

# DETERMINISTIC MAXIMUM LIKELIHOOD METHOD FOR THE LOCALIZATION OF NEAR-FIELD SOURCES

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## ABSTRACT

In this paper we proposed deterministic maximum likelihood approach for estimating the direction of arrival and range parameters of the near-field sources. Direct maximum likelihood estimation of near-field source parameters results in complicated multi-parameter optimization problems, we therefore reformulated the estimation problem in terms of actual-data sample, called the incomplete data and a hypothetical data set, called the complete data and then devised the Expectation/Maximization iterative method for obtaining maximum likelihood estimates. The Expectation/Maximization algorithm decomposes the observed data into its components and then estimates the parameters of each signal component separately providing computationally efficient solution to the resulting optimization problem. The applicability and effectiveness of the proposed algorithm is illustrated by some numerical simulations.

## 1. INTRODUCTION

Most of the existing work on passive source localization rely on the assumption that all sources are located relatively far from the array (i.e., far-field). The far-field assumption implies that the waves emitted by the sources can be considered as plane waves at the sensor array and consequently each source location can be parametrized by only the azimuth (bearing) [1]. This assumption however is violated if the sources are located close to the array (i.e., near-field). The inherent curvature of the waveform in the near-field scenario, must be characterized by both the azimuth and range [2], [3], [4]. Therefore, the estimation of the unambiguous source location in the near-field case is more involved than the far-field case, since the parameter vector to be estimated is extended to include the source ranges. The near-field approach is widely used in radar, sonar, electronic surveillance, and seismic exploration applications.

It is therefore necessary to derive more sophisticated localization algorithms for estimating the azimuth as well the range. Recently, a total least squares ESPRIT like algorithm, based on the fourth-order cumulants was proposed in [2]. In [3], a high resolution algorithm that uses only second order statistics of the array outputs was developed. Due to many attractive properties of maximum

likelihood (ML) estimator such as consistency, asymptotic unbiasedness, and asymptotic minimum variance, we focus on ML method for estimating the parameters of the near-field sources in this paper. The assumptions on the narrow-band signal model lead to corresponding ML solutions. In this paper, we do not make any statistical assumption on the narrow-band source model and treat it as unknown but deterministic quantity when deriving the likelihood function. However, the direct calculations of ML parameter estimates from corresponding likelihood function is intractable due to the nonlinear nature of the likelihood function. We therefore reformulated the estimation problem in terms of the actual data sample, called the incomplete data, and a hypothetical data set, called the complete data and employed Expectation/Maximization (EM) iterative method for obtaining ML estimates. The most important feature of the EM algorithm is that it decouples the multi-dimensional search associated with the direct ML approach into searches in smaller-dimensional parameter subspaces, resulting a considerable simplification in the computations involved [5], [6].

## 2. NEAR-FIELD SIGNAL MODEL

Consider a near-field scenario in which narrowband signals from  $d$  sources received by an  $M$  element antenna array. Each source is located at azimuth/range coordinates  $\{\theta_i, r_i\}$  relative to sensor with index '0' as shown in Figure 1.

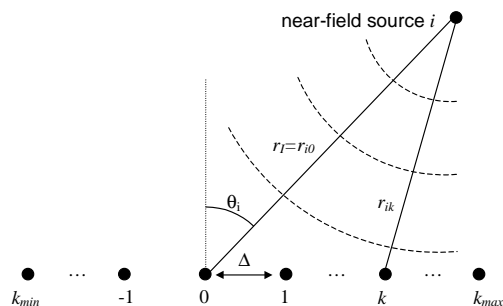


Figure 1: Near-field scenario with a uniform linear array

Assuming uniform linear array consisting of omnidirec-

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tional sensors with interelement spacing  $\Delta$ , we write the output of the  $k^{\text{th}}$  sensor with narrowband, co-channel signal as,

$$x_k(t_n) = \sum_{i=1}^d s_i(t_n) e^{j(\mu_i k + \zeta_i k^2)} + n_k(t_n), \quad 1 \leq t_n \leq N \quad (1)$$

