DIGITAL QUADRATURE SECAM ENCODING ALGORITHM

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ABSTRACT

SECAM color signal encoding require high frequency domain processing. High frequency pre-emphasis is the most demanding due to the required group delay tolerance of the notch filter and signal frequency: 3.900-4.756 MHz. This work is dedicated to the design of low frequency complex SECAM digital processing algorithm. Thus high frequency signals can be processed by low speed DSP or PLD. The signal carrier to bandwidth ratio of SECAM chroma signal is rather small (~1), so the utilization of quadrature signals instead of the Hilbert transform signals require additional examination of the resulting errors. This is done in this work theoretically and confirmed by mathematical modeling.

The results are: theoretical RMS error functions of the generated signal; complex digital filter with optimized up to one bit length multiply coefficients; PAL/SECAM/NTSC digital encoder algorithm; program tools and mathematical models to explore the static, dynamic and noise characteristics of digital PAL/SECAM encoder and signal statistics.

1. INTRODUCTION

Regarding the sampling theorem [1] the digital signal processing (DSP) require sampling frequency twice as the highest signal spectrum frequency (Nyquist frequency). In real DSP systems this ratio is 3-10 to provide high fidelity of output signal and simplicity of the analog anti-alias filter for A/D and reconstruction filter for D/A transformation. So radio signal DSP require high sampling frequency and therefore higher production expenses.

In case of signal carrier-to-bandwidth ratio (CBR) ~1000 processing by envelop will make the same result at the significantly lower speed requirement. Envelop signal can be determined regarding Hilbert transformation. It can be shown that Hilbert transform of COS-signal is quadrature one (SIN) and vice versa [2]. SECAM signal CBR is close to one. But statistically the effective SECAM signal spectra width is more narrow. So with some assumption quadrature signal can be used for complex processing of the SECAM color signals which CBR is about one. But it is necessary to examine the error caused by this assumption.

2. QUADRATURE SECAM ENCODER

2.1 Complex signal generating algorithm

Let's take a look at the radio signal processing algorithm. Output signal can be presented by inverse Fourier transform

$$s_{out}(t) = \int_{-\infty}^{+\infty} S(j\omega) K(j\omega) \exp(j\omega t) d\omega , \qquad (1)$$

$S(j\omega)$ – frequency spectra (complex) of processed signal;

$K(j\omega)$ – frequency response (complex) of processing function.

After application of analytic signal

$$f_A(t)=f(t)+jf_Q(t),\ \ f_Q(t) = -\int_{-\infty}^{\infty} \frac{f(\tau)}{\tau - t} d\tau\ ,\ \ F_A(\omega) = 2F(\omega)\ ,$$

where $f_A(t)$ – analytic signal; $f_Q(t)$ – Hilbert transform of real $f(t)$ signal; $F_A(\omega)$ – Fourier transform of $f_A(t)$; $F(\omega)$ – frequency spectra of $f(t)$, and substitution $\omega = \omega_H + \Omega$, (1) will transform to analytic signal

$$z_{out}(t) \approx \exp(j\omega_H t) \int_{-\infty}^{+\infty} S_A(j\Omega) K_A(j\Omega) \exp(j\Omega t) d\Omega\ , \qquad (2)$$

$S_A(j\Omega)$ – Fourier transform of analytic signal; $K_A(j\Omega)$ – analytic frequency response of processing function.

Sign "≈" means possible error if processed signal has highest frequency higher then ω_H (wide band signal).

The radio output signal is a real part of (2) which one can get using the Fig. 1 processing algorithm.

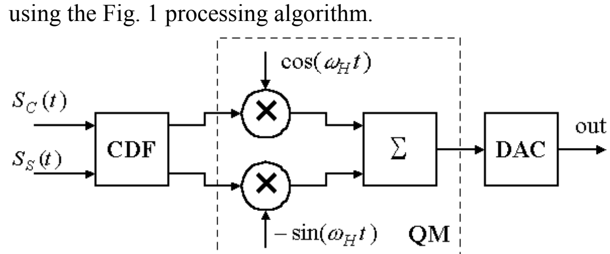

Figure 1. Complex algorithm of radio signal generation.

