# ROBUST INDEX ASSIGNMENT FOR QUANTIZATION OF LSF PARAMETERS TRANSMITTED OVER FINITE-MEMORY CONTAGION CHANNELS

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**ABSTRACT**

The paper analyzes the robustness of a new index assignment (IA) method for a particular class of Markov binary channels, namely the finite-memory contagion channels [1]. For this class of binary channels the channel distortion can be well approximated based on its dominant terms in the Hadamard transform domain. The IA method minimizes the distortion approximation and is shown to be very robust to changes in the parameters of the model. In the paper we prove the robustness of the proposed IA method, by applying it to the transmission of LSF parameters quantized as in G.729 standard.

## 1. INTRODUCTION

Index assignment techniques are called to reduce the effect of the channel errors, by arranging the codevectors in the codebook such that whenever the $n$-bit index $j$ is very likely to be received instead of $i$, the codevectors corresponding to $i$ and $j$ are close one to another according to some distortion measure.

It is well known that channel distortion evaluation can be speeded up by the use of computations in Hadamard transform domain [6]. When we specialize to finite-memory channels derived from Polya's contagion model, we proved in [4] that for ML decoding, the channel distortion is a sum of terms weighted by the eigenvalues $\lambda_\ell$ of the matrix of channel block transition probabilities:

$$D_C^{ML} \;=\; 2\sum_{\ell=1}^{N-1} ||\underline{t}_\ell||^2(1-\lambda_\ell) \tag{1}$$

where $\left[\underline{t}_0 \cdots \underline{t}_{N-1}\right] = \frac{1}{N} Z^T H_n$ and $Z$ is the matrix having as rows the codevectors and $H_n$ is the Hadamard matrix of size $2^n \times 2^n$. Using the dominant eigenvalues we approximate the channel distortion and obtain a performance index, *Hadamard assignment index*, very robust to the changes in the parameters of the finite memory contagion model.

## 2. INDEX ASSIGNMENT FOR A GIVEN CODEBOOK AND CHANNEL

### 2.1. The finite-memory contagion channel

The finite-memory contagion model was first introduced for communication channels by Alajaji and Fuja [1], who also derived the capacity of the channel. There are only three parameters in the model: $M$ is the "memory" of the channel, $\varepsilon$ is the probability of an error $\varepsilon = P(X_t = 1)$, and $\delta$ determines the correlation between errors at different times (the correlation coefficient is $\mathrm{cor}(X_t, X_{t-i}) = \delta/(1+\delta)$ for $i < M$).

We give here without proof a theorem characterizing the eigenvalues of the matrix of channel block transition probabilities $Q$.

**Theorem 1** *For any $M, n$ and $0 < \varepsilon < 0.5$, the dominant eigenvalue $\lambda^*$ of $Q$ and the indices for which $\lambda_\ell = \lambda^*$ are:*

*a) for $0 < \delta < 1 - 2\varepsilon$, $\lambda^* = 1 - 2\varepsilon = \lambda_\ell$, for all $\underline{\ell} \in \mathcal{I}_Q = \{\underline{\ell} \in \{0,1\}^n | w(\underline{\ell}) = 1\} \overset{not}{=} \mathcal{I}_1$*

*b) for $\delta > 1 - 2\varepsilon$, $\lambda^* = \frac{(1-2\varepsilon)^2+\delta}{1+\delta} = \lambda_\ell$ for all $\underline{\ell} \in \mathcal{I}_Q = \mathcal{I}_{2,M}$ where $\mathcal{I}_{2,M}$ is the set of all $\underline{\ell} \in \{0,1\}^n$ with Hamming weight $w(\underline{\ell})$ equal to 2, and such that in between the 2 ones of $\underline{\ell}$ there are at most $M-1$ zeros.*

### 2.2. Hadamard assignment index

**Definition 2.1** *The* Hadamard assignment index *of a codebook $Z$ for a channel with block transition probability matrix $Q$ is:*

$$0 \;\leq\; \gamma = \frac{\sum_{\underline{\ell} \in \mathcal{I}_Q} ||\underline{t}_\ell||^2}{\sigma_{VQ}^2} \;\leq\; 1 \tag{2}$$

*where $\sigma_{VQ}^2 = \sum_{\ell=1}^{N-1} ||\underline{t}_\ell||^2$ is the codebook power [3] and does not depend on the IA.*

The Hadamard assignment index is a generalization over the class of finite-memory contagion channels of the linearity index [6], defined for BSC. It provides similar bounds for the channel distortion, which we discuss in the next proposition and its corollaries.

