NONSTANDARD MODELS OF ELECTRICAL-ELECTRONIC CIRCUITS WITH INSTANTANEOUS ON/OFF SWITCHINGS

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ABSTRACT

Nonstandard models of electrical-electronics circuits with instantaneous on/off switching are proposed. Nonstandard analysis allows to ground new approaches of circuits models development when their analyses need only partial qualitative study as well bring out the calculation algorithm.

1. INTRODUCTION

Traditional assumption about instantaneous character of switching on/off processes in electrical-electronic circuits with switches leads to great calculation difficulties [1,2,3]. The most correct approach is to use sprayed parameters and apply the Theory of Transients in Distributed-Parameters Networks, but it is very complicated to solve very stiff differential equations for protracted time. It is especially important if due to switching new circuits with inductors (and sources of current) or capacitors (and voltage sources) are created. Due to Kirchhoff's laws currents across inductors and voltages on capacitors must be changed instantly. As a consequence, difficulties to find new initial values of currents across inductors and voltages on capacitors are arisen. But the utmost difficulties are coming out during the physical interpretation of results obtained. In this case we must assume the instantaneous changes of energy, stored in magnetic fields of inductors $W_m(t) = \dfrac{L_j i_j^2(t)}{2}$ and in electric fields of capacitors $W_e(t) = \dfrac{C_j u_j^2(t)}{2}$, and, as a consequence, the infinite character of power sources acting at the moments of switching. Besides, we have to use δ-function to find voltages on inductors $(u_l = L\dfrac{di_L}{dt})$ and currents across the capacitors $(i_c = C\dfrac{du_c}{dt})$ at that switching moments. But they are not the functions in the proper sense of the word but the functionals. It's impossible to obtain the infinite power in the real electric circuits as well these switching on/off processes are not perfect pulses. Consequently, lack of correspondence between the mathematical model of switching and its real physical process is obvious. At the same time refusal from the above mentioned assumption (about instantaneous character of switching on/off) complicates the circuits mathematical models. It increases their equivalent circuit dimension, changes the character of equations and, as a consequence, requests to orient to more complicated methods of calculations up to typical stiff problems statement. Besides, such refusal requests for additional information to refine the circuit's model and its parameters. That kind of problems during calculation may take place if the only fact we know about process of on/off switching is its negligibly short period of time.

2. DISCUSSION

Due to above mentioned nonstandard analysis or the so called "working mathematics" application gives more adequate model. Let's consider instantaneous switching on in the circuit that consists of capacitors (Fig.1.a). It is possible to apply two approaches. Here is used the first one according to the nonstandard model of this accumulating element with interrupting voltage and current. Let's consider that the switch is closed at t =0 (Fig.1.a). For this case we can find:

$$i_j(h) = G_j * u_j(h) + J_j(h), \quad G_j = \frac{G_j}{h},$$

$$J_j(h) = \frac{G_j * u_j(0)}{h}, \quad j = 1,2,3.$$

where G – equivalent conductivity, J – equivalent current source, t=*h* – the period of switching that considered just as a small value according to "working mathematics" principles or negligibly small if nonstandard analysis is used.
Thus, now capacitors models exclude the fact of instantaneous switching on due to taking into account its real duration of life *h*. However, there is no any voltages calculations difficulties after the switch close because the model becomes pure algebraic expression and can be presented as resistors contained analogue of capacitor. Calculation of capacitor voltages at the moment after the switch close for the circuit shown on Fig.1a converts into calculation of only resistor contained circuit (Fig.1.b) with scaled parameters:

$$G_{*j} = G_j h = G_j, \quad J_{*j} = J_j(h)h = G_j u_j(0), \; j = 1,2,3.$$

According to the node voltages method:

$$u_j(h) = U = C_1 * u_1(0) = \frac{\sum_{j=1}^{3} J_{*j}}{\sum_{j=1}^{3} G_{*j}} =$$

$$= \frac{C_1 * u_1(0) + C_2 * u_2(0) + C_3 * u_3(0)}{C_1 + C_2 + C_3}$$

The just obtained expression does not depend on switching period h, which according to problem conditions is negligibly small. Thus, different nonstandard model application in switching process analysis for only inductors or capacitors contained circuits avoid from the usual difficulties of the well-known classic mathematical approach. Besides, no need to use δ-functions for inductor voltages and capacitors currents models as well the energy storage are depicted by means of uninterrupted functions.

