"raised-cosine", and the optimal rolloff obtained previously by Panayirci and Tugbay[4]. Performance is measured both in terms of the "residual-tail energy" against excess bandwidth and in terms of the probability of error against sampling time for a given fixed signal-to-noise ratio.

II. FORMULATION OF THE PROBLEM

Our main concern is to design a Nyquist type and a bandlimited signal $s(t), t \in (-\infty, +\infty)$ of bandwidth W which maximizes its energy in the time interval $(-\sigma T, \sigma T)$, given by

$$E_i = \int_{-\sigma T}^{\sigma T} s^2(t)dt \tag{3}$$

under the constraint that the total signal energy,

$$E_0 = \int_{-\infty}^{\infty} s^2(t)dt \tag{4}$$

is constant, where σ is an arbitrary positive constant. Let $S(f)$ denote the Fourier transform of $s(t)$. Then the Nyquist constraint $s(kT) = 0$ for $k = \pm 1, \pm 2, \ldots$, can be inserted into the problem in the frequency domain by the first Nyquist criterion:

$$\sum_{k=-\infty}^{\infty} S(f + \frac{k}{T}) = A \tag{5}$$

where A is any positive real constant. Furthermore, since we limit consideration to bandlimited signals of bandwidth W, we express the bandwidth in terms of a rolloff factor γ which is the relative amount of bandwidth in excess of Nyquist band, $1/2T$,

$$\gamma = \frac{W - 1/2T}{1/2T}. \tag{6}$$

It is convenient to write $S(f)$ in terms of a normalized rolloff function $B(f)$

$$S(f) = A\left[\; P(f) + B(f - \tfrac{1}{2T}) + B^*(-f - \tfrac{1}{2T}) \;\right] \tag{7}$$

where $B(f) = 0$ for $|f| > \gamma/2T$ and $P(f)$ is the required shape for the case $\gamma = 0$.

$$P(f) = \begin{cases} 1 & |f| > 1/2T \\ 0 & \text{otherwise} \end{cases} \tag{8}$$

The Nyquist constraint is equivalent to $B(f) = -B^*(-f)$. Moreover, it can be shown that $S(f)$ is a real and even function [3], hence we take $B(f)$ to be real and odd. Therefore, (7) can be written as

$$S(f) = A\left[\; P(f) + B(f - \tfrac{1}{2T}) - B(f + \tfrac{1}{2T}) \;\right]. \tag{9}$$

Then, taking inverse Fourier transform of $S(f)$, and taking into account that $B(f)$ is an odd function, we obtain the following for $s(t)$.

$$s(t) = A\left[\; p(t) - 4b(t)\sin(\pi t/T) \;\right] \tag{10}$$

where

$$b(t) = \int_0^{\gamma/2T} B(f)\sin(2\pi f t)df. \tag{11}$$

The problem of selecting the optimal signal $s(t)$ under the constraints introduced above, reduces to finding the rolloff function $B(\cdot)$ and A which maximize the functional $J \equiv J[B(\cdot), A]$ defined by

$$J = E_i - \lambda E_o$$

where the constant λ is called a *Lagrange multiplier.* Using the periodically nonuniform sampling method as explained in Section 1, the positive part of $B(f)$ can be reconstructed from its samples as follows:

$$B(f) = \sum_{n=0}^{M/2} s_{2n}\Psi_0(f - n\Delta) + \sum_{n=0}^{M/2-1} s_{2n+1}\Psi_1(f - n\Delta) \tag{12}$$

where $\Psi_0(f) = \phi(f + \Delta/2) - \phi(f)$ and $\Psi_1(f) = 2\phi(f)$ are the synthesizing functions, $\phi(f)$ is linear splines with the length of Δ given by (2), and M is an even positive number. Figure 1. shows how to construct the positive part of $B(f)$ using periodically nonuniform sampling method. Substituting (12) in (11), we have for $b(t)$,

$$\begin{aligned} b(t) = {} & s_0 \int_0^{\Delta} (\phi(f + \Delta/2) - \phi(f))\sin(2\pi f t)df \\ & + s_M \int_0^{\Delta/2} (\phi(f))\sin(2\pi t(f - (m-1)\Delta/2))df \\ & + \sum_{n=1}^{M/2-1} s_{2n} \int_{-\Delta/2}^{\Delta} (\phi(f + \Delta/2) - \phi(f))\sin(2\pi t(f + n\Delta))df \\ & + \sum_{n=0}^{M/2-1} s_{2n+1} \int_0^{\Delta} (2\phi(f))\sin(2\pi t(f + n\Delta))df. \end{aligned} \tag{13}$$

Performing integrations in (13), $b(t)$ can be expressed as

$$b(t) = \beta_0(t) + \sum_{k=1}^{M} s_k\beta_k(t) \tag{14}$$

where

$$\beta_0(t) = \frac{1}{\Delta(2\pi t)^2}\left[-3\sin(2\pi t\Delta/2) + \sin(2\pi t\Delta) + 2\pi t\Delta\right]$$

$$\beta_k(t) = \tag{15}$$

$$\begin{cases} \frac{2}{\Delta(2\pi t)^2}\left[\sin(2\pi\Delta t\frac{k+2}{2}) - 3\sin(2\pi\Delta t\frac{k+1}{2}) \right. \\ \left. +3\sin(2\pi\Delta t\frac{k}{2}) - \sin(2\pi\Delta t\frac{(k-1)}{2})\right] & \text{if } k \text{ even} \\ \frac{2}{\Delta(2\pi t)^2}\left[4\sin(2\pi\Delta t) - 2\sin(2\pi\Delta kt) \right. \\ \left. \sin(2\pi\Delta t\frac{M+2}{2}) - \sin(2\pi\Delta t\frac{M+1}{2})\right] & \text{if } k \text{ odd} \\ -\frac{1}{2\pi t}\cos(2\pi\Delta t\frac{M+1}{2}), & \text{if } k = M \end{cases}$$