**Proposition 2.1** *For a codebook with Hadamard index $\gamma$ the channel distortion is bounded by:*

$$2\sigma_{VQ}^2((1-\lambda^*) + \Delta\lambda^*(1-\gamma)) \leq D_C^{ML} \leq 2\sigma_{VQ}^2(1-\lambda^*\gamma)$$

*with $\Delta\lambda^* = \lambda^* - \max\{\lambda_\ell | \underline{\ell} \notin \mathcal{I}_Q,\ \ell > 0\}$.*

**Corollary 1** *If there exists an index assignment with $\gamma = 1$, then it ensures the minimum possible channel distortion*

$$D_{C_{min}}^{ML} = 2(1-\lambda^*)\sigma_{VQ}^2 \tag{3}$$

**Corollary 2** *All IAs with $\gamma > 1 - \Delta\lambda^*/\lambda^*$ give smaller distortion than any IA with $\gamma' < 1 - (1-\gamma)\lambda^*/\Delta\lambda^*$.*

In light of the above properties, finding the assignment matrix $Z$ maximizing $\gamma$ is similar to the minimization of channel distortion, and the larger $\gamma$ is obtained, the tighter are the inequalities in Proposition 1. The main advantages of using the Hadamard assignment index to design good index assignments is that we obtain robust IAs, as $\gamma$ depends on the channel only through $\mathcal{I}_Q$, and Theorem 1 shows that, for given $M$, $\mathcal{I}_Q$ remains the same over a wide range of $\delta$ and $\varepsilon$.

When the correlation factor $\delta$ is weak, $0 \leq \delta < 1 - 2\varepsilon$, the channel behaves like a BSC and the Hadamard index becomes the linearity index of [6]:

$$\gamma = \gamma_1 \overset{not}{=} \frac{\sum_{\underline{\ell} \in \mathcal{I}_1} ||\underline{t}_\ell||^2}{\sigma^2_{VQ}} \tag{4}$$

For a stronger correlation factor, $\delta > 1 - 2\varepsilon$, the Hadamard index is:

$$\gamma = \gamma_{2,M} \overset{not}{=} \frac{\sum_{\underline{\ell} \in \mathcal{I}_{2,M}} ||\underline{t}_\ell||^2}{\sigma^2_{VQ}} \tag{5}$$

and depends on $M$. The larger $M$ is, more terms (corresponding to Hadamard indices of Hamming weight 2) are added, which happens until $M = n - 1$, when the maximal set $\mathcal{I}_{2,n-1}$ is obtained, and for $M \geq n$ $\gamma_{2,M} = \gamma_{2,n-1}$. It is easy to see that for a $n$-bit codebook with a given IA, the following inequalities hold:

$$0 \leq \gamma_{2,1} \leq \gamma_{2,2} \leq \ldots \leq \gamma_{2,n-1} \leq 1 \tag{6}$$

As a consequence of the above inequalities, an IA with a large $\gamma_{2,1}$ should provide protection against channel errors irrespective to the value of $M$, as long as $M > 0$ and $\delta > 1 - 2\varepsilon$.

## 3. ROBUSTNESS OF IA FOR LSF TRANSMISSION

We experimentally study the index assignment for the predictive multistage-split VQ of line spectral frequencies (LSF) and start from the codebooks used in the standard G.729 [2]. The LSF coefficients are quantized in G.729 using three codebooks: a 10-dimensional 5-bit codebook, $\mathcal{C}^1$, and two 5-dimensional 5-bit codebooks, $\mathcal{C}^2$ and $\mathcal{C}^3$, respectively.

