IMPROVED DESIGN OF MIXED IIR/FIR QMF BANKS

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ABSTRACT

A new mixed IIR/FIR near-perfect-reconstruction quadrature mirror filter bank is proposed. The design is based on the method of M.Zhu and others. The modification consists in replacing the Butterworth filters in the analysis part of the bank with more general class of flat delay IIR filters due to Selesnick. A condition on the synthesis part for a perfect reconstruction is established, making it possible to realize a nearly perfect-reconstruction filter bank by using a nonlinear-phase FIR synthesis filter. The banks obtained in such a way have very low reconstruction errors. Experimental results are presented to support this claim.

I. INTRODUCTION

Two-channel quadrature mirror-image filter (QMF) banks have been widely used in 1-D and 2-D signal processing. Many design algorithms have been developed to obtain perfect or nearly perfect reconstruction (NPR) filter banks [1]. Most of these techniques focus on the design of FIR filter bank that does not suffer from instability and phase distortion. However, the resulting filters require a large number of coefficients to meet the magnitude specifications. It is well known that from the aspect of decreasing the number of coefficients, the IIR filter bank is more efficient. However, the phase distortion in these banks is unavoidable. Designing an IIR filter that has to meet both magnitude and phase specifications simultaneously is generally difficult.

Recently, a different approach for the design of NPR QMF banks has been presented by M. Zhu and others [2]. This approach employs all-pass based IIR filters and FIR filters to realize the analysis and synthesis parts of the bank, respectively. The phase distortion of the bank, which is caused by the nonlinear phase IIR analysis filters, is reduced by an efficient compensation provided by the FIR synthesis filters. Butterworth filters are used for the analysis part. By using the all-pass based IIR filters proposed by Selesnick [3] to implement the analysis part of the bank, we have obtained NPR QMF banks that have significantly better performance. The banks obtained in such a way have very low reconstruction errors.

The paper is organized as follows. Section II contains the formulation of the design problem along with an outline of the method of M.Zhu and others. The results obtained by implementing Selesnick's filters in the analysis part of the bank are given in Section III.

II. FORMULATION OF THE DESIGN PROBLEM

We assume the following notation for a two-channel QMF bank: H_0 and H_1 are the lowpass and highpass filters, respectively, of the analysis section, and G_0, G_1 are the corresponding filters of synthesis section. The perfect reconstruction condition can be expressed as

$$G_0(z)H_0(z)+G_1(z)H_1(z)=2z^{-d} \qquad (1)$$

where d is the overall delay of the bank.

A pair of complementary quadrature mirror filters H_0 and H_1 of order N can be realized efficiently [4, 5] by a parallel connection of two all-pass filters H_a and H_b as

$$H_0(z)=\frac{H_a(z^2)+z^{-1}H_b(z^2)}{2} \qquad (2)$$

$$H_1(z)=\frac{H_a(z^2)-z^{-1}H_b(z^2)}{2} \qquad (3)$$

when N is odd.

The two all pass filters are given by

$$H_a(z^2) = \prod_{k=1}^{N_1} \frac{a_{2k-1}+z^{-2}}{1+a_{2k-1}z^{-2}} \tag{4}$$

$$H_b(z^2) = \prod_{k=1}^{N_2} \frac{a_{2k}+z^{-2}}{1+a_{2k}z^{-2}} \tag{5}$$

The coefficients a_i and the order N can be determined from the specifications of the analysis filters. The values of N_1 and N_2 depend on the value of N.

The design problem is formulated as follows. Let

$$H_0(e^{j\omega}) = \left|H_0(e^{j\omega})\right| e^{j\phi_0(\omega)} \tag{6}$$

be a frequency response of the low-pass analysis filter H_0. The low-pass synthesis filter G_0 is to be designed such that its phase response can compensate the nonlinear phase response of H_0, i.e.,

$$\hat{G}_0(e^{j\omega}) = \left|\hat{G}_0(e^{j\omega})\right| e^{j\psi_0(\omega)} = \left|H_0(e^{j\omega})\right| e^{-j[\omega d+\phi_0(\omega)]}. \tag{7}$$

Of course, the frequency response $\hat{G}_0(e^{j\omega})$ is the desired frequency response for the low-pass synthesis filter. The coefficients of G_0 are obtained by minimizing the weighting squared error defined as

$$E = \sum_{k=0}^{M-1} W(\omega_k)\left|G_0(e^{j\omega_k}) - \hat{G}_0(e^{j\omega_k})\right|^2. \tag{8}$$

The objective function E is evaluated on a dense grid of M frequencies ω_κ ($k = 0, 1, \ldots, M-1$) linearly distributed in the range from $\omega = -\pi$ to $\omega = \pi - 2\pi\kappa/M$. A rough approximation $\tilde{G}_0(e^{j\omega})$ of the desired frequency response $\hat{G}_0(e^{j\omega})$ is computed as L-point DFT ($L \le M$) of M-point IDFT of $\hat{G}_0(e^{j\omega})$ that has been truncated to L taps. The weighting function is defined as

$$W(\omega) = \left|\tilde{G}_0(e^{j\omega}) - \hat{G}_0(e^{j\omega})\right|^2. \tag{9}$$

III. DESIGN

We propose a design of mixed IIR/FIR QMF bank that follows the main line of the method given in [2]. Our design differs in the analysis part of bank. We have replaced the filters of Butterworth with more general filters proposed by Selesnick [3]. These filters allow adjustment of the delay. Moreover, the tradeoff between the delay and the phase linearity can be chosen. It is worth to note that, with the classical Butterworth filter of degree N which is retrieved as a special case, it is not possible to adjust the delay (or phase linearity). For the details of the design procedure we refer to the original paper [3].

