Performance Analysis of Trellis Coded MSK in Two-Ray Fading Channels

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ABSTRACT In this paper, the performance of trellis coded MSK systems transmitting over two-ray slow frequency-nonselective Ricean fading channel is investigated. An upper bound on the pairwise error probability is derived by means of the Chernoff bounding technique. The effects of the fading on both the amplitude and phase of the received signal is considered as well as the Doppler shift and time delay between direct and reflected components. The bit error performances of different MSK systems are evaluated both analytically and by means of computer simulation.

I – INTRODUCTION

In the signal transmission over satellite or airplane to earth station or ground vehicle, the channel is modeled by two-ray multipath slow frequency-nonselective Ricean fading. The noisy signal at the receiving antenna has two, direct and reflected components with different path lengths and phase angles [1]. MSK modulation is often an appropriate choice for this type of channels with its suppressed spectral sidelobes and constant envelope [2].

In this paper, error performance of uncoded and trellis coded MSK systems transmitting over two-ray fading mobile channels are investigated. Ideal interleaving / deinterleaving is assumed and no channel state information is available at the receiver. An analytical pairwise error probability upper bound is derived for trellis coded MSK using the Chernoff bounding technique which reflects the combined effects of the fading on both the amplitude and phase of the received signal, Doppler shift and time delay between two components as well as their power ratios. It is assumed any channel tracking technique is not used at the receiver.

II – THE SYSTEM MODEL

In the interval $kT \le t \le (k+1)T$, an MSK signal is defined as,

$$s(t) = \sqrt{\frac{2E_S}{T}} \cos(w_o t + a_k \frac{\pi\, t}{2T} + \phi_k) \qquad (1)$$

where w_0 is the carrier angular frequency, ϕ_k is the constant phase in the kth interval and $a_k \in \{-1,+1\}$. E_S and T denote the symbol energy and the symbol duration, respectively.

In the considered channel model, the link between the transmitter and the receiver consists of two components: the direct or coherent one exposed to a Doppler shift due to the vehicle mobility. Assuming the local oscillator frequency is derived from this dominant component, meanly taking the carrier angular frequency at the receiver equal to that of the received direct component. The direct component can be expressed as,

$$r_{d,k}(t) = \sqrt{\frac{2E_S}{T}} \cos(w_C t + a_k \frac{\pi\, t}{2T} + \phi_k) \quad . \qquad (2)$$

The reflected component has an additional Doppler shift Δw and a time delay t_d compared to the direct component. This component is also effected by a fading amplitude $\rho_k \in [0,\infty]$ and a uniformly distributed phase shift $\theta_k \in [0,2\pi]$ which are assumed constant over a signaling interval. The reflected component can be given as,

$$r_{r,k}(t) = \sqrt{\frac{2E_S}{T}} \rho_k \cos((w_C + \Delta w)(t - t_d) + \frac{a_k \pi}{2T}(t - t_d) + \theta_k + \phi_k) . \qquad (3)$$

Defining $\psi_k = \theta_k - (w_C + \Delta w)t_d - \frac{a_k \pi}{2T} t_d$, (3) can be rewritten as,

$$r_{r,k}(t) = \sqrt{\frac{2E_S}{T}} \rho_k \cos((w_C + \Delta w + \frac{a_k \pi}{2T})t + \psi_k + \phi_k) . \qquad (4)$$

Note that ψ_k is also uniformly distributed over an interval of size 2π .

The reflected component can be rewritten as,

$$\begin{aligned} r_{r,k}(t) &= \sqrt{\frac{2}{T}}\, R_{C,k} \cos(w_C + \Delta w + \frac{a_k \pi}{2T})t \\ &\quad - \sqrt{\frac{2}{T}}\, R_{S,k} \sin(w_C + \Delta w + \frac{a_k \pi}{2T})t \end{aligned} \qquad (5)$$

where $R_{c,k}$ and $R_{s,k}$ are defined as,

$$R_{C,k} \overset{\Delta}{=} \sqrt{E_S}(a_{R,k}\cos\phi_k - a_{I,k}\sin\phi_k) \tag{6}$$
$$R_{S,k} \overset{\Delta}{=} \sqrt{E_S}(a_{R,k}\sin\phi_k + a_{I,k}\cos\phi_k)$$

where,

$$a_{R,k} \overset{\Delta}{=} \rho_k \cos\phi_k \tag{7}$$
$$a_{I,k} \overset{\Delta}{=} \rho_k \sin\phi_k \quad .$$

