

RECEIVER ALGORITHMS FOR OFDM SYSTEMS IN PHASE NOISE AND AWGN

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Abstract - In this paper we address the subject of reducing the impact of phase noise on a QAM-OFDM system transmitting over an AWGN channel. Phase noise is known to have two effects on OFDM systems, rotating each symbol by a different common phase rotation (CPR) and producing an intercarrier interference term (ICI) that adds to the channel noise. We present two novel algorithms that remove CPR and reduce the amount of ICI using different approaches to the ICI problem. Their performance in terms of bit error rate (BER) are evaluated by simulation using symbol by symbol or maximum likelihood sequence detection (MLSD). The results show that these algorithms can significantly reduce the BER floor of the system while still maintaining an acceptable throughput.

Keywords - Phase noise, OFDM, ICI, estimation, MLSD.

I. INTRODUCTION

Orthogonal Frequency Division Multiplexing (OFDM) has over the years become an attractive method for transmitting a high data rate with an efficient use of the bandwidth. This modulation has been implemented in many standards for multimedia content broadcasting, such as DVB and DAB, and wireless LAN application standards such as IEEE 802.11a and HiperLAN2. The bandwidth efficiency of OFDM gives it a real advantage compared to other multi-carrier systems, but it also sets more stringent requirements on the RF hardware specifications. Because carriers are kept very close to each other, OFDM is very sensitive to distortion that may remove the orthogonality between carriers, and thus result in intercarrier interference (ICI). When considering a practical system, one must take into account hardware imperfections such as oscillator phase noise. This interference is unavoidable (although careful design can minimize it), and acts as a random phase change from the oscillator RF frequency. When the modulation experiences phase noise, it encounters two problems: 1) a common phase rotation over all the carrier frequencies which rotates the entire signal space for a given OFDM symbol and 2) intercarrier interference due to the lost orthogonality between subcarriers. The problem of phase noise effect characterisation, modelling and impact analysis has been vastly covered in the literature. The subject of modelling phase noise is covered in the paper of Foschini et al [1]. BER analysis was covered by Tomba in [2] using a moment method to evaluate the BER

analytically. In [3], OFDM sensitivity to phase noise was quantified in terms of SNR loss for different levels of phase noise and frequency offset. A summary of OFDM theory and practice can be found in [4]. In this work we present estimation algorithms for reducing the effect of CPR and ICI. These methods are based on several OFDM symbol observations which allows to spread known pilots positions over several symbols, thus increasing the payload in each OFDM symbol. The first algorithm uses MMSE estimation theory to estimate phase noise and compensate for it given its statistical properties. The second algorithm is based on a power series expansion of the phase noise term where each coefficient of the series is ML-estimated.

This paper is organised as follow. We describe the system model in section II, where the phase noise model and the OFDM system are presented. In section III, the CPR estimation and ICI reduction methods are presented and discussed. Section V shows the performance of these algorithms for different phase noise scenarios and detection methods. Finally in section VI we discuss these results and give our conclusions.

II. SYSTEM MODEL

A. OFDM System

Figure 1 shows the considered OFDM system. From the information source, $\log M$ bits are mapped onto an M-QAM symbol $D_m(k)$ with average energy per bit E_b and average transmitted energy $E_s = \log M E_b$. N of these QAM symbols are fed into an IFFT block to produce the m th OFDM symbol as

$$s_m(n) = \sum_{k=0}^{N-1} D_m(k) e^{2j\pi \frac{kn}{N}} \quad (1)$$

Oscillator phase noise and additive white complex Gaussian noise are applied to produce the received signal

$$r_m(n) = s_m(n) e^{j\phi_m(n)} + w_m(n) \quad (2)$$

where $\phi_m(n)$ is modeled according to the phase noise model described in section II-B and $w_m(n)$ is a complex white Gaussian noise sample with variance N_0/N . The k th carrier is demodulated taking the FFT at the k th index:

$$R_m(k) = \frac{1}{N} \sum_{n=0}^{N-1} r_m(n) e^{-2j\pi \frac{kn}{N}} \quad (3)$$