*Experiment 1* In this experiment we reassign the indices of the three codebooks of G729, by using the HSA algorithm. We designed permutations (index assignments) for three channel configurations: i) for $M = 0$ (BSC case) we maximized $\gamma_1$ to obtain the permutation $\nu_1(\cdot)$, ii) for $M = 1$, $\delta > 1 - 2\varepsilon$ we maximized $\gamma_{2,1}$ to obtain the permutation $\nu_2(\cdot)$, and iii) for $M = 2$, $\delta > 1 - 2\varepsilon$ we maximized $\gamma_{2,2}$ to obtain the permutation $\nu_3(\cdot)$. We note that $\nu_1(\cdot), \nu_2(\cdot), \nu_3(\cdot)$ are triplets of permutations, one for each codebook of G.729. To find the above permutations for every codebook and channel configuration we ran HSA 10 times, initialized with a random permutation, and chose the permutation giving the largest Hadamard assignment index. Tab. 1 shows the Hadamard assignment indices of the tested permutations of the three codebooks, for the considered channel configurations. From the table we inferred that the index assignment used in the standard, $\nu_0(\cdot)$, most likely had been designed assuming the BSC hypothesis.

*Experiment 2* The four different sets of permutations $\nu_0(\cdot)$, $\nu_1(\cdot)$, $\nu_2(\cdot)$, $\nu_3(\cdot)$ are now tested with real data, transmitted over a simulated channel.

To evaluate the speech spectrum distortion over a noisy channel a commonly used criterion is the spectral distortion measure [5]:

$$SD = \sqrt{\int_0^1 [10\log_{10}(P_i(f)) - 10\log_{10}(\hat{P}_i(f))]^2 df} \tag{7}$$

where $P_i(f) = 1/|A_i(exp(j2\pi f)|^2$ is the unquantized magnitude response and the reconstructed magnitude response is $\hat{P}_i(f) = 1/|\hat{A}_i(exp(j2\pi f)|^2$, and $A_i(z)$ and $\hat{A}_i(z)$ are the unquantized and respectively reconstructed LPC polynomials for the $i$-th frame.

In our derivations of the Hadamard assignment index we used the Euclidian norm as a measure of error effects, but here we are going to prove experimentally that our IA technique offers a good protection against channel errors, even if we measure the performance by the average SD criterion which is clearly different of the Euclidian norm.

A set of speech files from TIMIT database (more than 234000 frames) was encoded with G.729, and the three codevectors necessary to encode the LSF for one frame were assigned the indices according to the set of permutations obtained by HSA. The resulting bitstream was corrupted with errors generated by each of the three Markov models described at i), ii) and iii). After decoding, the spectral distortion $SD$ was evaluated for each frame. In each test we compute the mean spectral distortion $\overline{SD}$, the proportion of frames with $2dB \leq SD \leq 4dB$ and the proportion of outliers with $SD > 4dB$. To test the robustness of our method, we considered in turn $M = 0, 1, 2$ and different values $\varepsilon \in \{0.01, 0.05, 0.1\}$, while the value of the correlation factor was kept the same, $\delta = 10$.

The simulation results are listed in Tab. 2. The absolute difference in $\overline{SD}$, for two different realization, keeping all the parameters fixed, was found to be less than $0.01dB$. We expected, considering the results from Tab. 1, to obtain the best results over a given simulated channel by using the permutation designed by maximizing the corresponding Hadamard assignment index.

When using a simulated channel with $M = 0$ (BSC), the permutation set $\nu_1(\cdot)$ outperforms $\nu_0(\cdot)$ by a small amount ($0.03dB$ to $0.16dB$). The improvement is not so important, and once more this is an indication that the original IA used in G729 has been optimized for the case of binary symmetric channels. On the other hand, the permutations $\nu_2(\cdot)$ and $\nu_3(\cdot)$ provide much worse spectral distortion values, which is consistent with the low $\gamma$ values from Tab. 1. This ranking is expected to hold in general (i.e. the maximization of $\gamma_1$ will lead to small $\gamma_{2,M}$), since $\mathcal{I}_1 \cap \mathcal{I}_{2,M} = \emptyset$.