Here $s_c(t)$, $s_s(t)$ – quadrature low frequency components of signal under processing; *CDF* – complex digital filter; *QM* – quadrature modulator; *DAC* – digital-to-analog converter.

$$s_{out}(t) = A(t)\cos\{\omega_{sc} t + \varphi(t) + \varphi_0\},$$

$$s_{\substack{c\\s}}(t) = {\cos \atop \sin}\{(\omega_{sc} - \omega_H)t + \varphi(t) + \varphi_0\}. \qquad (3)$$

2.2 Digital quadrature modulator

The direct implementation of Fig. 1 QM unit requires two high speed digital multiplies and sum unit. This can be simplified by setting to zero φ_0 (this is possible for SECAM signals) and making sampling frequency four times the ω_H . So the odd and even samples are respectively:

$$s_{out}(n_{odd}) = \mp s_s^{out}(n_{odd}),\ s_{out}(n_{ev}) = \pm s_c^{out}(n_{ev}),$$

where $s_{c,s}^{out}$ – QM input (CDF output) signals. So the algorithm of quadrature modulator has simplified to the presented at Fig. 2.

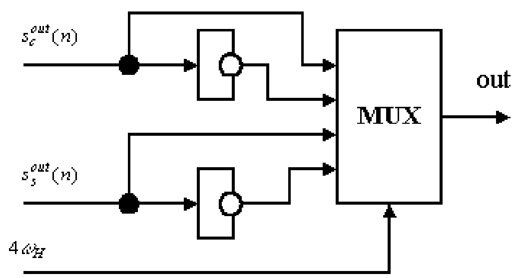

Figure 2. Quadrature digital modulator.

So two multipliers and sum unit are substituted by much simpler multiplexers.

2.3 Complex digital filter

SECAM high frequency pre-emphasis notch filter is determined by the following second order filter equation:

$$K(f) = \frac{1 + jmv}{1 + jwv}, \qquad (4)$$

where $v = f/f_0 - f_0/f$, $f_0 = \omega_0/2\pi$.

After applying to (4) the substitution $f = \frac{\omega}{2\pi} = \frac{\omega_H + \Omega}{2\pi}$; $\Omega = p/j$

Laplace transform yields

$$K(p) = K_0 \cdot \frac{p^2 + ap + c}{p^2 + bp + d} = K_C + jK_S, \qquad (5)$$

where $K_0 = m/w$; $a = \frac{2\pi \cdot f_0}{m} + j2\omega_H$;

$$c = (2\pi \cdot f_0)^2 - \omega_H{}^2 + j\frac{2\pi \cdot f_0}{m}\omega_H ;$$

b and *d* are equal to a and c respectively after the substitution *m* by *w*.

The bilinear transform of (5)

$$p = \gamma \cdot \frac{1 - z^{-1}}{1 + z^{-1}},\ \gamma = \Omega_r ctg(\frac{\Omega_r T}{2}),$$

where $z = \exp(j\omega T)$; Ω_r – reference frequency; T – input signal sampling period, yields the second order IIR digital filter

$$H(z) = H_0 \frac{1 + H_1 \cdot z^{-1} + H_3 \cdot z^{-2}}{1 + H_2 \cdot z^{-1} + H_4 \cdot z^{-2}}, \qquad (6)$$

where $H_0 = \frac{m}{w}\frac{M}{N}$; $M = \gamma^2 + a\gamma + c$, $N = \gamma^2 + b\gamma + d$;

$$H_1 = 2\frac{c - \gamma^2}{M},\quad H_2 = 2\frac{d - \gamma^2}{N};\quad H_3 = \frac{\gamma^2 - a\gamma + c}{M},$$

$$H_4 = \frac{\gamma^2 - b\gamma + d}{N}.$$

H_i , $i = 0...4$ – are complex digital filter coefficients.