$a_{R,k}$ and $a_{I,k}$ are two independent Gaussian random variables with zero mean and variance σ_f^2. Since the ratio of the power in direct component to that of the reflected component is $K = 1/E[\rho_k^2]$, where $E[.]$ denotes the expectation, σ_f^2 is related to K by $\sigma_f^2 = 1/(2+\pi/2)K$.

At the receiver, two orthonormal base functions are used to manage the coherent demodulation which are,

$$f_1(t) = \sqrt{\frac{2}{T}}\cos(w_C - \frac{\pi}{2T})t \tag{8}$$
$$f_2(t) = \sqrt{\frac{2}{T}}\cos(w_C + \frac{\pi}{2T})t \quad .$$

At the receiver, the correlator outputs are,

$$y_{i,k} = r_{r_i,k} + r_{d_i,k} + n_{i,k} \qquad i = 1,2 \tag{9}$$

where $r_{r_i,k}$ and $r_{d_i,k}$ are the correlator outputs due to the direct and the reflected components for the ith correlator, respectively. $n_{i,k}$'s are the zero mean independent Gaussian distributed random noise with variance σ_N^2.

The direct and reflected components at the corelator outputs can be given by the equation,

$$r_{d_i,k} = \int_{kT}^{(k+1)T} r_{d,k}(t)\, f_i(t)\, dt \tag{10}$$
$$r_{r_i,k} = \int_{kT}^{kT+t_d} r_{r,k-1}(t)\, f_i(t)\, dt + \int_{kT+t_d}^{(k+1)T} r_{r,k}(t)\, f_i(t)\, dt \quad .$$

After some simplifications, we obtain,

$$r_{r_1,k} = \alpha_3 R_{C,k-1} + \alpha_1 R_{C,k} - \alpha_4 R_{S,k-1} + \alpha_2 R_{S,k} \tag{11}$$
$$r_{r_2,k} = \alpha'_3 R_{C,k-1} + \alpha'_1 R_{C,k} - \alpha'_4 R_{S,k-1} + \alpha'_2 R_{S,k}$$

where,

$$\alpha_1 = \frac{\left[\sin(\Delta w + \frac{(a_k+1)\pi}{2T})T - \sin(\Delta w + \frac{(a_k+1)\pi}{2T})t_d\right]}{(\Delta w + \frac{(a_k+1)\pi}{2T})T}$$

$$\alpha_2 = \frac{\left[\cos(\Delta w + \frac{(a_k+1)\pi}{2T})T - \cos(\Delta w + \frac{(a_k+1)\pi}{2T})t_d\right]}{(\Delta w + \frac{(a_k+1)\pi}{2T})T}$$

$$\alpha_3 = \frac{\sin(\Delta w + \frac{(a_{k-1}+1)\pi}{2T})t_d}{(\Delta w + \frac{(a_{k-1}+1)\pi}{2T})T}$$

$$\alpha_4 = \frac{\left[1 - \cos(\Delta w + \frac{(a_{k-1}+1)\pi}{2T})t_d\right]}{(\Delta w + \frac{(a_{k-1}+1)\pi}{2T})T}$$

$$\alpha'_1 = \frac{\left[\sin(\Delta w + \frac{(a_k-1)\pi}{2T})T - \sin(\Delta w + \frac{(a_k-1)\pi}{2T})t_d\right]}{(\Delta w + \frac{(a_k-1)\pi}{2T})T}$$

$$\alpha'_2 = \frac{\left[\cos(\Delta w + \frac{(a_k-1)\pi}{2T})T - \cos(\Delta w + \frac{(a_k-1)\pi}{2T})t_d\right]}{(\Delta w + \frac{(a_k-1)\pi}{2T})T}$$

$$\alpha'_3 = \frac{\sin(\Delta w + \frac{(a_{k-1}-1)\pi}{2T})t_d}{(\Delta w + \frac{(a_{k-1}-1)\pi}{2T})T}$$

$$\alpha'_4 = \frac{\left[1 - \cos(\Delta w + \frac{(a_{k-1}-1)\pi}{2T})t_d\right]}{(\Delta w + \frac{(a_{k-1}-1)\pi}{2T})T} \quad .$$