For $M = 1$, the permutation set $\nu_2(\cdot)$ gives better results than $\nu_0(\cdot)$ and $\nu_1(\cdot)$, the improvement ranging from $0.08dB$ to $0.62dB$, while $\nu_3(\cdot)$ is only $0.02dB$ to $0.07dB$ worse than $\nu_2(\cdot)$. The good behavior of $\nu_3(\cdot)$ is due to the fact that $\mathcal{I}_{2,1} \subset \mathcal{I}_{2,2}$.

For $M = 2$, $\nu_3(\cdot)$ outperforms $\nu_0(\cdot)$ and $\nu_1(\cdot)$, by $0.09dB$ to $0.85dB$. However, there is little performance improvement of $\nu_3(\cdot)$ over $\nu_2(\cdot)$. This is a consequence of the fact that $\gamma_{2,2} \geq \gamma_{2,1}$ so good IAs for $M = 1$ are necessarily well suited also for $M = 2$ (see (6)).

We can conclude that a significant difference exists between memoryless channels and channels with memory: an index assignment tailored for BSC fails when data are transmitted over a channel with memory, and the other way around. For channels with memory, $\nu_2(\cdot)$ has a remarkable robustness property: it behaves quasi-optimally for $M = 1, 2$ and $\delta > 1 - 2\varepsilon$; furtheremore it does not only reduce significantly the $\overline{SD}$ achievable by $\nu_0(\cdot)$, but also more than halves the proportion of outliers.

*Experiment 3* In the previous experiment the correlation parameter $\delta$ was kept constant at a high value, but now we fix $M = 1$ and $\varepsilon = 0.05$, and change the correlation in the noise sequence, by varying $\delta$. The mean spectral distortion, the percentage of frames with $2 \leq SD \leq 4$, and the the percentage of frames with $SD > 4$ are plotted against $\delta$ in Fig. 1.

For $\delta \ll 1$, the correlation in the noise sequence is negligible, and the channel is well approximated by a BSC. That is why $\nu_1(\cdot)$ gives the best protection, and $\nu_2(\cdot)$ and $\nu_3(\cdot)$ are about $0.5dB$ worse.

For $\delta$ close to 1 there is a transition region, the channel is in the transition from the independent-like behavior towards a channel with significant memory.

The change in the ranking of eigenvalues of $Q$ appears at $\delta = 1 - 2\varepsilon = 0.9$ (see Theorem 1). This explains the general decreasing trend of the distortion and the fact that $\nu_2(\cdot)$ and $\nu_3(\cdot)$ have better and better distortion performance, outperforming $\nu_0(\cdot)$ and $\nu_1(\cdot)$.

