CALCULATING THE AUTOCORRELATION FUNCTION FOR MB-NB CODES

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ABSTRACT

Using the approach of Cariolaro e.a. [2] and Bilardi [3] we devised a general formula to calculate the autocorrelation function for mI-nO line codes using the kernel $d \cdot \Pi^k$ and the output or codeword matrix A. We checked its validity for a particular code namely CMI (Coded Mark Inversion). The results perfectly match those found in the literature.

1. INTRODUCTION

The calculations of autocorrelation function (AKF) of line codes can be mechanised and be done using matrices [1], [2], [3], [4]. Thus, we can write:

$$R(k) = \mathrm{Tr}\left(d \cdot \Pi^k \cdot Z\right) \qquad (1)$$

where d is a diagonal matrix containing stationary probabilities

$$d = \mathrm{diag}\left\{p(1), p(2), \cdots p(I)\right\} \qquad (2)$$

and I is is the number of states in the state transition diagram, Π is the state transition probability matrix, Z is a correlation matrix and Tr stands for trace (the sum of the elements on the main diagonal).

If the output symbols corresponding to the I states are a_i, $i = 1, 2, \cdots I$, then the element z_{ij} of the matrix Z is given by:

$$z_{ij} = a_i a_j \qquad (3)$$

The AKF can also be calculated using the kernel $d \cdot \Pi^k$ and the output or codeword matrix A [2]. We can replace the matrix Z in eq.(19) by multiplying the kernel $d \cdot \Pi^k$ on the left by A^* and on the right by A, where A^* denotes transpose conjugate [3].

$$R(k) = A \cdot (d \cdot \Pi)^k \cdot A^* \qquad (4)$$

Another way to calculate the coefficients $R(k)$ of the AKF is based on the entry vector e and departure vector d that are I-dimensional [4]. In this case

$$R(k) = e \cdot \Pi^{k-1} \cdot d' \qquad (5)$$

The i-th component of the entry vector e is the weighted sum of symbols that can be transmitted on entering the state i.

$$e_i = \sum_{k=1}^{I} p_k \cdot t_{ki} \cdot a_{ki} \qquad (6)$$

where p_k is the steady state probability associated to state k, t_{ki} is the transition probability from state k to state i and a_{ki} is the simbol to be transmitted when leaving the state k for state i. If there are more symbols associated with this transition, then an appropriate average should be used instead.

The components of vector d consist of weighted sums of the symbols that can be transmitted on departing the current state i. So, the ith entry of the vector d can be written as:

$$d_i = \sum_{k=1}^{I} t_{ik} a_{ik} \qquad (7)$$

2. AKF FOR SIGNALS WITH 1B2B BLOCK STRUCTURE

We shall consider a simple case of a mB-nB code (m Binary - n Binary) with $m = 1$ and $n = 2$. This coder maps each input bit into two output symbols. This halves the signaling interval and doubles the output symbol rate of the coder. As a consequence, we shall have to evaluate the AKF not only for integer values of T, but also for $kT + T/2$ or $(k + 1/2)T$. The situation corresponding to the evaluation of $R(k + 1/2)$ is depicted in Fig. 1. Since the delay involved has not an integer value, two distinct situations arise:

1. The waveforms to be correlated belong to the n and $n + k$ bit intervals (second half of x_{n+k} which is denoted by a_{j2} and first half of x_n which is termed a_{i1});

2. The waveforms involved in correlation belong to the n and $n + k + 1$ bit intervals (first half of x_{n+k+1}

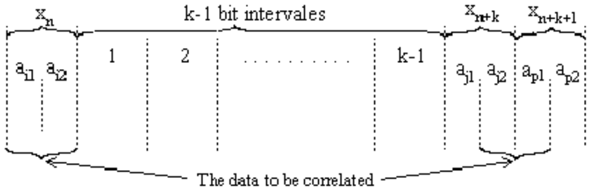

Figure 1: Evaluation of $R(k+1/2)$

which is denoted by a_{p1} and second half of x_n which is termed a_{i2}).

Applying the same procedure as before we can write for the values of AKF evaluated at integer multiples of T:

$$R(k) = \mathrm{Tr}\left(d \cdot \Pi^k \cdot Z\right) \tag{8}$$

where as before Z is a correlation matrix on the full bit interval. A generic z_{ij} element, found at the intersection of line i and column j is given by

$$z_{ij} = \frac{a_{i1}a_{j1} + a_{i2}a_{j2}}{2} \tag{9}$$

where a_{i1} and a_{i2} are the first and respectively the second output symbols transmitted in the state i. For the 1B1B case 1I1O (1 Input 1 Output) code, we have

$$a_{i1} = a_{i2} = a_i \quad \text{and} \quad a_{j1} = a_{j2} = a_j \tag{10}$$

and z_{ij} is given by rel. (9).