Using the method of Zhu and others to design the banks having analysis filters proposed by Selesnick leads to very efficient filter banks. We present two examples of banks realized with 15-tap and 18-tap FIR synthesis filters, respectively. The flatness parameters of the analysis filters are specified to 3 at $\omega = 0$ and 3 at $\omega = \pi$. A fifth-order Butterworth filter is used for comparison. The value for M is chosen to be 100. The magnitude responses and the aliasing errors are shown in Fig.1 and Fig.2 , respectively.

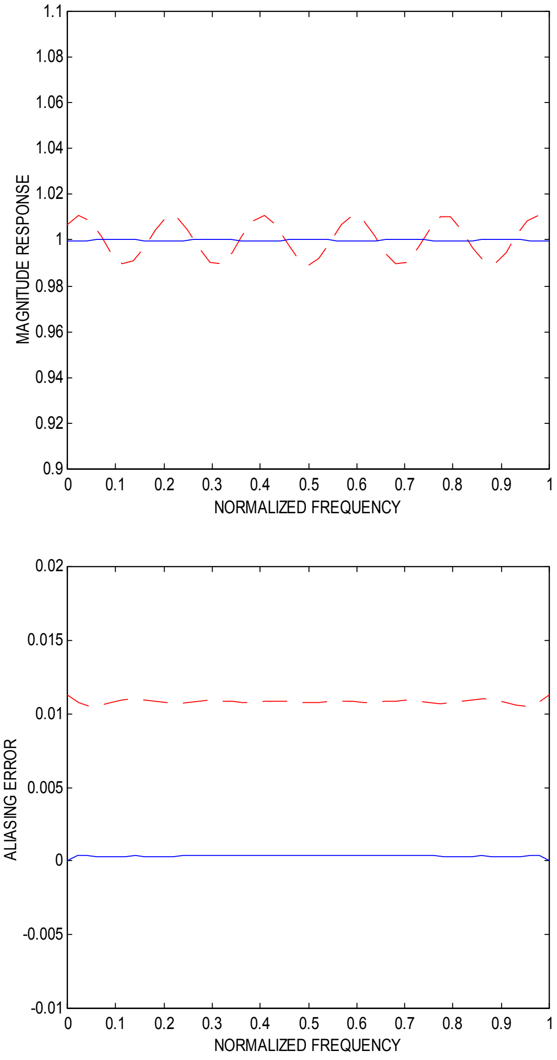

Fig. 1. Magnitude responses and aliasing errors for 15-taps FIR synthesis filters.The dotted lines correspond to the five-order Butterworth analysis filters.

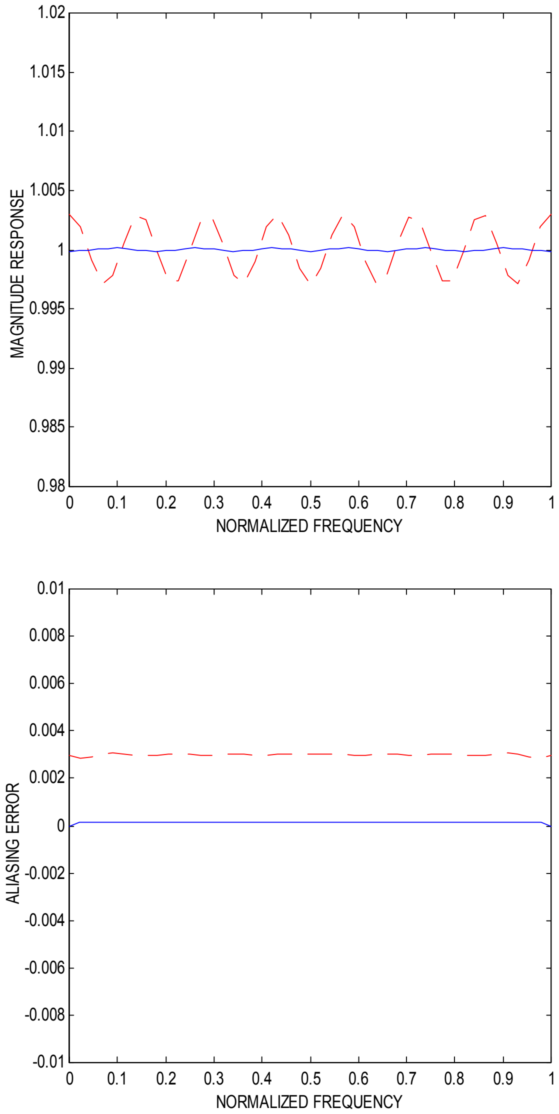

Fig. 2. Magnitude responses and aliasing errors for 18-taps FIR synthesis filters.The dotted lines correspond to the five-order Butterworth analysis filters.

It is seen that the aliasing error exhibits a constant behavior. Table 1 lists the maximum magnitude and aliasing errors.

TABLE 1: COMPARISON OF THE PROPOSED IMPROVEMENT

	Maximum magnitude error		Maximum aliasing error	
	$L = 15$	$L = 18$	$L = 15$	$L = 18$
Butterworth	0.01080	0.00300	0.01130	0.00310
Selesnick	0.00038	0.00013	0.00039	0.00015

IV. CONCLUSION

An improved design of mixed IIR/FIR quadrature mirror filter banks based on the method of Zhu, Ahmed and Swamy has been developed. The improvement consists in replacing IIR filters in the analysis part of the bank. As a result, we have obtained near-perfect-reconstruction QMF banks having significantly smaller aliasing and magnitude errors.

V. REFERENCES

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