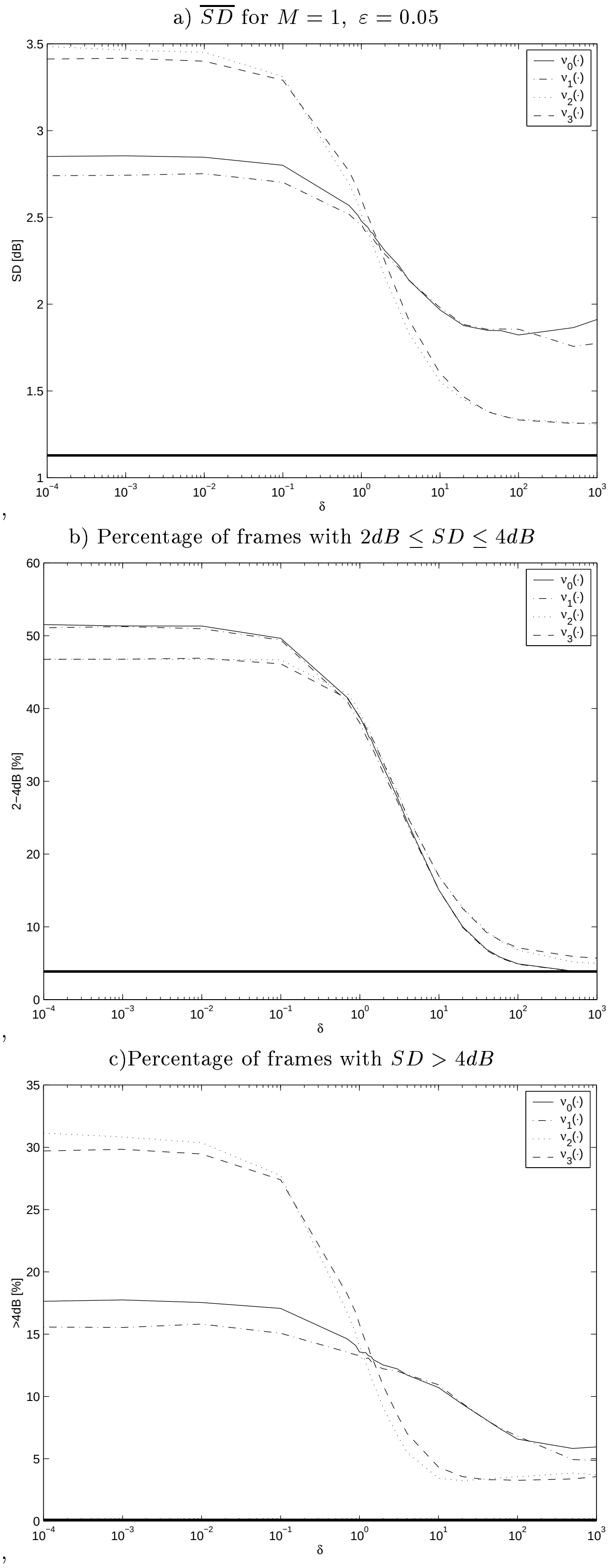


Figure 1: Simulation results for varying $\delta$, keeping $M = 1$, $\varepsilon = 0.05$ constant. The thick continuous levels in each plot represent the values corresponding to error-free transmission (see first column in Tab. 2)

## 4. REFERENCES


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| | $\nu_0(\cdot)$ | | | $\nu_1(\cdot)$ | | | $\nu_2(\cdot)$ | | | $\nu_3(\cdot)$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | $\mathcal{C}^1$ | $\mathcal{C}^2$ | $\mathcal{C}^3$ | $\mathcal{C}^1$ | $\mathcal{C}^2$ | $\mathcal{C}^3$ | $\mathcal{C}^1$ | $\mathcal{C}^2$ | $\mathcal{C}^3$ | $\mathcal{C}^1$ | $\mathcal{C}^2$ | $\mathcal{C}^3$ |
| $\gamma_1$ | 0.58 | 0.57 | 0.59 | **0.75** | **0.76** | **0.77** | 0.01 | 0.08 | 0.02 | 0.01 | 0.07 | 0.08 |
| $\gamma_{2,1}$ | 0.07 | 0.09 | 0.08 | 0.02 | 0.03 | 0.03 | **0.72** | **0.67** | **0.66** | 0.39 | 0.50 | 0.43 |
| $\gamma_{2,2}$ | 0.10 | 0.17 | 0.20 | 0.03 | 0.04 | 0.06 | 0.74 | 0.69 | 0.68 | **0.76** | **0.72** | **0.71** |

