GENERALIZED PSPICE AND SIMULINK MODELS FOR THE CONTINIOUS-TIME SIMULATIONS OF ASTABLE CELLULAR NEURAL NETWORKS

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ABSTRACT

The Cellular Neural Network (CNN) structure is a class of dynamic neural networks and is firstly proposed by L.O. Chua in 1988. It finds very attractive applications in many fields – especially in image processing. In this paper, generalized continious time models of CNNs for PSPICE *and* SIMULINK *are proposed and then with these models, as numerical examples, an oscillator and a chaotic signal generator which based on astable biased CNNs are simulated. The results are verified with [3],[4]. The mentioned models can be used in analyzing the transient behaviours and in the educational demonstrations of CNNs.*

1. CELLULAR NEURAL NETWORKS

CNN is a class of dynamic neural networks [1]. Unlike the Hopfield Network, in this structure a cell can only interact directly with its neighbours having a neighbourhood relationship given in (1).

$$N_r(i,j)=\{C(k,l) \mid \max\{|k-i|,|l-j|\}\le r, 1\le k\le M, 1\le l\le N\} \tag{1}$$

The equivalent circuit of a cell in a CNN structure is shown in figure - 1.

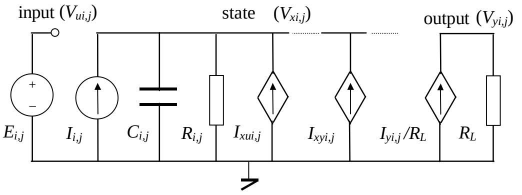


Figure – 1: *Equivalent circuit of a CNN cell.*

Where the I_{xy} and I_{xu} dependent current sources are controlled by the outputs and inputs of the $C(k,l)\in N_r(i,j)$ neighbours of the cell respectively. The voltage controlled current source I_y – whose equation will be given in the next section – is a nonlinear (piecewise - linear) function of the $v_{xi,j}$ state and can be realized with a simple op-amp circuit.

2. CIRCUIT EQUATIONS OF THE CNN

In order to derive the state equation of a cell, we apply the KCL to $v_{xi,j}$ node, therefore, equation below will be found:

$$0=-I_{i,j}+C_{i,j}\frac{dv_{xi,j}}{dt}+\frac{1}{R}v_{xi,j}-\sum_{C(k,l)\in N_r(i,j)}A(i,j;k,l)\cdot v_{yk,l}-\sum_{C(k,l)\in N_r(i,j)}B(i,j;k,l)\cdot v_{uk,l} \tag{2}$$

Where **A** and **B** are weight coefficients. The activation function shown in figure – 2 can be represented by the output equation given in (3).

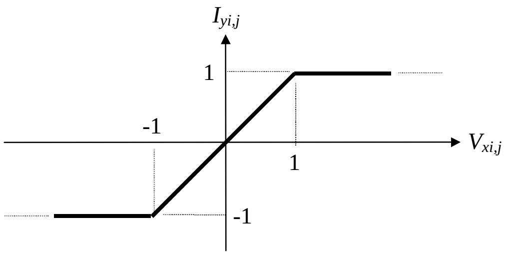


Figure – 2: *Activation function (piecewise-linear func.) of a CNN cell.*

$$v_{yi,j}=y(v_{xi,j})=\frac{1}{2}\left[|v_{xi,j}+1|-|v_{xi,j}-1|\right] \tag{3}$$

If an *MxN* CNN structure including *NM* cells is considered, we can re-write the whole state equations in the matrix form of:

$$\dot{\mathbf{x}} = -\mathbf{x} + \mathbf{A} * y(\mathbf{x}) + \mathbf{B} * \mathbf{u} + \mathbf{I} \tag{4}$$

where (*) is the two-dimensional convolution operator. **"A"** is called *cloning template*, **"B"** is called *control template* and **"I"** is called *threshold*.

