Optimum Signal Design for Data Communications Based on Linear Splines

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Abstract- *a new method is given for the optimal design of band-limited Nyquist-type signal shapes for data communications, which maximizes its energy in a given time interval. The method is based on the periodically nonuniform sampling (PNS) theory making use of the linear splines. The PNS theory is an extention of the Shannon sampling theorem to the subspaces used in the multiresolution analysis of wavelet theory. The computation is straightforward and the constraint for intersymbol interference is shown to be easy to include in the problem. A numerical example is given and performance of the optimal signal shapes is compared with that resulting from the use of previously obtained signal shapes in the literature. It is also concluded that the optimal signal shapes thus obtained are almost immune to small offsets at the sampling instants*

I. Introduction

In digital transmission systems, it is possible to select signal shapes which produce no intersymbol interference when the signal is sampled properly by using Nyquist-I criterion. It is well known that the *sinc* pulse shape satisfies Nyquist's criterion with minimum possible bandwidth for a given data rate. However, it is not practical due to the infinite cutoff slope of its Fourier transform, to its slow time decay and its high sensitivity to errors in receiver's sampling time. For signals with a bandwidth greater than the half signaling rate, however, the criterion does not yield a unique signal shape. Our main objective in this paper is to design a band-limited signal shape, satisfying the Nyquist-I constraint, which maximizes the ratio of its energy E_i in the time interval $(-\sigma T, \sigma T)$ to its total energy E_o, where σ is any nonzero positive real number and T is the signaling interval. It is clear that maximization of the signal energy in the main lobe leads to substantial reductions in interference. Hence this new class of signals can be used advantageously for high-speed digital transmission systems. Without intersymbol interference constraint the problem was solved by Slepian and Pollack[1]. The optimum signal shape, in this case, is a member of the prolate spheroidal wave functions. A different version of this problem in which the optimal design of Nyquist-type signals of finite duration was investigated in [2]. The present problem has been solved previously by a completely analytical approach [3]. In [4], a simpler and numerically oriented method is presented to obtain the same signal shapes where the optimum signal shape was characterized by its frequency domain rolloff shape and the frequency spectrum of the signal is approximated by a sequence of rectangular pulses equally spaced in frequency.

Wavelet transform has become recently a cutting edge technology in signal analysis and signal compression areas [5]. Its links to multiresolution signal decomposition technique, namely subband filter banks in discrete domain with the continuous time functions, has been well established [6]. Wavelet transform has been introduced as a new way of looking at the sampling theorem. In this paper, we propose a new method for the solution of problem based on the periodically nonuniform sampling(PNS) theory studied by Djokovic and Vaidyannattan [7] which make use of the linear splines. The PNS theory is an extention of the Shannon sampling theorem to the subspaces used in the multiresolution analysis in wavelet theory. It is based on finding synthesizing functions in the Reproducing Kernel Hilbert space for given sample instants, which are compactly supported and polynomial expansion of scaling function. Based on PNS theory, any continuous function $s(t)$ can be characterized in terms of its samples taken at arbitrarily sampling instances, $\{t_n\}$, as follows.

$$s(t) = \sum_{\text{n even}} s(t_n)\Psi_0(t-nT) + \sum_{\text{n odd}} s(t_n)\Psi_1(t-nT) \quad (1)$$

where, $\Psi_0(t)$ and $\Psi_1(t)$ are called synthesizing functions. They are defined by $\Psi_0(t) = \phi(t+1) - \phi(t)$, $\Psi_1(t) = 2\phi(t)$,(see [8]), where, $\phi(t)$ is a scaling function.

In this paper a popular type of scaling function $\phi(t)$ having a compact support, namely a spline function of duration Δ will be employed in the following form:

$$\phi(f) = \begin{cases} \frac{2}{\Delta} f, & \text{for } 0 \le f < \Delta/2, \\ \frac{2}{\Delta}(\Delta - f), & \text{for } \Delta/2 \le f \le \Delta \\ 0, & \text{otherwise.} \end{cases} \quad (2)$$

The novelty with respect to that previous work on the same problem is that this new approach yields very fast and more accurate results and the signal shape so obtained can be easily implemented by recent new signal processing techniques. It is adaptive in the sense that it can be used without any modification for any source bit-rate merely by changing the clock signal. To demonstrate the validity of the results, the performance of the optimal signal shape is compared with that resulting from the use of popular