Table 1: The values of the Hadamard assignment indices $\gamma_1$, $\gamma_{2,1}$, $\gamma_{2,2}$ of the codebooks of G.729 with different permutations (index assignments): $\nu_0(\cdot)$ is the original IA in G.729; $\nu_1(\cdot)$ is the IA obtained using HSA for $M=0$; $\nu_2(\cdot)$ is the IA obtained using HSA for $M=1, \delta > 1-2\varepsilon$; and $\nu_3(\cdot)$ is the IA obtained using HSA for $M=2, \delta > 1-2\varepsilon$. The maximization of Hadamard assignment indices is approximately equivalent with the minimization of the channel distortion under the following conditions: $\gamma_1$ for BSC ($M=1$), $\gamma_{2,1}$ for $M=1, \delta > 1-2\varepsilon$, and $\gamma_{2,2}$ for $M=2, \delta > 1-2\varepsilon$.

| Channel parameters | | | $M=0$ | | | $M=1, \delta=10$ | | | $M=2, \delta=10$ | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| | | $\varepsilon=0$ | $\varepsilon=0.01$ | $\varepsilon=0.05$ | $\varepsilon=0.1$ | $\varepsilon=0.01$ | $\varepsilon=0.05$ | $\varepsilon=0.1$ | $\varepsilon=0.01$ | $\varepsilon=0.05$ | $\varepsilon=0.1$ |
| $\nu_0(\cdot)$ | $\overline{SD}[dB]$ | 1.1289 | 1.55 | 2.86 | 4.08 | 1.30 | 1.97 | 2.77 | 1.29 | 1.91 | 2.66 |
| | 2-4 dB[%] | 3.86 | 18.1 | 51.6 | 47.6 | 6.5 | 15.0 | 21.9 | 5.0 | 9.2 | 12.7 |
| | >4 dB[%] | 0.09 | 2.1 | 17.7 | 44.9 | 2.1 | 10.7 | 21.6 | 1.9 | 9.0 | 17.5 |
| $\nu_1(\cdot)$ | $\overline{SD}[dB]$ | 1.1289 | **1.51** | **2.74** | **3.92** | 1.30 | 1.98 | 2.78 | 1.29 | 1.90 | 2.67 |
| | 2-4 dB[%] | 3.86 | 17.1 | 51.3 | 50.1 | 6.5 | 15.1 | 21.4 | 5.0 | 9.0 | 12.6 |
| | >4 dB[%] | 0.09 | **1.8** | **15.5** | **41.1** | 2.2 | 11.0 | 21.9 | 1.9 | 8.9 | 17.5 |
| $\nu_2(\cdot)$ | $\overline{SD}[dB]$ | 1.1289 | 1.70 | 3.49 | 5.13 | **1.22** | **1.55** | **1.95** | 1.21 | 1.49 | 1.83 |
| | 2-4 dB[%] | 3.86 | 20.7 | 46.4 | 31.8 | 6.7 | 16.9 | 27.2 | 5.6 | 11.7 | 18.0 |
| | >4 dB[%] | 0.09 | 4.1 | 31.3 | 64.2 | **0.7** | **3.4** | **7.7** | 1.0 | 4.4 | 8.9 |
| $\nu_3(\cdot)$ | $\overline{SD}[dB]$ | 1.1289 | 1.68 | 3.42 | 5.06 | 1.23 | 1.60 | 2.02 | **1.20** | **1.48** | **1.81** |
| | 2-4 dB[%] | 3.86 | 20.5 | 46.9 | 32.4 | 6.8 | 17.0 | 26.9 | 5.5 | 11.6 | 18.2 |
| | >4 dB[%] | 0.09 | 3.8 | 29.8 | 63.3 | 0.9 | 4.3 | 9.0 | **0.9** | **4.1** | **8.5** |

Table 2: Simulation results obtained from sending indices of the codebooks of G.729 over a channel modeled on contagion, for different model parameters. The indices are either assigned as in the original codebooks, $\nu_0(\cdot)$, or are reassigned with IAs constructed using HSA, for 3 channel conditions: $\nu_1(\cdot)$ for BSC, $\nu_2(\cdot)$ for $M=1$, $\delta > 1-2\varepsilon$, and $\nu_3(\cdot)$ for $M=2$, $\delta > 1-2\varepsilon$. The evaluation is done over more than 234000 frames of speech.