To calculate $R(k+1/2)$ we proceed in the same manner and we can write

$$\begin{aligned} R\left(k+\tfrac{1}{2}\right) &= \mathrm{Tr}\left(d \cdot \Pi^k \cdot X\right) + \mathrm{Tr}\left(d \cdot \Pi^{k+1} \cdot Y\right) = \\ &= \mathrm{Tr}\left(d \cdot \Pi^k \cdot \left(X + \Pi \cdot X^T\right)\right) \end{aligned} \tag{11}$$

or

$$R\left(k+\frac{1}{2}\right) = \mathrm{Tr}\left(d \cdot \Pi^k \cdot S\right) \tag{12}$$

where

$$S = X + \Pi \cdot X^T \tag{13}$$

The matrices X and Y (transpose of X) are correlation matrices on the half bit interval. The element x_{ij}, found at the intersection of line i and column j is given by

$$x_{ij} = \frac{a_{i2} \cdot a_{j1}}{2} \tag{14}$$

while

$$y_{ij} = \frac{a_{i1} \cdot a_{j2}}{2} \tag{15}$$

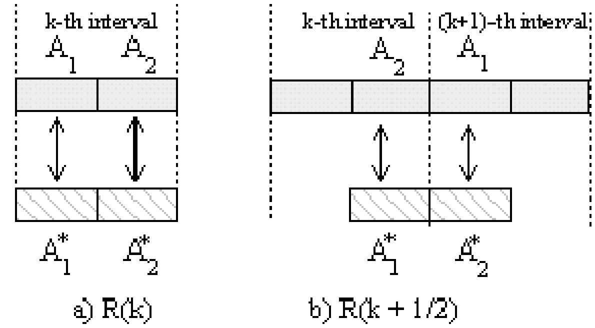

Figure 2: Illustration of correlation of 1B2B codes

It is obvious that $y_{ij} = x_{ji}$ and it results that

$$Y = X^T \tag{16}$$

where again the superscript T denotes the transpose of a matrix.

If we write two output matrices A_1 and A_2 that contain the first and respectively the second output symbol for each state from 1 to I (I is the number of states in the transition diagram), then we can express $R(k+1/2)$ also as

$$\begin{aligned} R\left(k+\tfrac{1}{2}\right) &= 0.5 \cdot A_1^* \cdot d \cdot \Pi^k \cdot A_2 + \\ &\quad +0.5 \cdot A_2^* \cdot d \cdot \Pi^k \cdot A_1 \end{aligned} \tag{17}$$

In the general case of 1I2O codes with I states the matrices A_1 and A_2 are of the form:

$$A_1 = \begin{bmatrix} a_{11} \\ a_{21} \\ \vdots \\ a_{I1} \end{bmatrix} \quad A_2 = \begin{bmatrix} a_{12} \\ a_{22} \\ \vdots \\ a_{I2} \end{bmatrix} \tag{18}$$

To calculate $R(k)$ we can also start from relation (4) which was written for the 1I1O case and based on Fig. 2 a and b we get similar relations to (4):

$$\begin{aligned} R(k) &= 0.5 \cdot A_1^* \cdot d \cdot \Pi^k \cdot A_1 + \\ &\quad +0.5 \cdot A_2^* \cdot d \cdot \Pi^k \cdot A_2 \\ R\left(k+\tfrac{1}{2}\right) &= 0.5 \cdot A_1^* \cdot d \cdot \Pi^k \cdot A_2 + \\ &\quad +0.5 \cdot A_2^* \cdot d \cdot \Pi^k \cdot A_1 \end{aligned} \tag{19}$$

3. AKF FOR SIGNALS WITH MI-NO BLOCK STRUCTURE

To calculate R(k) we can also start from relation (4) which was written for the 1I-1O case. Denoting $d \cdot \Pi^k = U_k$

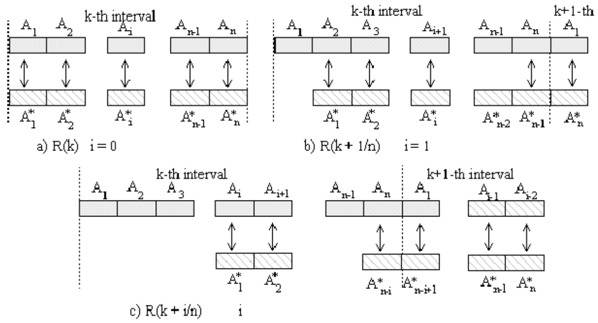

Figure 3: Calculation of AKF for mInO codes

for brevity's sake and based on fig. 3 we can write:

$$R(k) = \frac{1}{n}\left(A_1^* \cdot U_k \cdot A_1 + A_2^* \cdot U_k \cdot A_2 + \right. + \cdots + A_i^* \cdot U_k \cdot A_i + \left. + \cdots + A_n^* \cdot U_k \cdot A_n\right) \quad (20)$$