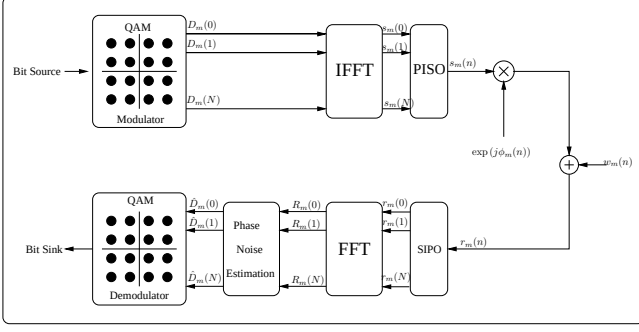


Fig. 1. Block diagram for an OFDM system with phase noise and AWGN.

Expanding (3) we obtain

$$R_m(k) = D_m(k) \frac{1}{N} \sum_{n=0}^{N-1} e^{j\phi_m(n)} + \frac{1}{N} \sum_{n=0}^{N-1} \sum_{\substack{l=0 \\ l \neq k}}^{N-1} D_m(l) e^{j\phi_m(n)} e^{j2\pi \frac{(l-k)n}{N}} + W_m(k) \quad (4)$$

where $W_m(k)$ is the FFT of $w_m(n)$, $n = 0, \dots, N-1$, which still is a white Gaussian random variable but its variance is now equal to N_0 . For convenience, let us define

$$X_m(l) = \frac{1}{N} \sum_{n=0}^{N-1} e^{(j\phi_m(n) + 2j\pi \frac{ln}{N})} \quad (5)$$

Equation (4) then becomes

$$R_m(k) = D_m(k) X_m(0) + \sum_{\substack{l=0 \\ l \neq k}}^{N-1} D_m(l) X_m(l-k) + W_m(k) \quad (6)$$

Here we can see more clearly the way phase noise affects an OFDM system: the desired QAM symbol $D_m(k)$ is rotated by a factor $X_m(0)$ which is independent of the carrier index. We will refer to this as the common phase rotation (CPR). The rest of the QAM symbols $D_m(l)$, $l \neq k$, are corrupting the wanted symbol because carriers are no longer orthogonal. The amount of ICI caused on the k th carrier by the l th carrier is represented by the factor $X_m(l-k)$. We conclude this section by expressing the content of the N subcarriers in a matrix form as

$$\mathbf{R}_m = \mathbf{X}_m \mathbf{D}_m + \mathbf{W}_m \quad (7)$$

where $\mathbf{R}_m = [R_m(0), R_m(1), \dots, R_m(N-1)]^T$ and $\mathbf{D}_m = [D_m(0), \dots, D_m(N-1)]^T$. $\mathbf{X}_m$ is an $N \times N$ matrix defined as the phase noise matrix whose elements are set as $\mathbf{X}_m(i, l) = X_m(l-i)$.

B. Phase Noise Model

Phase noise is described here as a sampled version of the Wiener process [2]

$$\phi(t) = \int_0^t \Delta(s) ds \quad (8)$$

The noisy carrier in its complex lowpass equivalent model $e^{j\phi(t)}$ has a Lorentzian power spectral density with a 3-dB bandwidth B controlled by the variance of the white Gaussian random variable $\Delta(s)$ [5]. For the purpose of analysis and simulation in a digital communication system we will use a discrete time random walk, also called Wiener-Lévy process. This process is the equivalent of the continuous process $\phi(t)$ sampled every T_s seconds. The n th point of the discrete process taken during the m th OFDM symbol is expressed as

$$\phi_m(n) = \phi((mN+n)T_s) = \sum_{u=0}^{mN+n} \Delta_u \quad (9)$$

where $T = NT_s$ is the OFDM symbol period. Δ_n is referred as the *stepsize* of the walk and is a zero mean white Gaussian random variable. Its variance sets the speed of the process and is equal to $\sigma_\Delta^2 = 2\pi BT/N$ [2]. The product BT is referred as the *phase noise rate* and expresses the relative double-sided bandwidth of the discrete time carrier $e^{j\phi_m(n)}$ with respect to the symbol period T .

