MATH 241 HW # 2

Due 12/10/2012

or

Due 14/10/2012
(if typed and uploaded as .pdf)

① Consider the following linear system of equations:

$$
\begin{aligned}
x_1 + 4x_2 - 2x_3 &= 1 \\
x_1 + 7x_2 - 6x_3 &= 6 \\
3x_2 + \alpha x_3 &= \beta
\end{aligned}
$$

(a) Determine all values of the constants α and β for which the given system has

(i) no solution;

(ii) a unique solution;

(iii) an infinite number of solutions.

(b) Find the solution set that has $x_3 = 1$.

② Prove that a linear system of equations cannot have exactly two solutions.

③ For each of the coefficient matrix $\underline{\underline{A}}$ and the right-hand side vector $\underline{b}$, find the reduced row echelon form of the augmented matrix (you may use the MATLAB command "rref"). Then, determine if the corresponding linear system of equations is consistent, and if so, find the solution.

(a) $\underline{\underline{A}} = \begin{bmatrix} 2 & -1 & 1 \\ -1 & -1 & -6 \\ 5 & -3 & 1 \end{bmatrix}$; $\underline{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \end{bmatrix}$ (b) $\underline{\underline{A}} = \begin{bmatrix} 5 & -6 & 1 \\ 2 & -3 & 1 \\ 4 & -3 & -1 \end{bmatrix}$; $\underline{b} = \begin{bmatrix} 4 \\ 1 \\ 5 \end{bmatrix}$ (c) $\underline{\underline{A}} = \begin{bmatrix} 1 & -1 & 2 & 1 \\ 2 & 1 & -3 & -1 \\ 4 & -1 & 1 & 1 \\ 1 & 2 & -5 & -2 \end{bmatrix}$; $\underline{b} = \begin{bmatrix} 1 \\ 2 \\ 3 \\ 1 \end{bmatrix}$

④ Prove the following identities about the vector spaces:

(a) The zero element, $\underline{0}$, is unique.

(b) $-\underline{0} = \underline{0}$

(c) $\forall \underline{u} \in \mathbb{R}^n$, $0\underline{u} = \underline{0}$

(d) $\forall \alpha \in \mathbb{R}$, $\alpha \underline{0} = \underline{0}$

(e) The additive inverse $-\underline{u}$ is unique for $\forall \underline{u} \in \mathbb{R}^n$

(f) $\forall \underline{u} \in \mathbb{R}^n$, $-\underline{u} = (-1)\underline{u}$

⑤ For real-valued set of vectors $V = \left\{ \underline{x} : \begin{bmatrix} x_1 \\ x_2 \end{bmatrix}, x_1 > 0, x_2 > 0 \right\}$, the vector addition and scalar multiplication are defined as follows:

$\forall \underline{x}, \underline{y} \in V$ and $c \in \mathbb{R}$ $\quad \underline{x} + \underline{y} = \begin{bmatrix} x_1 y_1 \\ x_2 y_2 \end{bmatrix}$, $c\,\underline{x} = \begin{bmatrix} x_1^c \\ x_2^c \end{bmatrix}$.

Show that V together with the above defined operations is a vector space.