where $\Delta = \gamma/TM$, $p(t) = \frac{1}{T}\,\mathrm{sinc}(\frac{t}{T})$ and $\mathrm{sinc}(t) = \sin(\pi t)/(\pi t)$. Then, in terms of (10) and (14), $s(t)$ can be written as

$$s(t) = A\left[\ f_0(t) + \textstyle\sum_{i=1}^{M} s_i f_i(t)\ \right] \qquad (16)$$

where

$$f_0(t) \;=\; p(t) - 4\sin(\frac{\pi t}{T})\beta_0(t) \qquad (17)$$

$$f_i(t) \;=\; -4\sin(\frac{\pi t}{T})\beta_i(t),\ i = 1, 2, \ldots, M. \qquad (18)$$

The energy of the signal $s(t)$ in the interval $(-\sigma T, \sigma T)$ can be expressed as a function of the coefficients $\{\ s_i\}$ and A, as follows:

$$\begin{aligned} E_i &= \int_{-\sigma T}^{\sigma T} s^2(t)dt \\ &= A^2\left[\ c_0 + 2\textstyle\sum_{i=1}^{M} a_i s_i + \sum_{i=1}^{M}\sum_{j=1}^{M} b_{ij} s_i s_j\ \right] \end{aligned}$$

where

$$\begin{aligned} c_0 &= \int_{-\sigma T}^{\sigma T} f_0^2(t)dt, a_i = \int_{-\sigma T}^{\sigma T} f_0(t) f_i(t)dt, \\ b_{ij} &= \int_{-\sigma T}^{\sigma T} f_i(t) f_j(t)dt,\ \ i, j = 1, 2, \ldots, M. \end{aligned}$$

On the other hand, the total energy of the signal can be computed as

$$\begin{aligned} E_0 &= \int_{-\infty}^{+\infty} s^2(t)dt \\ &= A^2\left[c_1 - 4\sum_{i=1}^{M} d_i s_i + 4\sum_{i=1}^{M} d_i s_i^2\right] \end{aligned}$$

where

$$c_1 = \frac{1}{T} + \frac{\Delta}{4} \ \text{ and, } d_i = \begin{cases} \Delta/4, & \text{if } i = M \\ \Delta/2, & \text{otherwise.} \end{cases}$$

The problem of selecting the optimal signal $s(t)$ which maximizes its energy in the time interval $(-\sigma T, \sigma T)$ subject to the constraint that the total energy is constant, reduces to finding the coefficients $s_1, s_2, \ldots, s_M$ and A which maximize

$$I = I(s_1, s_2, \ldots, s_M; A) = E_i - \lambda E_0.$$

Setting

$$\frac{\partial I}{\partial s_i} = 0,\ \text{for } i = 1, 2, \ldots, M, \quad \text{and} \quad \frac{\partial I}{\partial A} = 0,$$

yields the set of linear equations

$$\left(\ 2a_i + 2\textstyle\sum_{j=1}^{M} b_{ij} s_j\ \right) - 4\lambda d_i(2s_i - 1) = 0$$

$$c_0 + 2\sum_{i=1}^{M} a_i s_i + \sum_{i=1}^{M}\sum_{j=1}^{M} b_{ij} s_i s_j$$

$$-\lambda\left[c_1 - 4\sum_{i=1}^{M} s_i d_i(1 - s_i)\right] = 0$$

By solving the above equations simultaneously, the optimal coefficients $s_1^0, s_2^0, \ldots, s_M^0$ and the optimal energy ratio $\lambda = (E_i/E_o)_{opt}$ are obtained.

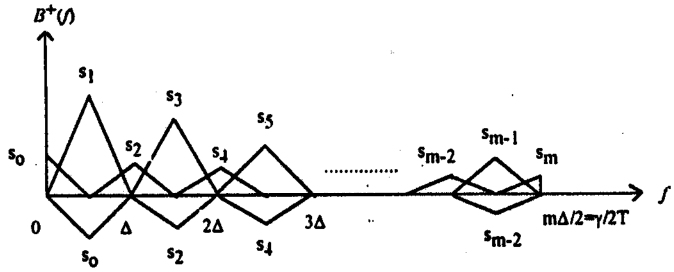

Fig. 1. Approximation of positive part of $B(f)$.

III. NUMERICAL EXAMPLES AND PERFORMANCE ANALYSIS

The optimal signals were obtained for the different values of σ, γ, where $M = 20$ and the resulting optimal signals was plotted in Figure 2. in time domain. Finally, the resulting optimal energy ratios λ_0 were computed and the residual-tail energy, $10\log(1 - \lambda_0)$ in dB, was compared to that of raised-cosine signal and of the signal shapes obtained in [4], for different values of the parameters, σ and γ. These results, summarized in Table 1., show that the optimal pulse shape obtained by the new method presented in this paper achieves an extremely high energy compaction and this increases especially as the value of γ increases.

Table 1
RESIDUAL-TAIL ENERGY

γ	σ	$10\log(1 - E_i/E_0)$		
		raised-cos	opt.pulse*	opt.pulse**
0.25	1.0	-13.059	-15.945	- 16.07
0.50	1.0	-17.128	-23.372	- 29.90
0.75	1.0	-23.223	-31.205	- 42.23
1.00	1.0	-32.887	-36.448	- 43.35
0.50	0.5	-8.473	-9.988	- 10.24

(*) Optimal pulse obtained by [4]
(**) Optimal pulse obtained by linear splines

The error performance of our approach has been also compared with the others. We computed the bit error rate(BER) of a binary baseband data transmission system, in which the received signal may be characterized by

$$r(t) = \sum_{k=-L}^{L} a_k s(t - kT) + n(t)$$

where $s(\cdot)$ describes the pulse shape, T is the time between pulses and the a_k are independent binary random variables taking values -1 and +1 with equal probabilities. The noise process $n(t)$ is Gaussian, real and independent of a_k; it is assumed to be stationary with zero mean and