The time-domain digital filter equation is defined by two functions:

$$y_c(n) = -H_2^c \cdot y_c(n-1) + H_2^s \cdot y_s(n-1) - H_4^c \cdot y_c(n-2) + H_4^s \cdot y_s(n-2) + \\ + x_c(n) + H_1^c \cdot x_c(n-1) - H_1^s \cdot x_s(n-1) + H_3^c \cdot x_c(n-2) - H_3^s \cdot x_s(n-2),$$

$$y_s(n) = -H_2^s \cdot y_c(n-1) - H_2^c \cdot y_s(n-1) - H_4^s \cdot y_c(n-2) - H_4^c \cdot y_s(n-2) + \\ + x_s(n) + H_1^s \cdot x_c(n-1) + H_1^c \cdot x_s(n-1) + H_3^s \cdot x_c(n-2) + H_3^c \cdot x_s(n-2),$$

Here $x_{c,s}(n)$, $y_{c,s}(n)$ – in-phase (and quadrature) input (and output) signals respectively.

Regarding the above equations the digital filter structure is presented at Fig. 3.

One of the most important SECAM high frequency pre-emphasis filter characteristic is group delay. It is stated by the formula

$$\tau(\omega) = -\frac{d\phi(\omega)}{d\omega} \qquad (7)$$

Here $\phi(\omega)$ – phase of (6). The calculation of (7) yields the function bellow.

$$\tau(F) = 1/f_T \times$$

$$\times \left[\frac{K_1 \cdot \sin\left(2\cdot\pi\cdot\frac{F}{f_T}\right) - K_3 \cdot \cos\left(2\cdot\pi\cdot\frac{F}{f_T}\right)^2 + P_1(F)\cdot\cos\left(2\cdot\pi\cdot\frac{F}{f_T}\right) + K_4}{K_2 \cdot \sin\left(2\cdot\pi\cdot\frac{F}{f_T}\right) + K_3 \cdot \cos\left(2\cdot\pi\cdot\frac{F}{f_T}\right)^2 + P_2(F)\cdot\cos\left(2\cdot\pi\cdot\frac{F}{f_T}\right) + K_5} - \right.$$

$$\left. - \frac{K_6 \cdot \sin\left(2\cdot\pi\cdot\frac{F}{f_T}\right) - K_8 \cdot \cos\left(2\cdot\pi\cdot\frac{F}{f_T}\right)^2 + P_3(F)\cdot\cos\left(2\cdot\pi\cdot\frac{F}{f_T}\right) + K_9}{K_7 \cdot \sin\left(2\cdot\pi\cdot\frac{F}{f_T}\right) + K_8 \cdot \cos\left(2\cdot\pi\cdot\frac{F}{f_T}\right)^2 + P_4(F)\cdot\cos\left(2\cdot\pi\cdot\frac{F}{f_T}\right) + K_{10}} \right]$$