Let's consider the second approach to the problem without application nonstandard models of any accumulating element. During every switching on (off) in the capacitors (inductors) contained circuit the switch itself is substituted by negligibly small resistance $r_* \in I$ (or negligibly small conductance $g_* \in I$). It helps to prevent from new only capacitors or inductors contained circuits or loops that may come out after the switching. Such assumption is correct because behavior patterns of these elements in the real electric fields not ideal. Uninterrupting character of changes (of capacitors voltages or currents through the inductors) in resistor-inductor or resistor-capacitor circuits are guaranteed now. Calculation of transient variables can be carried out by any traditional method. In this case researcher does not tie with concrete choice of switch parameters but takes into consideration only the fact that all these parameters are negligibly small (but not zero).

Let's use above-mentioned approach to switching model for circuit shown on Fig.1a. The model of the circuit is shown on Fig.1b. Transient process takes place just around t =0 neighborhood due to its small time constants (or standard parts of state variables set up around t=0) and stays constant for short period of time

$$\text{st}\, u_j(t) = \text{const.}, \quad \text{if st}\, t > 0,\; j = 1,2,3.$$

Consequently,

$$\text{st}\, u_1(t) = \text{st}\, u_2(t) = \text{st}\, u_3(t) = U, \quad \text{if st}\, t > 0.$$

Having integrated Kirchhoff's law equation $\sum_{j=1}^{3} C_j \frac{du_j}{dt} = 0$ for standard parts of variables we obtain:

$$st \sum_{j=1}^{3} \int_0^t C_j \frac{du_j}{dt} dt = 0$$

or

$$st\left[\sum_{j=1}^{3} C_j u_j(t) - \sum_{j=1}^{3} C_j u_j(0)\right] = 0$$

With the aim of all voltages calculation let's draw up equations systems:

$$stu_1(t) = stu_2(t) = stu_3(t) = U =$$

$$= \sum_{j=1}^{3} C_j stu_j(t) = \sum_{j=1}^{3} C_j stu_j(0)$$

if st t > 0.

This system can be corresponded with resistors contained circuit shown on Fig.1b, if

$$stu_j(t) = u_j, \quad j = 1,2,3.$$

In this case capacitors voltages are set up after negligibly small period of time. Their standard parts are well nown:

$$u_1(t) = u_2(t) = u_3(t) = U =$$

$$= \frac{C_1 u_1(0) + C_2 u_2(0) + C_3 u_3(0)}{C_1 + C_2 + C_3}$$

where $u_j(t) = stu_j(t), \quad j = 1,2,3.$

The latter approach is less formal than the previous one, but more correct from physics point of view. It is obvious that decrease of energy stored in accumulating elements is caused by heat losses in the protecting resistors.

The nonstandard analysis application allows to develop the formal approach for switching calculations in the above mentioned circuits based on resistors contained equivalent circuits. According to this approach new capacitors voltages calculations are substituted with $G_* I_*$-elements with parameters:

$$G_* = C, \quad J_* = Cu(0),$$

where u(0) are capacitors voltages just before switching. We can see that capacitors voltages are equal to the voltages obtained on the corresponding elements in resistors contained circuits. Similarly with the aim of currents across inductors calculation inductors are substituted by $R_* E_*$-elements with parameters:

$$R_* = L, E_* = Li(0),$$

where *i*(0) are currents through inductors just before switching.

For current calculation across every L_j after switching off in the circuit shown on Fig. 2a the model in the form of resistors contained circuit (Fig.2.b) is drawn up $R_{*1} = L_1, R_{*2} = L_2, E_{*1} = L_1 i_1(0), E_{*2} = L_2 i_2(0) = 0$

Currents values across L_j are equal to ones across the corresponding elements in resistors contained circuits. Thus, for the given example:

$$i_1(t) = i_2(t) = \frac{L_1 i_1(0)}{L_1 + L_2}, \; 0 < t \in R .$$

Description of switching by means of nonstandard analysis is adequate to coming out qualitative information.