where  $s_i(t_n)$  denotes the complex envelope of the  $i^{\text{th}}$  source signal,  $n_k(t_n)$  is an additive complex Gaussian noise. The parameters  $\mu_i$  and  $\zeta_i$  are the functions of the azimuth  $\theta_i$  and range  $r_i$  of the  $i^{\text{th}}$  source

$$\mu_i = -\frac{2\pi\Delta}{\lambda} \sin \theta_i, \quad \zeta_i = \frac{\pi\Delta^2}{\lambda r_i} \cos^2 \theta_i \quad (2)$$

where  $\lambda$  is the wavelength of the source wavefronts. For a collection of observed  $M$  sensor outputs  $\mathbf{x}(t_n) = [x_{k_{\text{min}}}(t_n), \dots, x_{k_{\text{max}}}(t_n)]^T$ , the matrix formulation of (1) is obtained as follows

$$\mathbf{x}(t_n) = \mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta}) \mathbf{s}(t_n) + \mathbf{n}(t_n) \quad (3)$$

where  $\mathbf{n}(t_n) = [n_{k_{\text{min}}}(t_n), \dots, n_{k_{\text{max}}}(t_n)]^T$  is zero-mean complex Gaussian random vector, with known spatial covariance  $\mathbf{I}$ ,  $\mathbf{s}(t_n) = [s_1(t_n), \dots, s_d(t_n)]^T$  is the collection of  $d$  impinging signals to a column vector,  $\mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta}) = [\mathbf{a}(\mu_1, \zeta_1), \dots, \mathbf{a}(\mu_d, \zeta_d)]$  is the array steering matrix in the near-field case which is known as a function of unknown set of parameters  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\} = \{(\mu_1, \zeta_1), \dots, (\zeta_d, \mu_d)\}$  and  $\mathbf{a}(\mu_i, \zeta_i)$  is the  $i^{\text{th}}$  array steering vector in the following form

$$\mathbf{a}(\mu_i, \zeta_i) = \begin{bmatrix} e^{j(k_{\text{min}}\mu_i + k_{\text{min}}^2\zeta_i)} \\ \vdots \\ 1 \\ e^{j(\mu_i + \zeta_i)} \\ e^{j(2\mu_i + 4\zeta_i)} \\ \vdots \\ e^{j(k_{\text{max}}\mu_i + k_{\text{max}}^2\zeta_i)} \end{bmatrix}. \quad (4)$$

The estimation problem we deal with in this paper can be formulated as follows. Given the array data  $\mathbf{x}(t_n)$  for  $1 \leq t_n \leq N$ , jointly estimate the parameters  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\} = \{(\mu_1, \zeta_1), \dots, (\zeta_d, \mu_d)\}$  of  $d$  sources and  $\mathbf{s}(t_n)$  to obtain source locations and signals.

### 3. MAXIMUM LIKELIHOOD ESTIMATION

The problem we address in this section is the ML estimation of the direction of arrival (DOA) and range  $\{\theta_i, r_i\}$ , or equivalently  $\{\mu_i, \zeta_i\}$  parameters of near-field sources. Different assumptions on the source signal  $s_i(t_n)$  lead to corresponding ML solutions. We focus on the deterministic ML (DML) approach in the sequel. In this case we do not make any statistical assumption on the source signal and treat it as unknown but deterministic quantity. Since the noise vector  $\mathbf{n}(t_n)$  is assumed to be additive, Gaussian with covariance matrix  $\mathbf{I}$ , the probability density function of the sensor outputs  $\mathbf{x}$  indexed by source signals  $\mathbf{s}(t_n)$  and unknown parameter vector  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\}$  can be written as

$$f(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\zeta}, \mathbf{s}) = \pi^{-NM} (\det \mathbf{I})^{-N}$$

$$\times \exp \left( - \sum_{t_n=1}^N [\mathbf{x}(t_n) - \mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta}) \mathbf{s}(t_n)]^H \times \mathbf{I}^{-1} [\mathbf{x}(t_n) - \mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta}) \mathbf{s}(t_n)] \right) \quad (5)$$

and its negative log-likelihood function (after neglecting unnecessary terms) is given by

$$\ell(\mathbf{x}; \boldsymbol{\mu}, \boldsymbol{\zeta}, \mathbf{s}) = \sum_{t_n=1}^N [\mathbf{x}(t_n) - \mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta}) \mathbf{s}(t_n)]^H \times [\mathbf{x}(t_n) - \mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta}) \mathbf{s}(t_n)]. \quad (6)$$