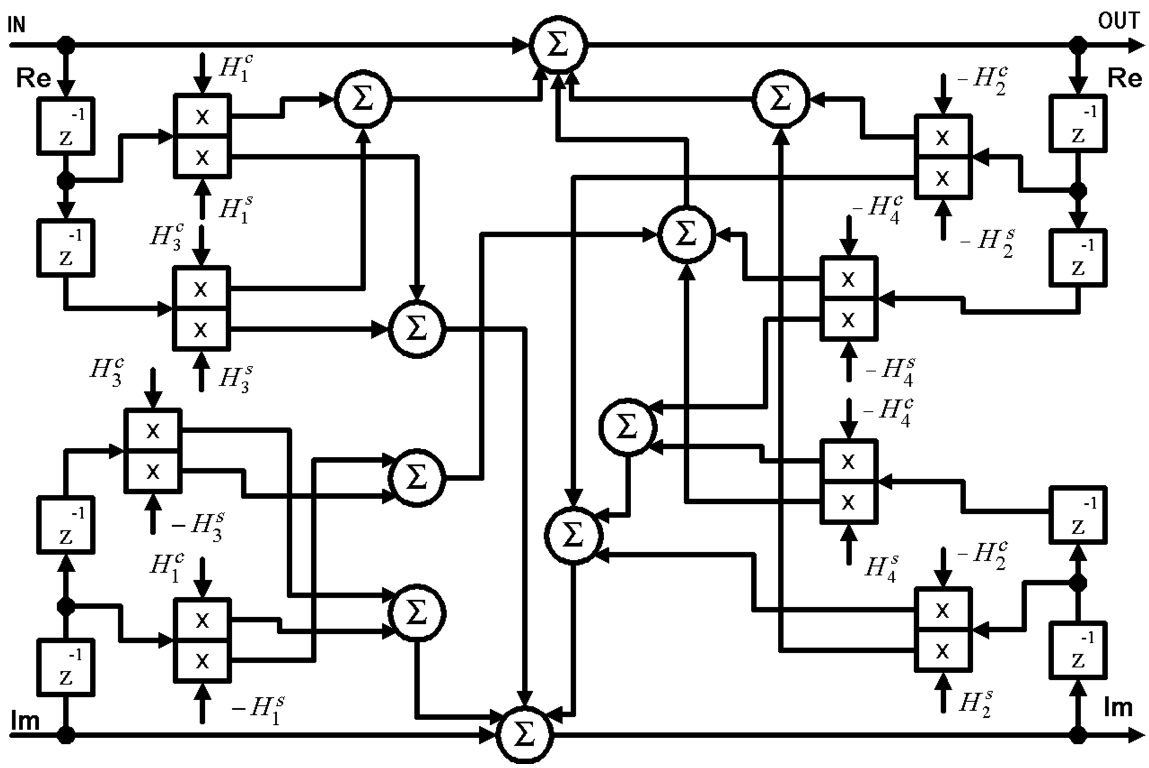

Figure 3. Complex digital filter structure.

Here $F = \frac{\Omega}{2\pi}$; f_T - signal sampling frequency;

$K_1 = -a_c \cdot c_s - 3 \cdot a_s + a_s \cdot c_c$;

$K_2 = 2 \cdot a_c \cdot c_s + 2 \cdot a_s - 2 \cdot a_s \cdot c_c$;

$K_3 = 4 \cdot c_c$; $K_4 = -2 - a_s^2 + 2 \cdot c_c - a_c^2$;

$K_5 = 1 + c_c^2 + c_s^2 + a_s^2 - 2 \cdot c_c + a_c^2$;

$K_6 = -b_c \cdot d_s - 3 \cdot b_s + b_s \cdot d_c$;

$K_7 = 2 \cdot b_c \cdot d_s + 2 \cdot b_s - 2 \cdot b_s \cdot d_c$

$K_8 = 4 \cdot d_c$; $K_9 = -2 - b_s^2 + 2 \cdot d_c - b_c^2$;

$K_{10} = 1 + d_c^2 + d_s^2 + b_s^2 - 2 \cdot d_c + b_c^2$;

$C_1 = -c_s \cdot a_s - 3 \cdot a_c - a_c \cdot c_c$;

$C_2 = 2 \cdot a_c + 2 \cdot a_c \cdot c_c + 2 \cdot c_s \cdot a_s$;

$D_1 = -b_s \cdot d_s - 3 \cdot b_c - b_c \cdot d_c$;

$D_2 = 2 \cdot b_c + 2 \cdot b_c \cdot d_c + 2 \cdot b_s \cdot d_s$;

$P_1(F) = C_1 - 4 \cdot c_s \cdot \sin\left(2 \cdot \pi \cdot \frac{F}{f_T}\right)$;

$P_2(F) = C_2 + 4 \cdot c_s \cdot \sin\left(2 \cdot \pi \cdot \frac{F}{f_T}\right)$;

$P_3(F) = D_1 - 4 \cdot d_s \cdot \sin\left(2 \cdot \pi \cdot \frac{F}{f_T}\right)$;

$P_4(F) = D_2 + 4 \cdot d_s \cdot \sin\left(2 \cdot \pi \cdot \frac{F}{f_T}\right)$;

$x_c = \mathrm{Re}(x)$, $x_s = \mathrm{Im}(x)$, here x=a, b, c, d.

2.4 Estimation of signal generation error

Digital signal processing causes quantization noise [3]. Complex processing makes it 1.167 as large [4] due to quadrature processing. Regarding the residue theorem

$$\oint_C F(z)dz = 2\pi j \sum_k \mathrm{Res}\left[F(z), p_k\right], \qquad (8)$$

and theorem of the infinite series

$$\sum_{m=0}^{\infty} h_i^2(m) = \frac{1}{2\pi j} \oint H_i(z) H_i(z^{-1}) z^{-1} dz, \qquad (9)$$

here h_i – samples of digital filter H_i impulse response (path from noise source to the filter output),

we can find the noise variance:

$$\sigma_0^2 = 1.167\sigma^2 \sum_m \mathrm{Res}\left[H(z)H(z^{-1})z^{-1}, p_m\right] \qquad (10)$$

here $\sigma^2 = 2^{-2b}/12$ – variance of each noise source (assumption is made of the equal quantisation bit number b), b – number of bits for input signal coding; m – number of function poles inside unit circle of the Argand diagram.