III. PHASE NOISE COMPENSATION SCHEMES

We aim at compensating phase noise by removing the phase noise matrix $\mathbf{X}_m$ from equation (7) before detection. Below we describe two methods to estimate this matrix; one is a classical MMSE approach aiming at minimizing the estimation error, while the other is based on a power series approximation of the phase noise. Both methods take advantage of the fact that $X_m(l)$ is correlated in time and in frequency. From (5) after some straightforward algebra we obtain

$$E[X_m(k)X_m^*(l)] = \frac{1}{N^2} \sum_{u,v=0}^{N-1} e^{-\frac{1}{2}[(m-k)N+u-v]\sigma_\Delta^2} e^{j2\pi \frac{ku-lv}{N}} \quad (10)$$

The estimators use observations from L OFDM symbols. These symbols contain each K pilots. Figure 2 shows an illustration of a frame with pilots inserted.

A. MMSE solution

The structure of the phase noise matrix $\mathbf{X}_m$ is such that there actually are only $2N-1$ unknowns in the matrix, namely $X_m(-N+1), \dots, X_m(0), \dots, X_m(N-1)$. Defining $\mathbf{\Omega} = [X_m(-N+1), \dots, X_m(0), \dots, X_m(N-1)]^T$ as the vector of unknowns, $\mathbf{\Omega}$ can be estimated by MMSE methods using the correlation between $\mathbf{\Omega}$ and the observed signal $\mathbf{Y} = [\mathbf{Y}_m^T \mathbf{Y}_{m-1}^T, \dots, \mathbf{Y}_{m-L+1}^T]^T$, where $Y_m(k) = R_m(k)/D_m(k)$, $k = 0, \dots, N-1$ is the m th received OFDM

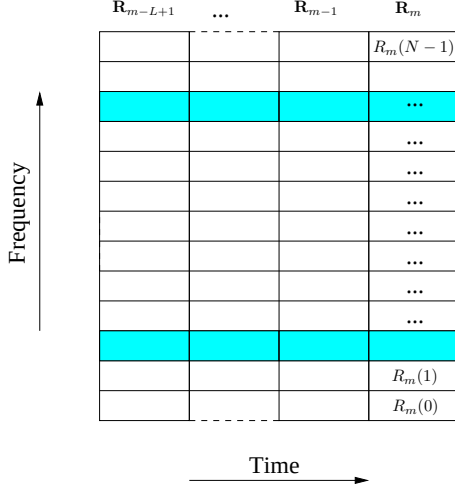


Fig. 2. Frame used for estimation of the phase noise matrix $\mathbf{X}_m$. The shaded slots show an example of the pilot positioning.

symbol vector after removal of the data dependency. The estimate of $\mathbf{\Omega}$ is obtained by [6]

$$\hat{\mathbf{\Omega}} = \mathbf{C}_{\mathbf{\Omega}\mathbf{Y}} \mathbf{C}_{\mathbf{Y}\mathbf{Y}}^{-1} \mathbf{Y} \quad (11)$$