or

$$R(k) = \frac{1}{n}\left(\sum_{j=1}^{n} A_j^* \cdot U_k \cdot A_j\right) \quad (21)$$

$$R\left(k+\tfrac{1}{n}\right) = \frac{1}{n}\left(A_1^* \cdot U_k \cdot A_2 + \right. + \cdots + A_i^* \cdot U_k \cdot A_{i+1} + + \cdots + A_{n-1}^* \cdot U_k \cdot A_n + \left. + \cdots + A_n^* \cdot U_k \cdot A_1\right) \quad (22)$$

or

$$R\left(k+\frac{1}{n}\right) = \frac{1}{n}\left[\sum_{j=1}^{n}\left(A_j^* \cdot U_k \cdot A_{j+1}\right) + A_n^* \cdot U_k \cdot A_1\right] \quad (23)$$

Further

$$R\left(k+\tfrac{2}{n}\right) = \frac{1}{n}\left[\sum_{j=1}^{n-2}\left(A_j^* \cdot U_k \cdot A_{j+2}\right) + \right. \left. + \sum_{j=1}^{2}\left(A_{n-j+1}^* \cdot U_k \cdot A_{2-j+1}\right)\right] \quad (24)$$

and

$$R\left(k+\tfrac{i}{n}\right) = \frac{1}{n}\left[\sum_{j=1}^{n-i}\left(A_j^* \cdot U_k \cdot A_{j+i}\right) + \right. \left. + \sum_{j=1}^{2}\left(A_{n-j+1}^* \cdot U_k \cdot A_{i-j+1}\right)\right] \quad (25)$$

with $i = 1, 2, \cdots n-1$.

4. EXAMPLES

We shall use the formulas derived above to illustrate the calculation of autocorrelation function values for a particular 1B-2B Code, namely CMI (Coded Mark Inversion) [1], [5], [6]. Its state transition diagram is represented in figure 4. In CMI code a "mark" is alternatively coded by HH and LL. A "space" is coded by LH. We can write:

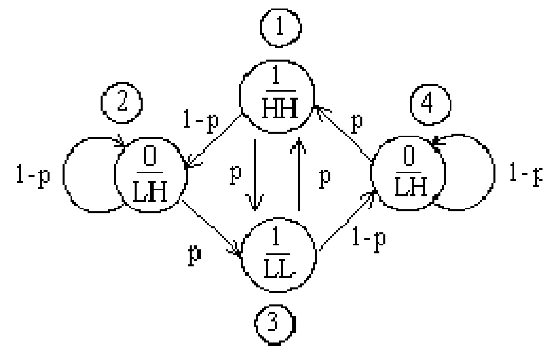

Figure 4: State diagram for CMI code

$$d = \begin{bmatrix} p/2 & 0 & 0 & 0 \\ 0 & (1-p)/2 & 0 & 0 \\ 0 & 0 & p/2 & 0 \\ 0 & 0 & 0 & (1-p)/2 \end{bmatrix}$$

$$\Pi = \begin{bmatrix} 0 & 1-p & p & 0 \\ 0 & 1-p & p & 0 \\ p & 0 & 0 & 1-p \\ p & 0 & 0 & 1-p \end{bmatrix}$$

$$A_1 = [1, -1, -1, -1]$$

$$A_2 = A_2^* = [1, 1, -1, 1]$$

Using relations (17), (19) or (21) and (23), we get the following results:

$$\begin{aligned} R[0] &= 1 \\ R[1] &= 1-2p \\ R[2] &= 1-2p+2p^3 \\ R[3] &= 1-2p+4p^3 \end{aligned}$$

$$\begin{aligned} R[1/2] &= -(1-2p)^2 \\ R[3/2] &= -1+2p-2p^2+p^3 \\ R[5/2] &= -1+2p-2p^2+3p^3- \\ &\quad -2p^4 \\ R[7/2] &= -1+2p-2p^2+5p^3- \\ &\quad -8p^4+4p^5 \\ R[9/2] &= -1+2p-2p^2+7p^3- \\ &\quad -18p^4+20p^5-8p^6 \end{aligned}$$

They perfectly match those found in the literature [1,3].

5. CONCLUSIONS

The calculations of autocorrelation function (AKF) of line codes can be done using matrices. A survey of the possible methods was done.

Using the approach of Cariolaro e.a. [2] we devised a general formula to calculate the autocorrelation function values evaluated at discrete time moments for any mI-nO line code using the kernel $d \cdot \Pi^k$ and the output or codeword matrix A. This is valid for any number n of output symbols contained in an output word.

We checked its validity for a particular code namely CMI (Coded Mark Inversion). The results perfectly match those found in the literature.

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