3. PSPICE AND SIMULINK MODELS

For PSPICE simulation the generalized cell equivalent of a CNN shown in figure – 3 is proposed. For realization of the activation function in (3) the relationship between the R_1, R_2, R_3 and R_4 resistors in the model must satisfy the following relationship [2]:

$$R_4 = R_1,\ R_3 = R_2 \text{ and } 1 + R_2/R_1 = V_{cc} \tag{5}$$

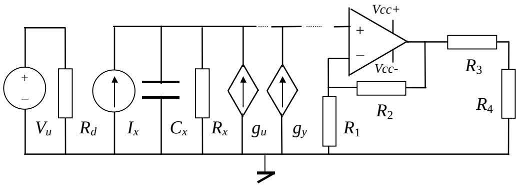


Figure – 3: *Proposed generalized cell equivalent for* PSPICE *simulation.*

SIMULINK is an effective graphical tool used for modeling both linear and nonlinear dynamic systems. With this thought in mind, a CNN cell can also be simulated by this program. The SIMULINK continious-time model of a cell is depicted in figure – 4.

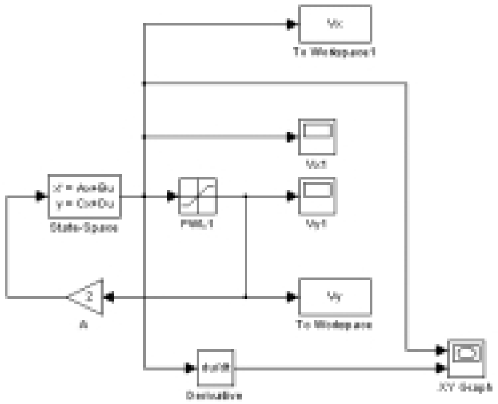


Figure – 4: *Proposed cell equivalent for* SIMULINK *simulation.*

Assuming we have a dynamical system whose state equation is:

$$\dot{\mathbf{X}} = -\mathbf{S} \cdot \mathbf{X} + \mathbf{A} \cdot \mathbf{Y} \tag{6}$$

Where **X** is the state variable vector, **Y** is the output vector and **A**, **S** are coefficient matrices. Substituting (2) in (6), we will have the following state equations of the whole system of dimension 1x3:

$$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} \frac{1}{R_1C_1} & \frac{-s_{12}}{C_1} & \frac{-s_{13}}{C_1} \\ \frac{-s_{21}}{C_2} & \frac{1}{R_2C_2} & \frac{-s_{23}}{C_2} \\ \frac{-s_{31}}{C_3} & \frac{-s_{32}}{C_3} & \frac{1}{R_3C_3} \end{bmatrix} \cdot \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} + \begin{bmatrix} \frac{A_{11}}{C_1} & \frac{A_{12}}{C_1} & \frac{A_{13}}{C_1} \\ \frac{A_{21}}{C_2} & \frac{A_{22}}{C_2} & \frac{A_{23}}{C_2} \\ \frac{A_{31}}{C_3} & \frac{A_{32}}{C_3} & \frac{A_{33}}{C_3} \end{bmatrix} \cdot \begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} \tag{7}$$

Any dynamical system with the state equations above can be modeled by using the generalized PSPICE and SIMULINK equivalents of the CNN cell circuit.

4. NUMERICAL EXAMPLES

In this section, two astable CNNs are simulated with the proposed PSPICE and SIMULINK models. The first one is a 1x2 structure which makes an oscillation. The second one is a chaotic signal generator based on a 1x3 CNN.

Firstly, with the structure below we can generate oscillations by using the astable biased CNNs [3]:

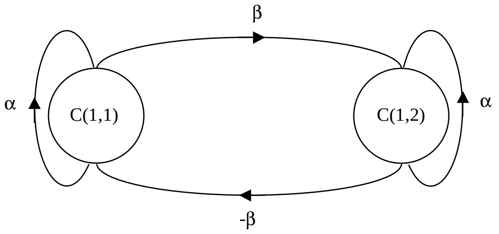
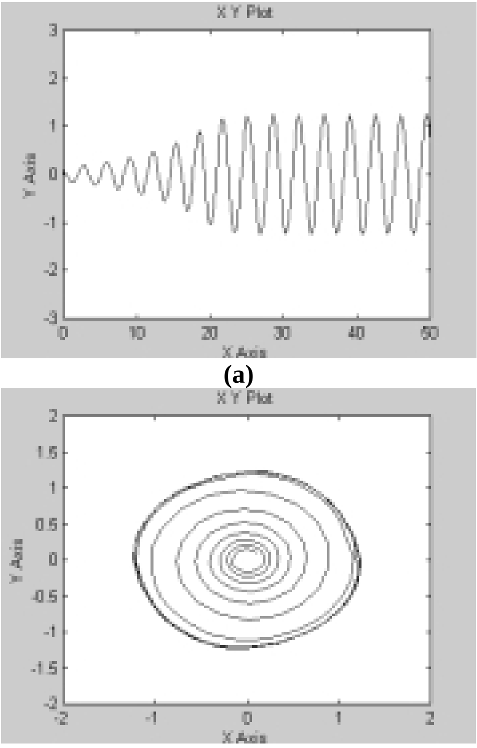