where $\mathbf{C}_{\mathbf{\Omega}\mathbf{Y}}$ is the cross-correlation matrix between $\mathbf{\Omega}$ and the observation vector $\mathbf{Y}$, and $\mathbf{C}_{\mathbf{Y}\mathbf{Y}}$ is the autocorrelation matrix of the observed signal $\mathbf{Y}$. The calculation of the correlation matrices $\mathbf{C}_{\mathbf{\Omega}\mathbf{Y}}$ and $\mathbf{C}_{\mathbf{Y}\mathbf{Y}}$ are given in an appendix. Once $\hat{\mathbf{\Omega}}$ has been obtained, the corresponding estimate of the phase noise matrix $\hat{\mathbf{X}}_m$ can be formed. One remaining problem to solve is the one of data dependencies. For the $L - 1$ previous OFDM symbols, decisions are used for removing data dependencies at their respective position in the observation vector. As for the current OFDM symbol $\mathbf{R}_m$, a fully decision directed method would be too sensitive to the amount of CPR in $X_m(0)$ and would result in poor estimation. Our proposal is to use a two stage estimation. We first produce a coarse estimate $\hat{\mathbf{X}}_m$ of $\mathbf{X}_m$ using equation (11) on a reduced observation set where $\mathbf{Y}_m^T$ only contains the data from the pilot symbols. This estimate is reliable enough to make tentative decisions on the non-pilot subcarriers of the current symbol. Using these tentative decisions as the actual transmitted data we produce our final estimate $\hat{\mathbf{X}}_m$ based on the full vector of L OFDM symbols.

B. Solution based on a power series

An approximation of the phase noise process is to model it in a power series as

$$\phi_m(n) = \sum_{\lambda=0}^{\epsilon} \phi_{m\lambda} n^{\lambda} \quad (12)$$

For small phase noise rates, only a small number of terms are needed.

1) *ML estimation for a linear approximation:* For the case of $\epsilon = 1$, the phase noise term is $e^{j\phi_m(n)} = e^{j(\phi_{m0} + \phi_{m1}n)}$. As it is commonly stated in the analysis of phase noise [2], we will assume that phase noise evolves slowly inside one OFDM symbol so that we can approximate $e^{j\phi_{m1}n}$ by the shortened Taylor expansion $1 + j\phi_{m1}n$. Substituting this expression of $\phi_m(n)$ into equation (4), the k th carrier is demodulated as

$$R_m(k) = D_m(k)X_m(0) + K_m \sum_{l=0, l \neq k}^{N-1} D_m(l) \sum_{n=0}^{N-1} n e^{2j\pi(l-k)\frac{n}{N}} + W'_m(k) \quad (13)$$

where $K_m = j e^{j\phi_{m0}} \phi_{m1} / N$. In this equation, the AWGN noise term $W'_m(k)$ accounts for the channel noise as well as the approximation error coming from the truncation of the power series. The first term of the equation is the desired data rotated by the CPR. The second term is the ICI. The estimation procedure here is split in two steps. First we estimate the CPR in an MMSE manner using equation (11) and the method described in section III-A with the parameter vector consisting only of $X_m(0)$. Once CPR has been estimated, we obtain a first estimate of the transmitted data for the current symbol $\mathbf{R}_m$. The only unknown left in equation (13) is the parameter K_m for which an estimate can be obtained by ML methods. The probability distribution of the received signal $\mathbf{R}_m$ for a given data set $\mathbf{D}_m$ and ICI matrix $\mathbf{X}_m$ is multivariate N -complex Gaussian. The maximum likelihood (ML) estimate of K_m is obtained by minimizing the log-likelihood function

$$\Lambda = [\mathbf{R}_m - \boldsymbol{\mu}_m]^H \mathbf{C}^{-1} [\mathbf{R}_m - \boldsymbol{\mu}_m] \quad (14)$$

where the k th element of the mean vector $\boldsymbol{\mu}$ is given by

$$\mu(k) = D_m(k)X_m(0) + K_m \sum_{\substack{l=0 \\ l \neq k}}^{N-1} D_m(l)Z_m(l-k) \quad (15)$$

For convenience in notation, let us define two vectors $\mathbf{A}_m$ and $\mathbf{B}_m$ so that $\boldsymbol{\mu} = \mathbf{A}_m + K_m \mathbf{B}_m$. Minimization of Λ yields the optimum K_m estimate

$$\hat{K}_m = \frac{\mathbf{B}_m^H \mathbf{I}_N (\mathbf{R}_m - \mathbf{A}_m) + (\mathbf{R}_m - \mathbf{A}_m)^H \mathbf{I}_N \mathbf{B}_m}{2 \mathbf{B}_m^H \mathbf{I}_N \mathbf{B}_m} \quad (16)$$