The ML estimate of the parameters  $\hat{\mathbf{s}}(t_n)$  and  $\{\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\zeta}}\}$  is a choice of parameters  $\mathbf{s}(t_n)$  and  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\}$  which locally maximizes the log-likelihood function (6). Typically the maximization problem of (6) is solved in two steps. First, maximize (6) with respect to  $\mathbf{s}(t_n)$ , keeping  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\}$  fixed. The first step results in closed form solution, since the maximization with respect to  $\mathbf{s}(t_n)$  is a linear least-squares problem. However, the second step, maximization of the log-likelihood function with respect to  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\}$ , results in nonlinear optimization problem. Based on these observations, the steps of the deterministic ML estimation approach is summarized as follows:

**Step 1:** For given  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\}$ , the values of  $\hat{\mathbf{s}}(t_n)$  that maximizes (6) is

$$\hat{\mathbf{s}}(t_n) = \mathbf{A}^\dagger(\boldsymbol{\mu}, \boldsymbol{\zeta}) \mathbf{x}(t_n). \quad (7)$$

where  $\mathbf{A}^\dagger(\boldsymbol{\mu}, \boldsymbol{\zeta}) = [\mathbf{A}^H(\boldsymbol{\mu}, \boldsymbol{\zeta}) \mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta})]^{-1} \mathbf{A}^H(\boldsymbol{\mu}, \boldsymbol{\zeta})$  is the pseudo-inverse of the array steering matrix  $\mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta})$ .

**Step 2:** Substitute  $\hat{\mathbf{s}}(t_n)$  back into (6) and solve for  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\}$ , which yields

$$\{\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\zeta}}\} = \arg \max_{\{\boldsymbol{\mu}, \boldsymbol{\zeta}\}} \text{tr} [\mathbf{A}(\boldsymbol{\mu}, \boldsymbol{\zeta}) \mathbf{A}^\dagger(\boldsymbol{\mu}, \boldsymbol{\zeta}) \hat{\mathbf{K}}_x] \quad (8)$$

where  $\hat{\mathbf{K}}_x = \frac{1}{N} \sum_{t_n=1}^N \mathbf{x}(t_n) \mathbf{x}(t_n)^H$  is the sample covariance matrix of  $\mathbf{x}(t_n)$  and  $\text{tr}[\cdot]$  denotes the matrix trace operation.

In **Step 2**, obtaining ML estimate of  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\}$  from (8) is a complicated multiparameter optimization problem and does not yield to a closed form solution. Solutions of such problems usually requires numerical methods, such as the methods of Scoring, Newton-Raphson or some other gradient search algorithm. However, for the problem at hand, these numerical methods tend to be computationally complex.

Fortunately, the formulation of the estimation problem at hand in terms of the actual data sample (incomplete-data) and a hypothetical data set (complete-data) allows us to apply the EM algorithm. The EM algorithm offers a simple indirect method for iteratively approximating the ML estimate. To be able to easily apply the EM algorithm the complete data must be chosen in such a way that: -the complete data log-likelihood function is easily maximized and -the complete data log-likelihood function can be easily estimated from the incomplete data. Based on these conditions, we should specify complete-data and its associated log-likelihood function. A natural choice for the complete-data would be

$$\mathbf{y}_l(t_n) = \mathbf{a}(\mu_l, \zeta_l) s_l(t_n) + \mathbf{n}_l(t_n) \quad (9)$$

where  $\mathbf{n}_l(t_n)$  is the Gaussian noise vector belongs to  $l^{th}$  signal. The complete-data vector  $\mathbf{y}_l(t_n)$  is the set of  $N$  samples of  $d$  independent Gaussian vectors with the  $l^{th}$  vector  $\mathbf{y}_l(t_n)$  having mean  $\mathbf{a}(\mu_l, \zeta_l) s_l(t_n)$  and with identical covariance  $\frac{1}{d}\mathbf{I}$ .

