

variance  $\sigma_n^2$ . The bit-error rate is computed by the direct method given by Aaron and Tufts [8], based on the truncated pulse train approximation whose total pulse length is equal to  $(2L + 1)T$ . We computed the BER as a function of the sampling-time deviation  $\tau$  for our optimal pulse and compared it with corresponding curves for the raised-cosine, modified triangular rolloff pulse shapes and signals generated by Panayirci and Tuğbay [4]. Since the BER curves in these examples are even functions of  $\tau$ , so that only positive sampling time deviations are shown in Figure 3. For all cases, the signal/noise ratio defined at the nominal sampling time by  $\eta = |s(\tau)|/\sigma_n$  is 6.361, corresponding to a probability of error of  $10^{-10}$  in the absence of intersymbol interference. These curves demonstrate the definite superiority of our results. In order to see this fact more clearly, In Fig. 4, the eye diagrams associated with the optimal signal shape and the raised-cosine pulse shape is given.

#### IV. CONCLUSIONS

We have proposed an efficient method for designing optimal band-limited Nyquist signals based on generalized sampling theory. Maximum energy in the time interval  $(-\sigma T, \sigma T)$ , where  $1/T$  is the signaling rate and  $\sigma$  is a positive real constant, is achieved as compared to the energy outside this interval. The constraint for intersymbol interference may easily be included in the problem. The bit error rate is computed as a function of the sampling time deviation  $\tau$ . For all cases, the other signal shapes demonstrate the definite superiority of our results. One of the important conclusions drawn from the results of the numerical example in Section 3, is that the optimal pulse shapes thus obtained also provide maximum immunity to timing offsets at the sampling instants, especially small rolloff factors. In determining the optimal pulse shape, we only used Djokovic and Vaidyanathan's periodically nonuniform sampling method with linear splines. One can perform the same scheme with different scaling functions, such as cubic splines. Another way is to use local averages method with different scaling functions and analysis filters, namely using Binomial QMF's which satisfy the perfect reconstruction. Using longer tap Binomial QMF's may result in better performance against the others.

#### V. REFERENCES

- [1] D. Slepian, H.O. Pollak, "Prolate spheroidal wave functions, Fourier analysis and uncertainty-I", *Bell Syst. Tech. J.*, Vol.40, pp. 43-63, 1961.
- [2] E.Panayirci, N. Tugbay, "Class of Optimum Signal Shapes in Data Transmission", *Proc. IEE*, Vol.135, Pt.F, No.3, pp.272-276, 1988.
- [3] E.Panayirci, N. Tugbay, " Optimum Design of Finite Duration Nyquist Signals", *Signal Processing*, Vol. 6, No. 1, pp.57-64, 1984.
- [4] E.Panayirci, N. Tugbay, "Energy Optimization of Band-limited Nyquist Signals in the Time Domain", *IEEE Trans. Commun.*, COM-35, pp.427-434, 1987.
- [5] A. N. Akansu and R.A. Haddad, *Multiresolution Signal — Decomposition*, Academic Press Inc., 1992.
- [6] I. Daubetchies, " Orthonormal Bases of Compactly Supported Wavelets", *Commun. Pure Apply. Math.*, vol.41, pp.909-996, Nov. 1989.

[7] I. Djokovic, P.P. Vaidyanathan, "Generalized Sampling Theorems in Multiresolution Subspaces", *Technical Report*, SP EDICS number. 2.4.4, 1994.

[8] M.R. Aaron, D.W Tufts, " Intersymbol Interference and Error Probability", *IEEE Trans. Inf. Theory*, IT-12, pp.26-34, 1966.

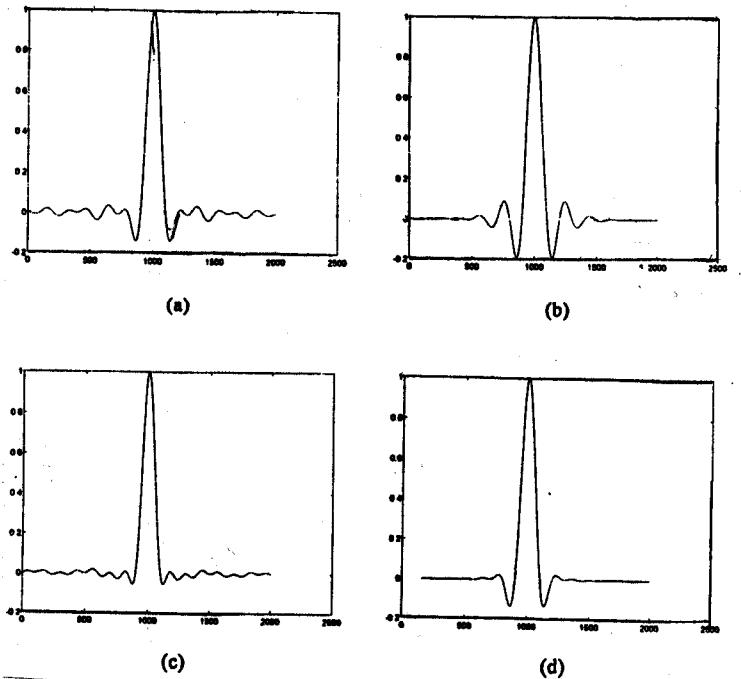


Fig. 2 The optimal signal shapes  $s(t)$  in  $t \in [-10, 10]$  for (a)  $\gamma = 0.25, \sigma = 1$  (b)  $\gamma = 0.5, \sigma = 1$ ; Raised-cosine in  $t \in [-10, 10]$  for (b)  $\gamma = 0.25$  (d)  $\gamma = 0.5$

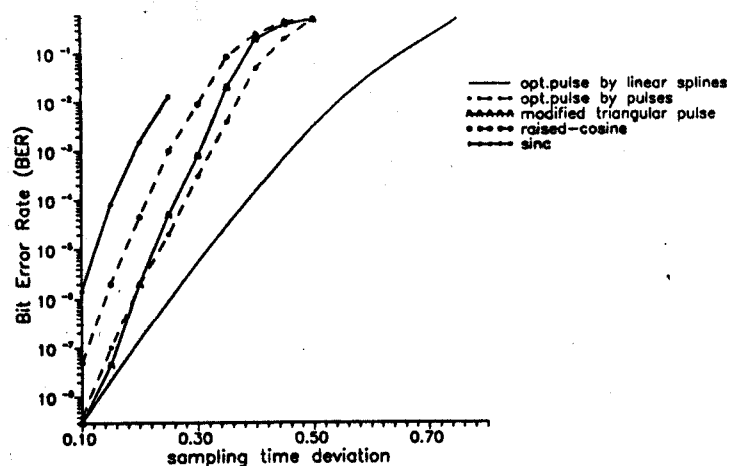


Fig. 3 BER against sampling time deviation, SNR= 16 dB,  $\gamma = 0.5, \sigma = 1$