

A¹ SYSTEMATIC PROCEDURE TO MODEL MEASURED DATA OBTAINED FROM A PASSIVE PHYSICAL DEVICE BY MEANS OF ITS DARLINGTON EQUIVALENT

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Abstract

A Systematic procedure is presented to model measured data, obtained from an actual passive one port by means of its Darlington equivalent. In other words, measured data is modelled as a lossless two port in unit termination. Proposed method will find practical applications in determining physical behaviour of one port devices particularly for power transfer which is essential for designing communication systems at microwave/millimeter wave frequencies.

1. Introduction

One of the major issues in designing high frequency communication systems is to determine the physical limitations of commercially available devices for power transfer. The one port device is regarded as a dissipative complex load. The load may be described as either a measured immittance data or a reflection coefficient over the frequencies of interest. Precise theoretical power transfer limitations of the load may be determined by accessing to the Analytic Gain Bandwidth Theory [1-5]. In this case, circuit model of the load is essential. There may be some other valid reasons to come up with a circuit model of the measured data obtained from a passive one port terminal to observe the physical behaviour of the systems under consideration. Under any circumstances, herein this work, the practical problem is defined as *"to model the given data as a lossless two port in resistive termination which is called the Darlington equivalent of the physical device"*(figure 1).

The common exercise to model the measured data starts with the choice of an appropriate circuit topology. Then, the element values of the chosen topology are determined to best fit the data by means

of an optimisation program. Although this trial is straight forward, it presents serious difficulties. First, the optimisation is heavily non-linear in terms of the element values that may result in local minimas or may not converge at all. Secondly, there is no established process to initialise the element values of the chosen circuit topology. Worst of all **"the optimum choice of the circuit topology which best describes the physical device is in question"**.

Fortunately, these problems are overcome employing the modelling technique introduced in this work. In the next sections, we present the proposed technique to model physical devices using lumped elements. However, the techniques can easily be extended to handle models with mixed elements (lumped and distributed elements) employing the realisable, two variable, driving point network functions representations [6].

2. An Immitance Based Model Of A Physical Device

In this approach, measured data of the physical device is taken as a positive real immittance function $F(j\omega_i) = R(\omega_i) + jX(\omega_i)$ over the frequencies ω_i , where the subscript i designates the index of the test frequencies. $F(j\omega)$ could either be an impedance $Z(j\omega)$ or an admittance $Y(j\omega)$.

In general, any positive real function $F(s)$ can be written in terms of its minimum and the foster part functions;

$$F(s) = F_m(s) + F_f(s) \quad (1)$$

where $s = \sigma + j\omega$ is the usual complex domain variable,

$F_m(s)$ is the minimum part of $F(s)$, which is free of $j\omega$ poles,

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$F_f(s)$ is the Foster part of $F(s)$, which only includes $j\omega$ poles.

On the $j\omega$ axis,

$$F(j\omega) = R(\omega) + jX(\omega) \quad (2a)$$

$$F_m(j\omega) = R_m(\omega) + jX_m(\omega) \quad (2b)$$

$$F_f(j\omega) = jX_f(\omega) \quad (2c)$$

In the above representation, it is clear that

$$R(\omega) = R_m(\omega) \quad (3a)$$

$$X(\omega) = X_m(\omega) + X_f(\omega) \quad (3b)$$

Since $F_m(s)$ is a minimum function which contains no poles on the $j\omega$ axis, its imaginary part $X_m(\omega)$ is related to the real part $R_m(\omega)$ by Hilbert transformation relation;

$$X_m(\omega) = H\{R(\omega)\} \quad (3c)$$

where $H\{\cdot\}$ designates the Hilbert Transformation Operation (HTO). That is

$$X_m(\omega) = (1/\pi) \int_{-\infty}^{+\infty} \{R(y)/[\omega - y]\} dy \quad (4)$$

In the immittance based modelling technique, the crux of the idea is to decompose the measured data into its minimum $F_m(j\omega) = R_m(\omega) + jX_m(\omega)$ and foster $F_f(j\omega) = jX_f(\omega)$ parts. Hence, the modelling process is carried out within two major steps; models for the minimum and the foster parts (Figure 2-3).

In order to extract the foster part $F_f(j\omega) = jX_f(\omega)$ from the original measured data, one has to generate $X_m(\omega)$ using the Hilbert transformation relation of (4).

Eventually, realisable analytical forms for the minimum immittance function $F_m(j\omega) = R_m(\omega) + jX_m(\omega)$ and the foster function $X_f(\omega) = X(\omega) - X_m(\omega)$ are obtained by means of an appropriate curve fitting algorithms and they are synthesised to yield the desired device model under consideration.

Based on the above explanation, the immittance technique to model the given physical device can be summarised in the following algorithm.

Immittance Base Device Modelling Algorithm:

Step 1:

Measure the physical device characteristics and prepare the driving point immittance table $F(j\omega_i) = R(\omega_i) + jX(\omega_i)$ over the test frequencies ω_i , $i=1,2,3,\dots,N$. The integer N designates the total number of sampling frequencies over which the measurements are made.

