

A LARGE SIGNAL INTEGRATION FORMULA

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ABSTRACT

A new integration method for highly oscillating circuits is proposed. The method is based on using finite (truncated) Fourier series for representation of signals. Nonlinear transformations are implemented in two phases: harmonic linearization using only one sine wave in phase space, and general Chebyshev transformation of waveforms to return into true time domain.

1. INTRODUCTION

Design of high quality oscillators and other narrow-band systems needs special modeling and simulation methods as direct integration of differential equations is too time consumable to achieve required accuracy. Precompiled large signal models may accelerate solution significantly and give higher accuracy because of using analytical solutions.

Classical approach for analysis of narrow-band systems is using harmonic balance assuming that non-linear components produce signals with complicated spectrum but linear part behaves as a narrow-band filter and returns near sine wave. Implementation of that needs special harmonic analysis applied periodically to signals produced by non-linear components. Depending on how non-linear components are modeled, two approaches exist: 1) direct time domain calculation of non-linear functions, e.g. [1] using of precompiled large signal models [2,3]. Anyway, modeling of highly oscillating systems needs a lot of memory and simulation time [4,5]. More complications appear because of possibility of chaos and difficulties in control of simulations [6].

This paper deals with large signal model based on harmonic input signal proposed in [7,8] that uses representation of excitation in the form $v(t) = v_0 + v_1 \cos p(t)$ where $p(t)$ is monotonic phase.

Therefore, assuming input signal of a non-linear component to be

$$v(t) = a_0 + a_1 \cos p(t) \quad (1)$$

we calculate output as

$$y(t) = y_0 + y_1 \cos p + y_2 \cos 2p + y_3 \cos 3p + \dots \quad (2)$$

where

$$\cos p(t) = \frac{v(t) - a_0}{a_1} \quad (3)$$

using precalculated harmonic linearization for known nonlinear transformation.

Note that choice of a_0 and a_1 is theoretically free but in fact, they should be evaluated as correctly as possible to minimize numerical problems. Evaluation of a_0 and a_1 is the first step of iteration when few harmonics can be used.

The second step is refinement where we fix values of a_0 , a_1 and, consequently, y_0 , y_1 , y_2, \dots . Nonlinear behavior appears now only in general Chebyshev transformation

$$\cos np = T_n(\cos p) \quad (4)$$

that can be calculated in frequency domain as $v(t)$ is represented by Fourier series.

Truncated Fourier series cannot be used to model transients. That is why the signal model is extended by half-frequency component:

$$v_0 + v_{1/2}^R \cos \frac{\omega t}{2} - v_{1/2}^I \sin \frac{\omega t}{2} + \sum_{k=1}^n (v_k^R \cos k\omega t - v_k^I \sin k\omega t)$$

for $0 \leq t \leq T = \frac{2\pi}{\omega}$

Presence of such component makes calculations a little bit more complicated but keeps harmonic nature of signals.

The equations describing integration can be derived from this general form of signals. It is assumed that integration step is equal to T that could be one or more periods but may also be part of oscillation period. That means using of very large time step, in some applications it exceeded by 1000 times time step used by conventional simulators.

2. LINEAR BENCHMARK

It is extremely important to have correct continuous-to-discrete-time mapping when modeling oscillating systems. Classical method to characterize integration formulas is using linear benchmark problem:

$$\dot{v}(t) = \lambda v(t)$$

Our integration formula defines values at the ends of iteration step as

$$v(0) = v_0 + v_{1/2}^R + \sum_{k=1}^n v_k^R \quad \text{and} \quad v(T) = v_0 - v_{1/2}^R + \sum_{k=1}^n v_k^R$$

and

$$\rho = \frac{v(T)}{v(0)} = \frac{1 + b_{1/2} - \sum_{k=1}^n b_k}{1 - b_{1/2} - \sum_{k=1}^n b_k}, \quad b_{1/2} = \frac{\pi^2}{2} \frac{\mu}{\pi^2 + \mu^2}$$

$$b_k = \frac{2\mu^2}{(4k^2 - 1)(2\pi k)^2 + \mu^2}, \quad \mu = \lambda T$$

For real values of λ , correct value must be $\rho = \exp(\mu)$. Comparison of this value, trapezoidal formula, and several cases of our formula is presented on Figure 1. To make the results distinguishable we use $\log \rho = \mu$. The curves show excellent behavior of the formula. One can see that even when 10 harmonics are used the coefficient is correct up to 7μ that means values of $\rho = \exp(\pm 7)$.