Motivation behind this choice is that, if one could somehow observe each of the incident waves separately, the estimation of its near-field parameters would be straightforward by performing  $d$  parallel maximizations. The incomplete data is the set of observations themselves. Thus, the relation between the complete-data  $\mathbf{y}_l(t_n)$  and the incomplete data  $\mathbf{x}_l(t_n)$  is given by

$$\mathbf{x}(t_n) = \sum_{l=1}^d \mathbf{y}_l(t_n). \quad (10)$$

In that case, the log-likelihood function of the complete-data  $\mathbf{y}_l(t_n)$  is

$$\ell_{\mathbf{y}}(\mathbf{y}; \boldsymbol{\mu}, \boldsymbol{\zeta}, \mathbf{s}) = - \sum_{t_n=1}^N \sum_{l=1}^d \left| \mathbf{y}_l(t_n) - \mathbf{a}(\mu_l, \zeta_l) s_l(t_n) \right|^2. \quad (11)$$

The EM algorithm is a two-step iterative procedure that makes use of the log-likelihood function of the complete data (11) to obtain ML estimates of near-field source parameters. For our problem, two-step procedure at the  $(p+1)^{th}$  iteration can be defined as follows:

**Expectation-Step:** Compute the expectation of the log-likelihood of the complete-data conditioned on previous estimates of  $\{\boldsymbol{\mu}^p, \boldsymbol{\zeta}^p\}$  and  $\mathbf{s}^p(t_n)$ ,

$$\ell_{\mathbf{y}_l}^{p+1} = E[\ell_{\mathbf{y}_l}(\mathbf{y}_l; \mu_l, \zeta_l, s_l) \mid \boldsymbol{\mu}^p, \boldsymbol{\zeta}^p, \mathbf{s}^p, \mathbf{x}]. \quad (12)$$

**Maximization-Step:** Maximize  $E[\ell_{\mathbf{y}_l}(\mathbf{y}_l; \boldsymbol{\mu}, \boldsymbol{\zeta}, \mathbf{s}) \mid \boldsymbol{\mu}^p, \boldsymbol{\zeta}^p, \mathbf{s}^p, \mathbf{x}]$  with respect to  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\}$  to obtain  $\{\boldsymbol{\mu}^{p+1}, \boldsymbol{\zeta}^{p+1}\}$ .

Since the complete-data  $\mathbf{y}_l(t_n)$  and the incomplete-data are related by a linear transformation (10), they are jointly Gaussian, and the conditional expectation required in **Expectation-Step** (12), can be computed by a straightforward modification of the well-known formulas for the conditional expectations in the Gaussian case [7]. Thus, the conditional mean is given by

$$\mathbf{y}_l^{p+1}(t_n) = \mathbf{a}(\mu_l^p, \zeta_l^p) s_l^p(t_n) + \frac{1}{d} [\mathbf{x}(t_n) - \mathbf{A}(\boldsymbol{\mu}^p, \boldsymbol{\zeta}^p) \mathbf{s}^p(t_n)] \quad (13)$$

It is important to note that the conditional mean (13) is the sum of two terms. The first term  $\mathbf{a}(\mu_l^p, \zeta_l^p) s_l^p(t_n)$  is the known signal component estimated at the  $p^{th}$  iteration, while the second term is an equal share of the total noise vector,  $\mathbf{n}(t_n) = \mathbf{x}(t_n) - \mathbf{A}(\boldsymbol{\mu}^p, \boldsymbol{\zeta}^p) \mathbf{s}^p(t_n)$  which is actually orthogonal to the signal subspace defined by  $\mathbf{A}(\boldsymbol{\mu}^p, \boldsymbol{\zeta}^p)$ .