Step 2:

Using (4), generate the imaginary part $X_m(\omega)$ of $F_m(j\omega) = R_m(\omega) + jX_m(\omega)$

Step 3a:

Find the realisable mathematical form for $R(\omega_i)$ as simple as possible. General form of $R(\omega)$ may be chosen as

$$R(\omega) = N(\omega)/D(\omega) \\ = [A_0 + A_1\omega^2 + A_2\omega^4 + \dots + A_q\omega^{2q}] / [B_0 + B_1\omega^2 + B_2\omega^4 + \dots + B_n\omega^{2n}] \quad (5)$$

where n designates the total number of elements in the minimum part of the circuit model, q is the total number of transmission zeros such that $q \leq n$.

In this representation, q and n are related to the complexity of the circuit model associated with the minimum part of the driving point immittance. In many practical cases, the minimum part of the circuit model may be chosen as a lossless ladder with transmission zeros at DC, infinity, and may be at finite frequencies or any combination of these. No matter where the transmission zeros are, $R(\omega)$ must always be non-negative. Therefore, coefficients A_i and B_i must be determined accordingly.

Step 3b:

Once $R(\omega)$ is determined, generate the realisable analytical form of the minimum function $F_m(s)$. In this step, Bode's or Gewertz's Procedure can be employed [1,7]. In order to ease the synthesis, the Gewertz's technique may be preferred over the others which yields the following form of $F_m(s)$.

$$F_m(s) = n(s)/d(s) \\ = [a_0 + a_1s + a_2s^2 + \dots + a_{n-1}s^{n-1}] / [b_0 + b_1s + \dots + b_ns^n] \quad (6)$$

Step 4:

Using (3b), obtain the foster part of $F(j\omega_i)$ as

$$X_f(\omega_i) = X(\omega_i) - X_m(\omega_i) \quad (7)$$

Step 5:

Find the mathematical realisable foster form for the data $X_f(\omega_i)$ as

$$X_f(\omega_i) = k_\infty \omega_i - k_0/\omega_i + \sum_{r=1}^p \{k_r \omega_i / \omega_i^2 - \omega_r^2\} \quad (8)$$

where the integer p is the total number of poles of the foster function $X_f(\omega_i)$. It should be noted that in the above representation, the residues' k_∞ , k_0 , and k_r 's must be all non-negative.

Once the mathematical form of $X_f(\omega)$ is obtained, it is straight forward to end up with the analytical, realisable form of $F_f(s)$;

$$F_f(s) = k_\infty s + k_0/s + \sum_{r=1}^p \{k_r s / s^2 + \omega_r^2\} \quad (9)$$

Step 6:

Synthesise the immittance function $F(s) = F_m(s) + F_f(s)$ as a lossless two-port in unit termination which in turn yields the desired model of the physical device.

In order to ease the implementation of the above algorithm, one should pay attention to the following remarks.

Remarks:

The either algorithm described above could be impedance based or admittance based. The choice may be arbitrary. However, one should bear in mind that, the structural model would differ with the initial choice. If the impedance based approach is chosen, the model will start with a series arm which includes all the $j\omega$ poles in series with the minimum reactance driving point input impedance (figure 2). If the one starts with admittance data, then, the model will include a shunt arm, which contains all the $j\omega$ poles of the admittance, in parallel with the minimum susceptance

driving point input admittance (figure 3). Practical choices can be made according to the physical behaviour of the device. Certainly, numeric of the problem is also important for the initial choice.

Evaluation of the Hilbert Transform Integral of (4) may create some problems.

A simple numerical procedure may be to consider $R(\omega)$ as the linear combination of line segments which connect the measured points $R_i = R(\omega_i)$ on the real part curve. With this representation of $R_m(\omega)$, $X_m(\omega)$ is also expressed in terms of the linear combination of the same so called the real part break points $R_i = R(\omega_i)$ as described in [8].

In Step 3, one simply utilises a curve fitting algorithm, which in turn, yields the coefficients of A_i and B_i of (5). Depending on the given data, there may be several practical forms for (5). For many cases, $N(\omega)$ is given by

$$N(\omega) = \omega^{2k} \quad (10)$$

In (10), $k=0$ chosen, if data for $R(\omega)$ starts at a reasonably large fixed value, $k>0$ chosen, if $R(\omega)$ tends to go zero at the lower end of the frequency band of measurement. The value of k depends on the roll-off speed of the measured data at the lower frequency band.

At the conference, implementation of the above algorithm will be exhibited with several, practical examples.

4. References:

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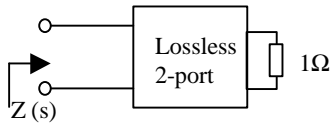


Figure 1: Darlington Representation of an impedance function $Z(s)$

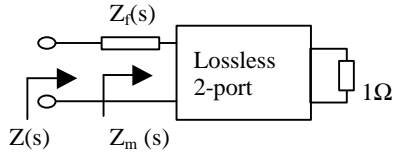


Figure 2:Extraction of the foster component $Z_f(s)$ from the impedance function $Z(s)=Z_m(s)+Z_f(s)$

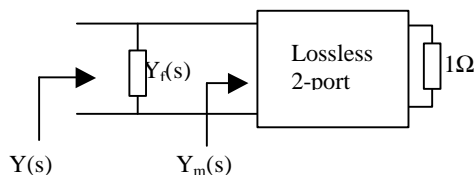


Figure 3: Extraction of the foster component $Y_f(s)$ from the admittance function $Y(s)=Y_m(s)+Y_f(s)$