3. CONCLUSION

Nonstandard analysis is laconic and reflects the intuitive ideas about such classes of circuits in mathematical form. That is why the wide application of above mentioned methods in theoretical electrical-electronic engineering are proposed.

[1] G. Seveke, P.Jonkin, A.Netushil, S. Strakhov (**1989**), Analysis of Electric Circuits, (translated from Russian by B.Kuznetsov), MIR Publishers, Moscow.

[2] David R. Cunningham, John A. Suller (**1991**), Basic Circuit Analysis, Houghton Mifflin Comp.

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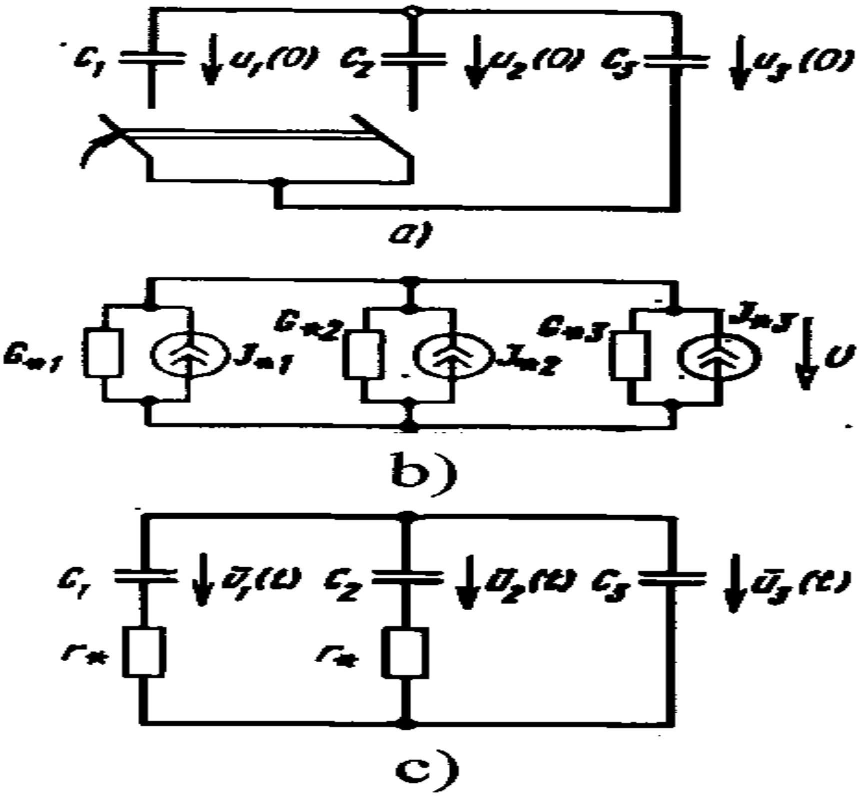

Fig. 1.

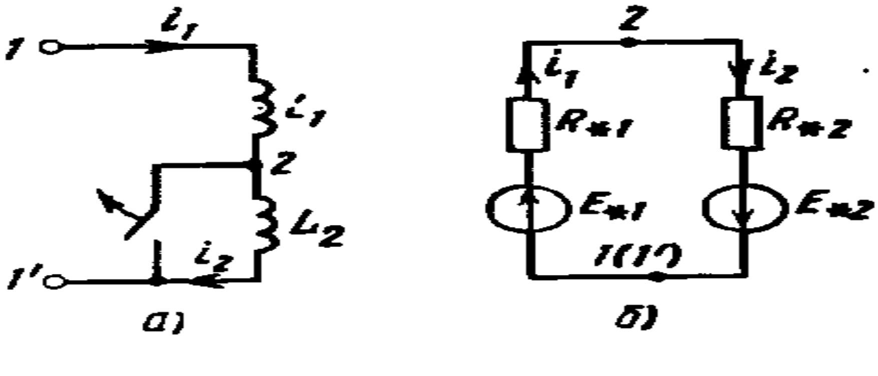

Fig.2.