The two correlator outputs due to the direct component are,

$$r_{d_1,k} = \frac{\sqrt{E_S}\sin((a_k+1)\frac{\pi}{2} + \phi_k)}{(a_k+1)\frac{\pi}{2}}$$
$$r_{d_2,k} = \frac{\sqrt{E_S}\sin((a_k-1)\frac{\pi}{2} + \phi_k)}{(a_k-1)\frac{\pi}{2}} \tag{12}$$

and the overall output of the correlators are,

$$\begin{aligned} y_{1,k} = r_{d_1,k} &+ (\sqrt{E_S}\alpha_1\cos\phi_k + \sqrt{E_S}\alpha_2\sin\phi_k)a_{R,k} \\ &+ (\sqrt{E_S}\alpha_2\cos\phi_k - \sqrt{E_S}\alpha_1\sin\phi_k)a_{I,k} \\ &+ (\sqrt{E_S}\alpha_3\cos\phi_{k-1} - \sqrt{E_S}\alpha_4\sin\phi_{k-1})a_{R,k-1} \\ &- (\sqrt{E_S}\alpha_4\cos\phi_{k-1} + \sqrt{E_S}\alpha_3\sin\phi_{k-1})a_{I,k-1} \\ &+ n_{1,k} \end{aligned} \tag{13}$$

$$y_{2,k} = r_{d_2,k} + (\sqrt{E_S}\alpha'_1 \cos\phi_k + \sqrt{E_S}\alpha'_2 \sin\phi_k) a_{R,k} + (\sqrt{E_S}\alpha'_2 \cos\phi_k - \sqrt{E_S}\alpha'_1 \sin\phi_k) a_{I,k} + (\sqrt{E_S}\alpha'_3 \cos\phi_{k-1} - \sqrt{E_S}\alpha'_4 \sin\phi_{k-1}) a_{R,k-1} - (\sqrt{E_S}\alpha'_4 \cos\phi_{k-1} + \sqrt{E_S}\alpha'_3 \sin\phi_{k-1}) a_{I,k-1} + n_{2,k} \quad . \tag{14}$$

The first terms in (13) and (14) are the deterministic signals and the other terms are a linear combination of Gaussian random variables with zero mean which is also a zero mean Gaussian random variable. Thus, it can easily be shown that,

$$E[(y_{1,k} - r_{d_1,k})(y_{2,k} - r_{d_2,k})] = 0 \quad . \tag{15}$$

Let's now apply the Chernoff bounding technique as was used in [1]. After some algebra, we obtain the analytical pairwise error probability upper bound for MSK in two-ray fading channel as,

$$P(\mathbf{s} \to \hat{\mathbf{s}}) \le \prod_k W^{|\mathbf{s}_k - \hat{\mathbf{s}}_k|^2} \tag{16}$$

where

$$W = \exp\{-1/8(E_S C^2 \sigma_f^2 + \sigma_N^2)\} \quad . \tag{17}$$

In (17), C^2 is defined as an expected value which is equal to,

$$C^2 = E[\alpha_1^2 + \alpha_2^2 + \alpha_3^2 + \alpha_4^2] = \frac{1+\cos(\Delta wT)}{(\Delta wT + \pi)^2} + \frac{1-\cos(\Delta wT)}{(\Delta wT)^2} \quad . \tag{19}$$

For given values of ΔwT and t_d, this upper bound decreases as the Euclidean distance between $\mathbf{s}$ and $\hat{\mathbf{s}}$ increases. From (16), it is seen that the free Euclidean distance is the primary performance criterion for two-ray fading channels as in the AWGN case.