Figure – 5: *An oscillator based on a 1x2 CNN.*

This structure called “CNN with Opposite Sign Templates” can be described by the differential equation system in (8):

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\end{bmatrix}=-\begin{bmatrix}1&0\\0&1\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\end{bmatrix}+\begin{bmatrix}\alpha&-\beta\\ \beta&\alpha\end{bmatrix}\cdot\begin{bmatrix}f(x_1)\\f(x_2)\end{bmatrix}\qquad(8)$$

If the parameters are chosen as α=1.1, β=2 and with the initial conditions $x_1(0)$=0.1 and $x_2(0)$=0.1 the results in figure – 6 are obtained.



(a)

(b)

Figure – 6: *Simulation results for the CNN given in Figure – 5:* ***(a)*** *(t,v_{x1}) plane;* ***(b)*** *(v_{x1},v_{x2}) plane.*

As shown in figure – 6(b), the phase shift between the two states of the cells is approximately 90^o. It is also observed that, α determines the magnitudes and β determines the frequencies of the states.

Secondly, if the following set of differential equations which can be realized with the CNN shown in figure – 7 are simulated, it will be seen that the states will act as chaotic signal generators [4].

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\end{bmatrix}=-\begin{bmatrix}1&0&0\\0&1&0\\0&0&1\end{bmatrix}\cdot\begin{bmatrix}x_1\\x_2\\x_3\end{bmatrix}+\begin{bmatrix}1.25&-3.2&-3.2\\-3.2&1.1&-4.4\\-3.2&4.4&1\end{bmatrix}\cdot\begin{bmatrix}y_1\\y_2\\y_3\end{bmatrix}\quad(9)$$

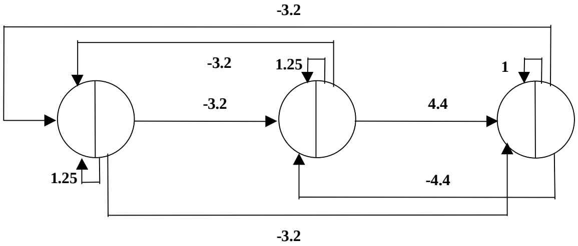


Figure – 7: *CNN based chaotic signal generator.*

The “Double Attractor” is observed from the simulation results is shown in figure – 8.

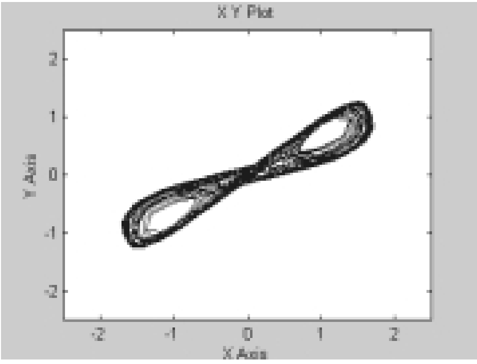


Figure – 8: *“Double Attractor” observed from the system in Figure – 7. Projection onto (v_{x1}, v_{x2}) plane.*

5. CONCLUSIONS

In this paper, Cellular Neural Networks are analyzed from system theory point of view and a generalized PSPICE and a SIMULINK model is proposed for the simulations of astable CNNs. With these models, an oscillator and a chaotic signal generator are simulated and the results are verified with the ones that were found in the literature. These models can be used in analyzing the transients and stabilities of the small dimensional CNNs, as well as for educational demonstrations. Specifically, the model for the chaotic CNNs can be used in “Chaotic Masking” applications for secure communications [5].

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