8)$.
(c) $\mathbf{v}_1 = (3, 1, 4)$, $\mathbf{v}_2 = (2, -3, 5)$, $\mathbf{v}_3 = (5, -2, 9)$, $\mathbf{v}_4 = (1, 4, -1)$.
(d) $\mathbf{v}_1 = (1, 2, 6)$, $\mathbf{v}_2 = (3, 4, 1)$, $\mathbf{v}_3 = (4, 3, 1)$, $\mathbf{v}_4 = (3, 3, 1)$.