III – PERFORMANCE ANALYSIS

The pairwise error probability upper bound obtained in (16) leads to the bit error probability upper bound when the transfer function bounding technique [4] is considered as in the AWGN case. The only difference compared to the AWGN case is replacing $W = \exp\{-1/4No\}$ by $W = \exp\{-1/8(E_S C^2 \sigma_f^2 + \sigma_N^2)\}$. Thus, the upper bound on the bit error probability is given by,

$$P_b \le \frac{1}{m} \left. \frac{\partial\, T(W,I)}{\partial\, I} \right|_{I=1, W=e^{-1/8(E_S C^2 \sigma_f^2 + \sigma_N^2)}} \tag{20}$$

where m is the number of data bits carried by one channel symbol and $T(W,I)$ is the transfer function of uncoded or trellis coded MSK scheme.

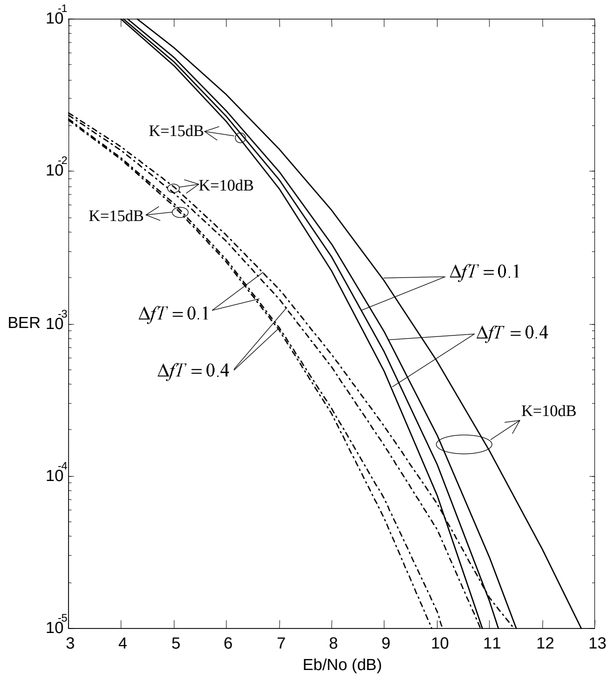

Figure 1. Bit error performance of serial MSK for different values of ΔfT (K=10 and K=15dB , no time delay, solid curves: upper bound, dashed curves: simulation)

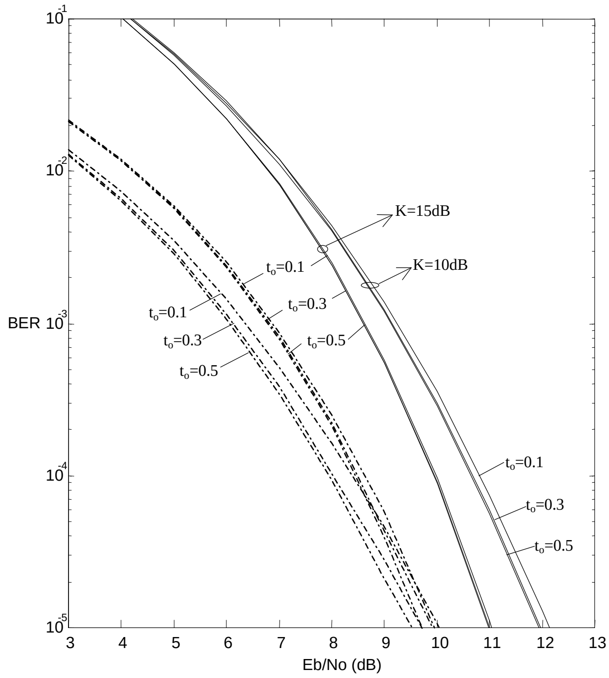

Figure 2. Bit error performance of serial MSK for different values of t_o (K=10 and K=15dB , solid curves: upper bound, dashed curves: simulation)

