MATH 241 - Fall 2012

HW #4

Due 02/11/2012

Due 04/11/2012 if typed and uploaded as .pdf

1) Obtain an orthonormal basis for the subspace of $\Re^4$ spanned by

$$\underline{v}_1 = \begin{bmatrix} 1 \\ 0 \\ -1 \\ 0 \end{bmatrix},\ \underline{v}_2 = \begin{bmatrix} 1 \\ 1 \\ -1 \\ 0 \end{bmatrix} \text{ and } \underline{v}_3 = \begin{bmatrix} -1 \\ 1 \\ 0 \\ 1 \end{bmatrix}.$$

2) Question 9.9.11 from your textbook[1].

3) Question 9.9.13 from your textbook, parts (a), (c) and (e).

4) Let $\boldsymbol{S}$ denote the subspace of $\Re^3$ consisting of all points lying on the plane with the equation $-x_1 - x_2 + x_3 = 0$.

(a) Determine a basis for $\boldsymbol{S}$.
(b) Extend your basis for $\boldsymbol{S}$ to obtain a basis for $\Re^3$.
(c) find the best approximation of the vector $\underline{b} = \begin{bmatrix} 2 \\ 1 \\ 0 \end{bmatrix}$ on this subspace.

5) Let the vector space $\boldsymbol{C}^0[a, b]$ be all real-valued functions that are continuous on the interval $[a, b]$. In this vector space the mapping $\langle f, g\rangle = \int_a^b f(x)\, g(x)\, dx$ defines an inner product. You can view $\langle f, g\rangle$ as $\underline{f} \cdot \underline{g}$ (i.e., dot product) for functions. Therefore,

(i) two functions are orthogonal if $\langle f, g\rangle = 0$,
(ii) $\|f\| = \left[\int_a^b \big(f(x)\big)^2\, dx\right]^{1/2}$

Determine an orthonormal basis for the subspace of $\boldsymbol{C}^0[-1, 1]$ spanned by the functions $f_1(x) = x$ and $f_2(x) = x^3$.

6) Question 9.10.3 from your textbook.

7) Let $\boldsymbol{M}_n(\Re)$ be a real vector space of all $n \times n$ matrices with real elements.

(a) Write a basis for $\boldsymbol{M}_3(\Re)$.
(b) What is the dimension of $\boldsymbol{M}_3(\Re)$?
(c) What is the dimension of $\boldsymbol{M}_n(\Re)$?

8) Let $\boldsymbol{S}$ be the set of all $n \times n$ diagonal matrices with zero trace (i.e., $\boldsymbol{s} = \big\{\underline{\underline{A}} \in \boldsymbol{S} : \underline{\underline{A}} = \{a_{i,j}\}_{n\times n}$ such that $a_{i,j} = 0$ if $i \neq j$ and $trace(\underline{\underline{A}}) = 0\ \big\}$).

(a) Show that $\boldsymbol{S}$ is a subspace.
(b) What is the dimension of $\boldsymbol{S}$?

9) Question 10.2.12 form your textbook, parts (a), (b).

10) Question 10.2.15 form your textbook, parts (a), (b).

[1] Textbook refers to Advanced Engineering Mathematics by Greenberg.