where $\mathbf{I}_N$ is the $N \times N$ identity matrix. K_m can be obtained by substituting $D_m(k)$ and $X_m(0)$ by their tentative decision and MMSE estimate, respectively, into equation (16). The estimate of the phase noise matrix is then formed using the CPR estimate in the diagonal, $\hat{\mathbf{X}}_m(i, i) = \hat{X}(0)$, and using the ICI estimate $\hat{\mathbf{X}}_m(i, l) = \hat{X}_m(l - i) = \hat{K}_m \sum_{n=0}^{N-1} n \exp(2j\pi(l-i)\frac{n}{N})$ when $i \neq l$.

2) *Generalization to higher orders:* The result of section III-B.1 can easily be generalised to higher order of powers in the series. We begin by generalizing the notation. The ICI

term corresponding to the λ th term in the power series is

$$\mathbf{X}_m^\lambda(i, l) = K_m^\lambda \sum_{n=0}^{N-1} n^\lambda \exp(2j\pi(i-l)\frac{n}{N}) \quad (17)$$

where $K_m^\lambda = j e^{j\phi_{m0}} \phi_{m\lambda} / N$. The total ICI present is equal to

$$\mathbf{X}_m(i, l) = X_m(l-i) = \sum_{\lambda=1}^{\epsilon} K_m^\lambda \mathbf{Z}_m^\lambda(i, l) \quad (18)$$

where we also define

$$\mathbf{Z}_m^\lambda(i, l) = Z_m^\lambda(l-i) = \sum_{n=0}^{N-1} n^\lambda \exp(2j\pi(l-i)\frac{n}{N}) \quad (19)$$

In order to estimate K_m^λ we take a kind of successive interference cancellation approach. We first make an estimate of K_m^1 leaving the remaining $\epsilon-1$ terms into the noise term W'_k , which we assume have a Gaussian pdf. Using this estimate we form another equation to estimate K_m^2 and so on until K_m^ϵ is estimated. The received vector $\mathbf{R}_m$ is a random variable with a conditional pdf which is N -variate Gaussian. The ML estimate for K_m^λ is equal to

$$\hat{K}_m^\lambda = \frac{(\mathbf{B}_m^\lambda)^H \mathbf{I}_N (\mathbf{R}_m - \mathbf{A}_m^\lambda) + (\mathbf{R}_m - \mathbf{A}_m^\lambda)^H \mathbf{I}_N \mathbf{B}_m^\lambda}{2(\mathbf{B}_m^\lambda)^H \mathbf{I}_N \mathbf{B}_m^\lambda} \quad (20)$$

where

$$A_m^\lambda(k) = D_m(k) X_m(0) + \sum_{l=0}^{N-1} \sum_{\beta=1}^{\lambda-1} K_m^\beta Z_m^\beta(l-k) D_m(l) \quad (21)$$

and

$$B_m^\lambda(k) = \sum_{l=0}^{N-1} D_m(l) Z_m^\lambda(l-k) \quad (22)$$

The phase noise matrix estimate is then obtained from $\hat{\mathbf{X}}_m = \sum_{\lambda=1}^{\epsilon} \hat{K}_m^\lambda \mathbf{Z}_m^\lambda + \mathbf{I}_N \hat{X}_m(0)$.

IV. DATA DETECTION METHODS

Data detection on the subcarriers can be made in two different manners, namely symbol by symbol or sequence detection. Since the N carriers of the OFDM symbol are not anymore orthogonal, maximum likelihood sequence detection (MLSD) should perform better than symbol by symbol detection in this system. However, we should bear in mind that MLSD raise some complexity issues when the number of subcarriers and the QAM constellation are large. The metric for decisions in a symbol by symbol method is

$$\hat{D}(k) = \arg \min_{D_m(k)} |(\hat{\mathbf{R}}_m - \mathbf{D}_m)_k|^2 \quad (23)$$

where $(\mathbf{x})_k$ return the k th position of the vector $\mathbf{x}$ and $\hat{\mathbf{R}}_m = \hat{\mathbf{X}}_m^{-1} \mathbf{R}_m$. For MLSD, the metric used is

$$\hat{\mathbf{D}}_m = \arg \min_{\mathbf{D}_m} \|\hat{\mathbf{R}}_m - \mathbf{D}_m\|^2 \quad (24)$$