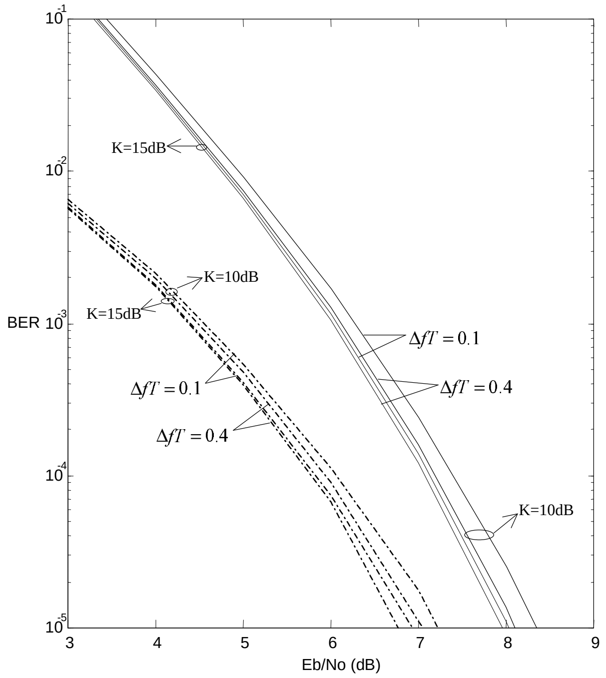

Figure 3. Bit error performance of trellis coded rate 1/2 MSK for different values of ΔfT (K=10 and K=15dB , no time delay, solid curves: upper bound, dashed curves: simulation)

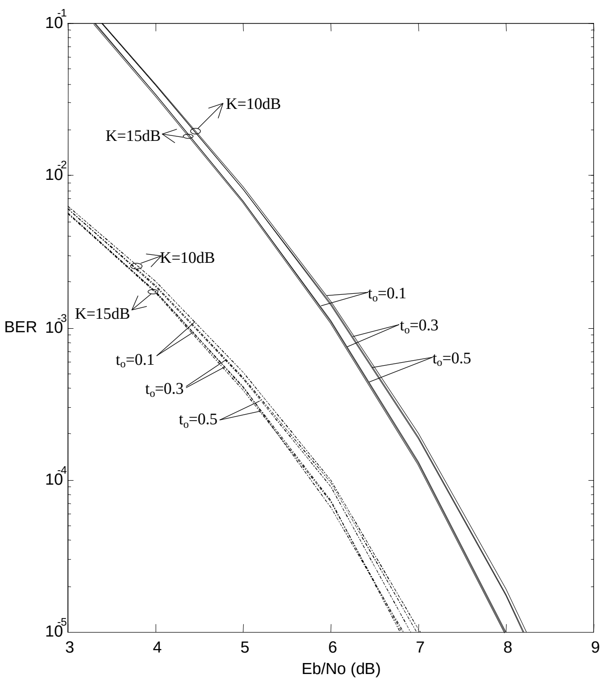

Figure 4. Bit error performance of trellis coded rate 1/2 MSK for different values of t_o (K=10 and K=15dB, solid curves: upper bound, dashed curves: simulation)

We evaluated the bit error performance of uncoded serial MSK given in [2] and trellis coded rate 1/2 MSK with generator polynomials in octal form [7,6], given in [3], in two-ray Ricean fading channels. Figs 1-4 show the bit error probability upper bound curves for different values of the time delay parameter $t_o = t_d / T$ and the Doppler shift ΔfT . Computer simulation results are also superimposed on these figures.

From these figures, we conclude that the bit error rate of the coded 1/2 rate MSK, as well as its sensitivity to the Doppler shift and time delay is lower than the uncoded MSK due to its increased free Euclidean distance.

On the other hand, the bit error probabilities for both of the considered MSK schemes decrease for the values of $t_o < 0.5$ and increases for $t_o > 0.5$.

IV – CONCLUSION

In this paper, we have evaluated the bit error performance of uncoded and coded 1/2 rate MSK systems transmitting over two-ray Ricean fading mobile channels. We have shown by deriving the pairwise error probability upper bound that the free Euclidean distance is the primary design criterion for this type of channels. The upper bound enables the bit error rate evaluation by the well-known transfer function technique.

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