In the **Maximization-Step**, we first substitute  $\mathbf{y}_l^{p+1}(t_n)$  back into (11), and then maximize the complete-data log-likelihood with respect to  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\}$  and  $s_l(t_n)$  which yields

$$(\mu_l^{p+1}, \zeta_l^{p+1}) = \arg \max_{\{\mu_l, \zeta_l\}} \frac{\mathbf{a}^H(\mu_l, \zeta_l) \hat{\mathbf{K}}_{\mathbf{y}_l}^{p+1} \mathbf{a}(\mu_l, \zeta_l)}{|\mathbf{a}(\mu_l, \zeta_l)|^2} \quad (14)$$

$$s_l^{p+1}(t_n) = \frac{\mathbf{a}^H(\mu_l^{p+1}, \zeta_l^{p+1}) \mathbf{y}_l^{p+1}(t_n)}{|\mathbf{a}(\mu_l^{p+1}, \zeta_l^{p+1})|^2} \quad (15)$$

where  $\hat{\mathbf{K}}_{\mathbf{y}_l}^p = \frac{1}{N} \sum_{t_n=1}^N \mathbf{y}_l^p(t_n) \mathbf{y}_l^p(t_n)^H$  is the sample covariance of the incomplete-data.

Based on this results, the steps of the proposed deterministic ML algorithm are summarized as follows:

1. Given  $\{\boldsymbol{\mu}^0, \boldsymbol{\zeta}^0\}$  and  $\mathbf{s}^0(t_n)$ ,  $p = 0$ ,
2.  $p = p + 1$ ,
  - Obtain  $\mathbf{y}_l^{p+1}(t_n)$  from (13),
  - Use  $\mathbf{y}_l^{p+1}(t_n)$  to compute  $\mathbf{K}_l^{p+1}$ , substitute  $\mathbf{K}_l^{p+1}$  in (14), and then solve (14) for  $\{\boldsymbol{\mu}^{p+1}, \boldsymbol{\zeta}^{p+1}\}$ ,
  - Substitute the estimates  $\{\boldsymbol{\mu}^{p+1}, \boldsymbol{\zeta}^{p+1}\}$  in (15), then compute  $s_l^{p+1}(t_n)$ ,
3. Continue this process until  $\{\boldsymbol{\mu}, \boldsymbol{\zeta}\}$  and  $\mathbf{s}(t_n)$  converges.

For a sufficiently good initialization, proposed algorithm converges rapidly to the ML estimate of  $\{\hat{\boldsymbol{\mu}}, \hat{\boldsymbol{\zeta}}\}$  and  $\hat{\mathbf{s}}(t_n)$ . Since the spatial structure of the array matrix is known, then the good initial estimates of the steering matrix can be obtained from MUSIC and ESPRIT algorithms.

One of the most striking feature of the proposed EM based deterministic ML algorithm is that it does not require any special care for parameter pairing problem. Since the proposed method decouples the complicated multi-parameter optimization into  $d$  separate ML optimization, each ML estimator maximizes its parameters  $\{\hat{\mu}_l, \hat{\zeta}_l\}$  and  $\hat{s}_l(t_n)$  separately.

## 4. SIMULATIONS

In order to verify the effectiveness and applicability of the proposed method, some numerical simulations are presented in the sequel. A Uniform linear array of  $M = 5$  sensors with inter-element spacing  $\Delta = \frac{\lambda}{2}$  was used to estimate the locations of two sources located at  $\{\theta_1, r_1\} = \{-5^\circ, 1.4\lambda\}$  and  $\{\theta_2, r_2\} = \{20^\circ, 3\lambda\}$ . In all simulations, the signal to noise ratio (SNR) was set to  $20dB$ . The results were compared with a similar stochastic EM based estimation method.