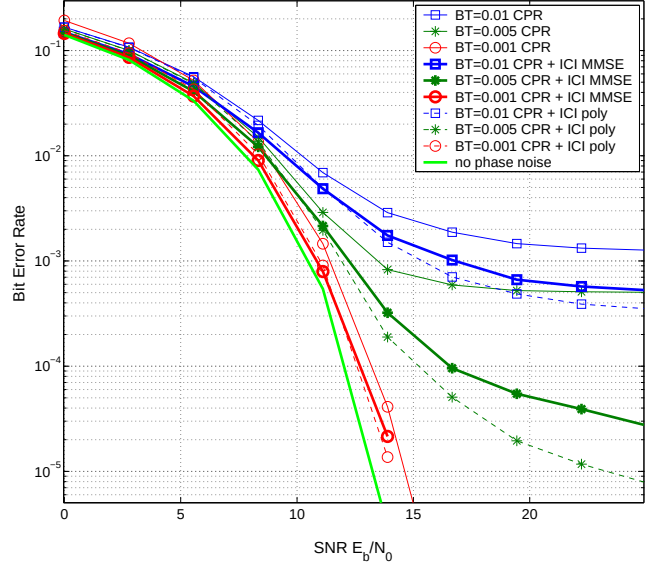


Fig. 3. Bit error rate of the ICI cancellation schemes. The dashed curve show the performance of the system without ICI cancellation. The lowest bold line shows the no phase noise case.

V. SIMULATION RESULTS

The MMSE and power series based algorithms were simulated on a 16-QAM OFDM system with 128 subcarriers and pilots symbols at position 0, 42, 84 and 127. $L = 2$ OFDM symbols were used for estimation and 6000 symbols were used for simulations. The phase noise rate was simulated at values of $BT = 0$ (no phase noise), $BT = 0.001$, $BT = 0.005$, and $BT = 0.01$. The power series algorithm is used with $\epsilon = 2$. The BER result of these simulations are shown in figure 3. Another simulation considering MLSD was also performed. Due to complexity issues, the scheme used here is BPSK-OFDM with $N = 16$ subcarriers. These simulation results are plotted in figures 4 and 5 and were done for phase noise rates of $BT = 0$ (no phase noise), $BT = 0.001$, $BT = 0.005$, and $BT = 0.01$. The results from these simulations show that both algorithms provide significant gains in the SNR compared to the CPR-only compensation scheme. The BER floor for a given amount of phase noise is greatly reduced. The MMSE algorithm performs comparatively worse than the power series algorithm, but has the advantage of being able to compute the estimator coefficients offline, provided we have the knowledge of signal to noise ratio and phase noise rate. The power series estimator has to compute (20) online by performing many matrix manipulations, compared to the MMSE estimator which estimates the unknown with only one matrix multiplication in equation (11).

VI. CONCLUSIONS

We have presented two phase noise estimators compensating both for CPR and ICI effects. The estimators make use

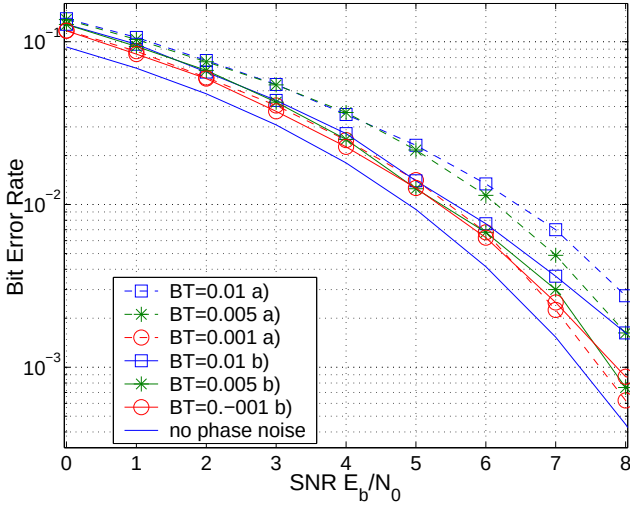


Fig. 4. Bit Error Rate for (a) symbol by symbol and (b) MLSD methods using the MMSE algorithm.