Complex eigenvalues are more important for analysis of oscillator but somewhat more complicate to describe. Dependencies of $\log \rho$ for $\mu = \sigma + j2\pi$ that correspond to 1st harmonic frequency and damping σ , changing from negative to positive region are shown on Figure 2 and Figure 3. They are compared with correct value $\log \mu = \log \sigma$ and again we can see very good behavior even for small number of harmonics involved.

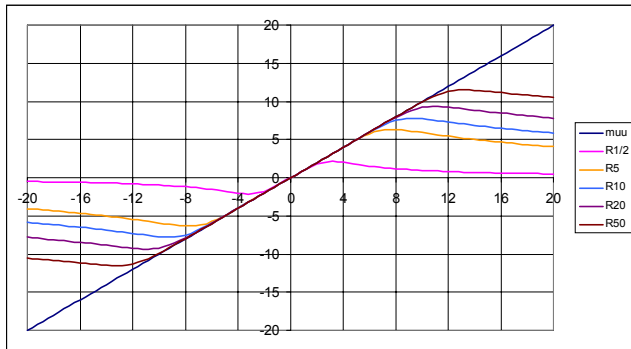


Figure 1. Real poles. Number of harmonics: 0 (half-frequency only), 5, 10, 20, and 50

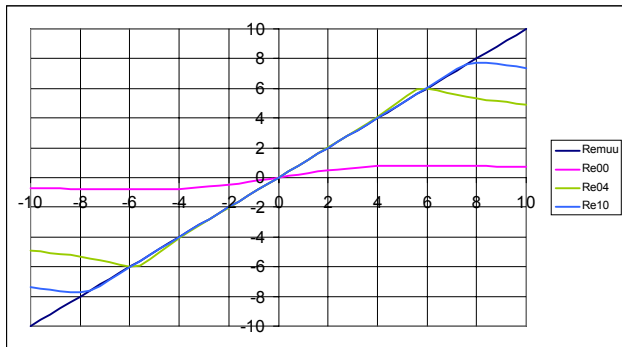


Figure 2. Complex poles: magnitude of ρ . Number of harmonics: 0 (half-frequency only), 4, and 10

Note that if eigenvalues are imaginary then $|\rho|=1$ that shows preserving of stability property. However, the value of frequency may be distorted. It can be checked by studying phase of ρ . This is demonstrated on Figure 4 where difference between correct frequency and the modeled one is shown for 5 harmonics. It is seen that for all 5 harmonics the formula is exact.

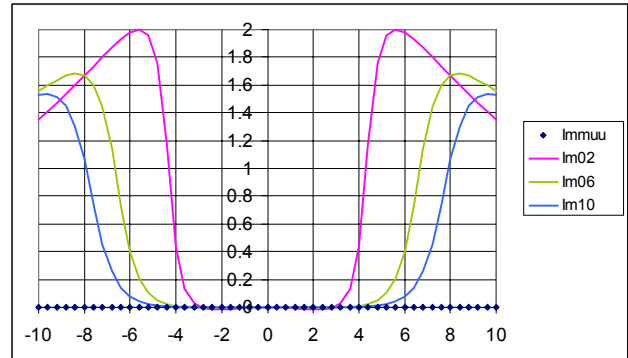


Figure 3. Complex poles: phase of ρ . Number of harmonics: 2, 6, and 10.

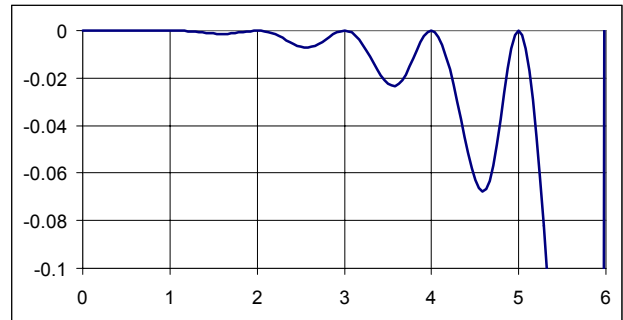


Figure 4. Frequency error. Number of harmonics is 5.

3. LARGE SIGNAL MODEL

Fundamental idea of the present method is the use of standardized large signal model as defined by (1)..(4). Assume that transfer characteristic of non-linear component is

$$y = F(v).$$

Then, considering for simplicity $v = Ax = A \cos(p)$, our approach means that its expansion of $F(v)$ into Chebyshev series is applied:

$$F(v) = \sum_{k=0}^{\infty} y_k(A) T_k(x), \quad x = \cos p$$

Such expansions are well known in approximation theory as very efficient formulas. Those expansions converge for $|\cos p| \leq 1$ only that may cause problems when we try to evaluate function $F(v)$ for $|\cos p| > 1$. However, when implementing our integration method, we really need to know only values of spectral components of $F(v)$ (equal to y_k when $x = \cos t$).

