MATH241 **Homework 3** **Fall 2012-2013**

(This homework is due on 19-10-2012, or on 21-10-2012 if typed and uploaded as .pdf)

Q1: Consider the vector space $\mathbb{R}^2$. Define the mapping

$$\langle \mathbf{x},\mathbf{y}\rangle = x_1y_1 - x_2y_2 \text{ for all vectors } \mathbf{x}=\begin{bmatrix} x_1 \\ x_2 \end{bmatrix} \text{ and } \mathbf{y}=\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} \in \mathbb{R}^2. \quad \text{------- (*)}$$

(a) Show that $\langle \mathbf{x},\mathbf{y}\rangle = \langle \mathbf{y},\mathbf{x}\rangle$.

(b) Show that $\langle \alpha\mathbf{x},\mathbf{y}\rangle = \alpha\langle \mathbf{y},\mathbf{x}\rangle$ *for any* $\alpha \in \mathbb{R}$.

(c) Show that $\langle (\mathbf{x}+\mathbf{y}),\mathbf{z}\rangle = \langle \mathbf{x},\mathbf{z}\rangle + \langle \mathbf{y},\mathbf{z}\rangle$.

(d) Determine all <u>non-zero vectors</u> satisfying $\langle \mathbf{x},\mathbf{x}\rangle = 0.$ Scetch them on $\mathbb{R}^2$.

(e) Determine all non-zero vectors satisfying $\langle \mathbf{x},\mathbf{x}\rangle < 0.$ Scetch this region on $\mathbb{R}^2$.

(f) Determine all non-zero vectors satisfying $\langle \mathbf{x},\mathbf{x}\rangle > 0.$ Scetch this region on $\mathbb{R}^2$.

(g) Is the mapping given by equation (*) a valid dot (i.e., inner) product?

Q2: Determine whether the following statements about subspaces are TRUE of FALSE. For each statement if it is true, provide a complete proof; and if it is false, provide a counter example that disputes the claim.

(a) If S_1 and S_2 are two subspaces of the vector space V, then $S_1 \cap S_2$ is also a subspace.

(b) If S_1 and S_2 are two subspaces of the vector space V, then $S_1 \cup S_2$ is also a subspace.

(c) If $S_1 \cap S_2$ is a subspace of the vector space V, then both S_1 and S_2 are must be subspaces of V.

Q3: Show that the set of vectors $\left\{ \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}, \begin{bmatrix} 3 \\ 4 \\ 5 \end{bmatrix}, \begin{bmatrix} 4 \\ 5 \\ 6 \end{bmatrix} \right\}$ does not span $\mathbb{R}^3$, but it does span the subspace of $\mathbb{R}^3$ consisting of all vectors lying in the plane with equation $x_1 - 2x_2 + x_3 = 0$.

Q4: Determine all values of constant k for which the set of vectors $S = \left\{ \begin{bmatrix} 1 \\ 1 \\ 0 \\ -1 \end{bmatrix}, \begin{bmatrix} 1 \\ k \\ 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \\ k \\ 1 \end{bmatrix}, \begin{bmatrix} -1 \\ 1 \\ 1 \\ k \end{bmatrix} \right\}$ is linearly independent (and hence, a basis) in $\mathbb{R}^4$.

Q5: Let S denote the subspace of $\mathbb{R}^3$ consisting of all points lying on the plane with the equation $2x_1 + x_2 - 6x_3 = 0$.

(a) Determine a basis for S .

(b) Extend your basis for S to obtain a basis for $\mathbb{R}^3$.

Q6: Let $V = P_3$ be the real vector space of all real-valued polynomials of degree less than 3 with real coefficients. Determine a linearly independent set of vectors that spans the same subspace of V as that spanned by the set of vectors given by $\{2+x^2, 4-2x+3x^2, 1+x\}$.

Q7: $V = Span\{1, \cos t, \cos(2t), \cos(3t)\}$ is a vector space. Find its dimension.

Hint: Use the orhogonality of trigonometric functions.