of knowledge of phase noise and information both along the time axis and the frequency axis, which is valuable since the throughput of the system can be increased by spreading the estimation procedure over several OFDM symbols. MLSD data detection is mentioned here as an interesting detection method since carriers are correlated by the ICI term. It however turns out that an MLSD detector might be too complex to implement in current technology. Using a reduced amount of carriers and a simple modulation, we demonstrated that this detection is anyway advantageous in the BER sense and therefore should not be set aside for future research. Since its performance in terms of BER are better we recommend using the power series algorithm over the MMSE solution for receivers where complexity is not an issue. If complexity is a concern the MMSE solution still gives a significant gain compared to the basic CPR compensation solution.

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APPENDIX I DERIVATION FOR THE MMSE ESTIMATOR

The vector $\mathbf{C}_{\Omega\mathbf{Y}}\mathbf{C}_{\mathbf{Y}\mathbf{Y}}^{-1}$ can be pre-calculated using analytical expressions for the cross-correlation and autocorrelation matrices. The cross-correlation matrix is a $2N - 1 \times LN$ matrix whose elements are expressed by

$$\mathbf{C}_{\Omega\mathbf{Y}} = E[\Omega\mathbf{Y}^*] \quad (25)$$

The autocorrelation matrix is expressed by

$$\mathbf{C}_{\mathbf{Y}\mathbf{Y}} = E[\mathbf{Y}\mathbf{Y}^*] \quad (26)$$

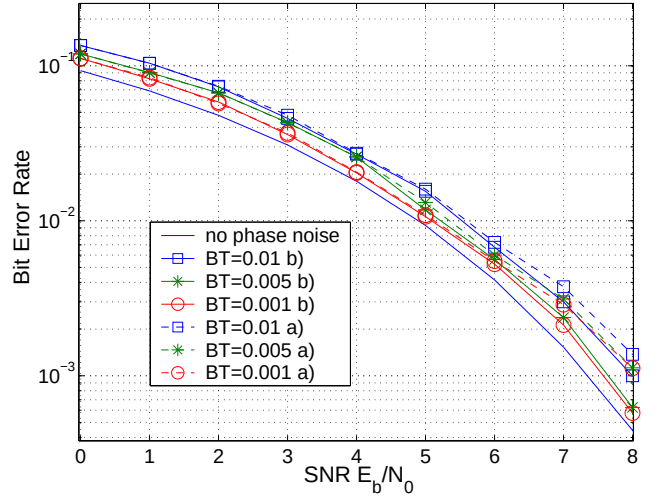


Fig. 5. Bit Error Rate for (a) symbol by symbol and (b) MLSD methods using the power series algorithm.

The content of these matrices in the a th row, b th column is:

$$\mathbf{C}_{\Omega\mathbf{Y}}(a, b) = \frac{1}{N^2} \sum_{u,v=0}^{N-1} e^{-\frac{1}{2}|L + \lfloor \frac{b}{N} \rfloor + u - v| \sigma_{\Delta}^2} e^{j2\pi \frac{(a - N + 1)u}{N}} \quad (27)$$

$$\mathbf{C}_{\mathbf{Y}\mathbf{Y}}(a, b) = \frac{1}{N^2} \sum_{u,v=0}^{N-1} e^{-\frac{1}{2}|\lfloor \frac{b}{N} \rfloor N - \lfloor \frac{a}{N} \rfloor N + u - v| \sigma_{\Delta}^2} + \delta(a - b) \sigma_w^2 \quad (28)$$

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