The residue for Figure 3 CDF (throw path "in – out") can be calculated by the following expressions:

$$\mathrm{Res}\, f(z) = \frac{\left(z^2 + H_1 \cdot z + H_3\right)\cdot\left(1 + H_1 \cdot z + H_3 \cdot z^2\right)}{\left(5 \cdot H_4 \cdot z^4 + C_1 \cdot z^3 + C_2 \cdot z^2 + C_3 \cdot z + H_4\right)} \qquad (11)$$

here $C_1=(4H_4H_2+4H_2)$, $C_2=(3H_4^2+3H_2^2+3)$, $C_3=(2H_4H_2+2H_2)$;

H_i, i =1…4 – CDF coefficients.

Applying the same procedure to the calculation of the noise coursed by digital filter multiplication results rounding one can get the following expression (for simplicity it is assumed the equal number of bits for all multiplication results presentation):

$$\sigma_0^2 = 8\frac{2^{-2b}}{12}(I_{DIR} + I_{CR}) = \frac{2^{-2b+1}}{3}(I_{DIR} + I_{CR}) \qquad (12)$$

where I_{DIR}, I_{CR} – integral calculations of CDF trannsfer function for direct [real (im) in – real (im) out] and cross [real (im) in – im (real) out] paths. The full expression and it's deriviation the reader can find in [5].

2.5 Modeling

The examined complex processing system and analytical results of noise estimation were tested in Matlab 5, Simulink II and MathCAD 7. Mathematical models, programs and test procedures were designed for this purpose. The test calculations were made for SECAM high-frequency pre-emphases band-stop filter (m=16, w=1.26, f_0=4.286 MHz) in *3900-4756* kHz band.

The modeling showed small difference of group delay (less then 20 nsec, Figure 4) and magnitude-frequency response (less then 1 dB) of CDF system with respect to analog prototype system. The digital filter recursive coefficients H_2, H_4 were rounded up to one significant (not zero) bits. The noise estimation was made by comparison of two systems – ideal and with respected quantisation. After statistical processing the estimation of the noise standard deviation was found. The difference of theoretical and model signal-to-noise ratio (SNR) was small (~1.4 dB, SNR=71 dB).

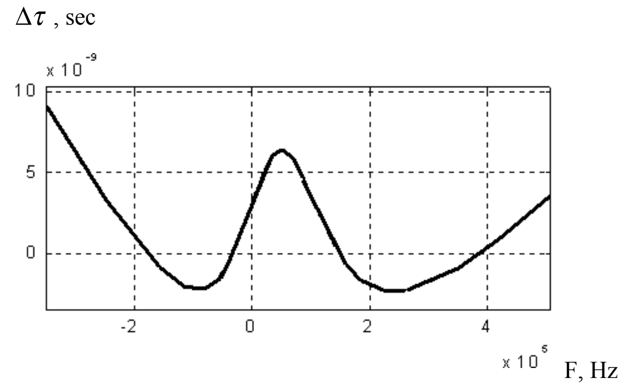

Figure 4. Difference of group delay between prototype filter and CDF vs. carrier deviation (f-f_0).

3. SUMMARY

The algorithm of the quadrature SECAM chroma signal generation allows making unified digital PAL/SECAM/NTSC encoder. Thus improving the quality and simplicity of the standard television signal generation at the transitional period to digital television. It is proven the possibility to generate and process digital signals even in case of non-symmetrical frequency pre-emphasis requirements by the low speed DSP. The goal is reducing main digital hardware speed requirements that is factor equal to carrier-to-signal bandwidth ratio. The minimization of filter coefficients quantisation bit number is especially attractive for PLD/FPGA implementation of encoder.

The received in this work digital noise deviation equations allow to calculate without modeling the resulting SNR and hardware requirements. The designed software tools can be used to explore the characteristics of complex digital filtering systems. The algorithm of complex signal processing can be applied for different types of processed signal.

4. REFERENCES

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