The proposed method was tested for  $K = 100$  independent trials first and the estimated locations of the sources are presented in Figure 2 (as dots). We tested the proposed method for different snapshots  $N$  (10 – 5000) ( $K = 10$  trials per each snapshot point). In each trial, the *RMSE* of the estimations for  $\{\theta_1, r_1\}$  and  $\{\theta_2, r_2\}$  were recorded and the corresponding results are presented in the Figure 3, Figure 5, Figure 4 and Figure 6, respectively, together with stochastic ML method results. The *RMSEs* are computed from  $RMSE_{\theta_i} = \sqrt{\frac{1}{K} \sum_{k=1}^K (\theta_i - \hat{\theta}_{i_k})^2}$  and

$RMSE_{r_i} = \sqrt{\frac{1}{K} \sum_{k=1}^K (r_i - \hat{r}_{i_k})^2}$  for  $i = 1, 2$ . Based on the simulation results some observations are now in order:-For small  $N$ , it is observed that the proposed method perform well. -Deterministic and Stochastic ML methods perform almost identical, however ML methods offer performance gains over higher order subspace based methods [1].

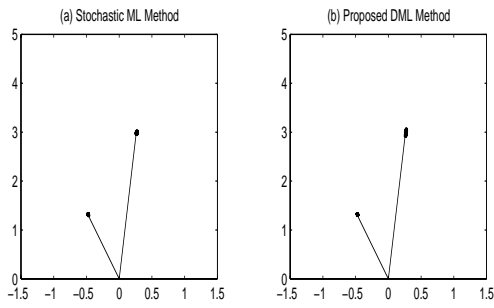


Figure 2: Estimated source locations along the array axis

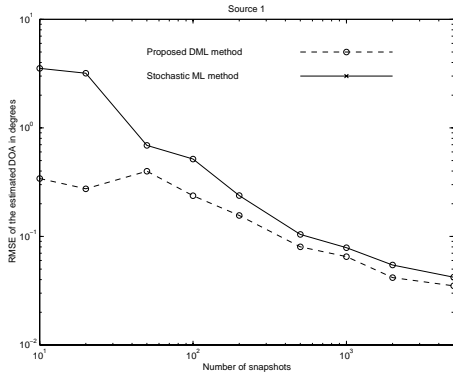


Figure 3: RMS error of the estimated DOA of source 1

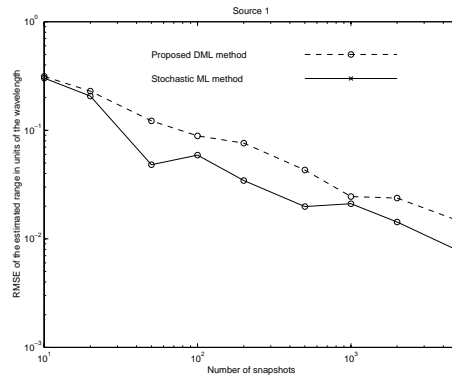


Figure 5: RMS error of the estimated range of source 1

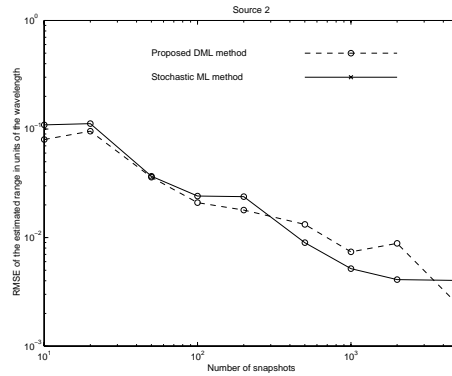


Figure 6: RMS error of the estimated range of source 2

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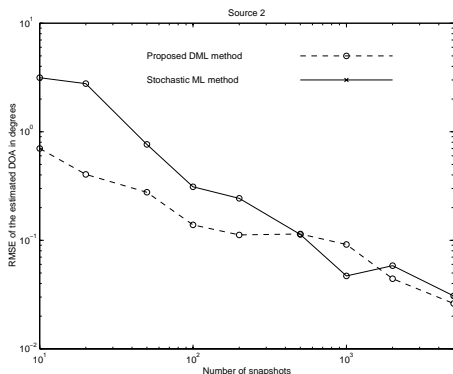


Figure 4: RMS error of the estimated DOA of source 2