If the function $F(v)$ is smooth, that is equivalent to fast decrease of y_k -s (spectrum!), then we do not need to worry about convergence. As an example, consider the most important non-linear function for electronic circuits - exponent. We have

$$e^{Ax} = I_0(A) + 2 \sum_{k=1}^{\infty} I_k(A) T_k(x)$$

where I_k are Bessel functions. It can be shown that we can substitute x by $a \cdot \cos(p)$ with $a > 1$ and obtain correct values of spectral components. Here we describe a very hard test. Let $A=30$ and $a=1.5$; if applied to pn-junction with $\varphi_j=26\text{mV}$, that would mean reference amplitude of voltage $30 \cdot 26=780\text{mV}$ and its deviation up to $1.5 \cdot 780=1170\text{mV}$ - huge for silicon! Spectrums of signal directly computed and evaluated by present method are same by at least 6 digits for 50 harmonics with values decreasing from $2\text{E}+18$ (1^{st}) to $2\text{E}+7$ (50^{th})! This is accuracy of transformation

$$\begin{aligned} e^{Aa \cos t} &= I_0(Aa) + 2 \sum_{k=1}^{\infty} I_k(Aa) T_k(\cos t) = \\ &= I_0(A) + 2 \sum_{k=1}^{\infty} I_k(A) T_k(a \cos t) \end{aligned}$$

Situation is different if the function $F(v)$ is not smooth. Hard example is absolute value $F(v)=|v|$. Chebyshev expansion converges very slowly and simple extension outside interval $[-1,1]$ is not possible. As usual in such cases, we have two ways to overcome the problem. First, we can always keep normalizing factor v_l high enough to be sure that $|\cos p|$ does not exceed 1. This may be expensive (to check instant values of signal in time domain), and therefore, we may prefer regularization of nonlinear function. This is equivalent to windowing of the "spectrum" $\{y_k\}$. Note that this is not practical problem as large signal models are compiled once and there is no need to recalculate them during solution. This example also supports the thesis of MOS model developers that non-smooth functions should not be used for computer modeling.

4. NOTE ON APPROXIMATIONS

We could use instead of Chebyshev polynomials other approximations eg other types of orthogonal polynomials. Indeed, if nonlinear function is expanded as

$$F(Ax) = f_0(A) + \sum_{k=1}^{\infty} f_k(A) P_k(x) \quad (5)$$

where $\{P_k(x)\}$ is a set of polynomials then substituting $x = \cos p$ we find that finite number of terms in (5) generate finite number of harmonics $\cos(np)$. However, when changing number of terms used, coefficients of those harmonics are to be recalculated. Only Chebyshev polynomials preserve equality of coefficients in harmonic expansion and $f_k(A)$.

The most important difference is in the approximation style. In case of Chebyshev polynomials approximation is convergent inside interval $[-1,1]$. That means assumption about limited amplitude of signals. It follows that exceeding those limits may cause problems as shown above. If using eg Hermitian polynomials with weighting function $\exp(-x^2)$ we obtain more smooth dependence upon amplitude, as approximation is convergent everywhere. However, as weight is decreasing very fast when x exceeds 1 we also obtain low quality approximation for signals with $A > 1$. Note that Wiener used Hermitian polynomials for approximation of non-linear transformation as he assumed stochastic signal to be general form of test signal.

5. BENEFITS AND DRAWBACKS

B1. Very large integration step can be used that reduces number of recursions.

B2. In highly oscillating circuits, time step can be made equal to period defined by zero-crossings. The value of period can be calculated with very high accuracy. Periods can be considered individually, also during transients.

B3. Number of evaluations of complicated non-linear functions can be reduced due to the use of large signal models.

B4. Rough solutions may help to accelerate convergence.

D1. The method is complicated if compared with classical integration formulas.

D2. Large signal models must be developed.

D3. It is not clear which algorithms should be used for fast spectrum calculation needed to evaluate Chebyshev transformations. This procedure is the most time consuming.

D4. It is not clear how complicated may become the method when applied to multidimensional non-linear functions, multi-tone systems, and large number of components.

D5. Specific iteration strategy is needed, probably using waveform relaxation methods.

6. References

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