A NON-LINEAR ALGORITHM FOR DIGITAL BEAMFORMING OF A WIDEBAND ACTIVE SONAR ARRAY

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ABSTRACT

A signal interpolation technique based on the Hilbert transform of a signal is proposed. This technique can be efficiently used for fractional time delay estimation. A useful application of the technique is demonstrated using examples of digital beamforming with wideband signals. Conventional wideband signal beamforming is computationally intensive due to the calculation of fractional time delays. It is shown that the proposed Hilbert transform based fractional time delay estimation provides simple and computationally efficient, yet very accurate results, and therefore, suitable for digital beamforming of wideband signals.

1. INTRODUCTION

Wideband signals are usually used in active sonar systems for the detection of underwater targets in the presence of noise and clutter. By using matched filtering techniques on the received echo signal, these signals can provide accurate estimates of the targets' ranges and velocities (or more precisely, range rates) [1]. Wideband signals such as the linear frequency modulated (LFM) signals are used because, by increasing the bandwidth of the transmitted signal, it is possible to increase the range resolution without affecting the velocity resolution. In addition to velocity and range estimation, target bearing can also be estimated using beamforming techniques used in array processing [2]. However, beamforming with wideband signals is a computationally intensive task. This is because the efficient beamforming technique of complex phase weight multiplication can not be used with wideband signals. The beamforming is thus achieved by delaying the sensor input signals. This is a time domain technique. Alternatively, wideband beamforming could be obtained in the frequency domain via FFT processing. But this again is a computationally intensive method.

In the time domain method, for high resolution beamforming, it will be necessary to use fractional input time delays. This is usually achieved using an interpolation and smoothing technique via over-sampling of the input signal. This, however, is not computationally efficient. In this paper, an efficient digital beamforming technique that can be used with input sonar signal is proposed. The beamforming is based on a non-linear interpolation technique based on the Hilbert transform of the input sensor signal.

2. WIDEBAND PROCESSING IN THE DELAY - SCALE DOMAIN

Consider a single discrete scatterer (target) situated at a range r_0 from the transmitter at time, t = 0 and moving at a constant velocity v towards the transmitter. Suppose the transmitted burst is described by a(t). Then the received backscatter from the single scatterer is given by,

$$g(t) \cong S_0 a\{(t - \tau_0)(\frac{c + v}{c - v})\} = S_0 a\{\alpha_0(t - \tau_0)\}, \qquad (1)$$

where $\alpha_0 = (c+v)/(c-v)$, and $\tau_0 = 2r_0/(c+v)$ is the time taken by the back-scattered signal to reach the sensor (round trip delay), and *c* is the velocity of sound in the medium under observation [1]. The factor S_0 describes the reflectivity of the scatterer at range r_0 (at t=0). It also includes the attenuation in the medium as well as the receiver gain. Furthermore, it is assumed that S_0 is independent of time. Note that, due to the scatterer motion, the transmitted pulse a(t) is compressed by a factor of α_0 .

For wideband signals, target detection is usually performed by processing g(t) via a matched filtering approach in the delay-scale (τ, α) domain. The delay-scale domain matched filter is described by the following equation.

$$\rho(\tau,\alpha) = \sqrt{\alpha} \int g(t) a^*(\alpha[t-\tau]) dt \quad . \tag{2}$$

(The limits of integration are $\pm \infty$ unless otherwise stated.)

Suppose the wideband auto-ambiguity function of the transmitted signal is denoted by $\chi(\tau, \alpha)$, i.e.

$$\chi(\tau,\alpha) = \sqrt{\alpha} \int a(t) a^*(\alpha[t-\tau]) dt , \qquad (3)$$

then equation (2) can be expressed as,

$$\rho(\tau,\alpha) = \frac{S_0}{\sqrt{\alpha_0}} \chi(\alpha_0[\tau - \tau_0], \frac{\alpha}{\alpha_0}) .$$
(4)

In the delay-scale plane, $\rho(\tau, \alpha)$ peaks at $\tau = \tau_0$ and $\alpha = \alpha_0$ which can be used to obtain the range and velocity of the target. Note that $\chi(\tau, \alpha = 1)$ is same as the autocorrelation function of a(t). The autocorrelation function of a(t).

3. BEAMFORMING AND RANGE -BEARING RESOLUTION PATTERN

Consider a linear sonar array of (M + 1) sensors with uniform spacing d. The array weighting coefficient corresponding to the m^{th} element is denoted by w_m and the range r_0 of the target is considered to be measured from the centre of the array. Suppose a plane wave signal source arrives at an angle θ measured with respect of the normal of the array axis and the output of the beamformer g(t), is processed via a delay-scale (τ, α) domain matched filter as described in equation (2).

Consider the beamformer with no pre-steering delays. Equation (4) can now be extended as follows.

$$\rho_{s}(\tau,\alpha,\theta) = \sum_{m=-M/2}^{M/2} \left(\frac{S_{0}}{\sqrt{\alpha_{0}}} \right) w_{m} \chi(\alpha_{0}[\tau - \tau_{0} + \frac{md\sin\theta}{c}], \frac{\alpha}{\alpha_{0}})$$
(5)

Evaluating the above equation at $\tau = \tau_0$ and $\alpha = \alpha_0$ provides the signal energy of the detection process, i.e.

$$|\rho_{s}(\tau_{0},\alpha_{0},\theta)|^{2} = \frac{|S_{0}|^{2}}{\alpha_{0}} \left| \sum_{m=-M/2}^{M/2} w_{m} \chi(\frac{m\alpha_{0}d\sin\theta}{c},1) \right|^{2}$$
$$= \frac{|S_{0}|^{2}}{\alpha_{0}} \times B(\theta)$$
(6)

where $B(\theta)$ defines an array directivity pattern (beam pattern) which is dependent on the wideband ambiguity function of the transmitted signal. Evaluating equation (5) at $\alpha = \alpha_0$ the following could be obtained.

$$\rho_s(\tau, \alpha_0, \theta) = \left(\frac{S_0}{\sqrt{\alpha_0}}\right) \sum_{m=-M/2}^{M/2} w_m R(\alpha_0[\tau - \tau_0 + \frac{md\sin\theta}{c}]).$$
(7)

The above can be considered as a range-bearing resolution pattern. In a manner similar to the ambiguity function, the function in equation (7) describes the achievable range and bearing resolutions of the sonar system. Note that the evaluation of the beam pattern and the range-bearing resolution pattern needs the implementation of equations (6) and (7) which requires delaying of the input signal by a fraction of the sampling period. In this paper the time delays require to evaluate equations (6) and (7) are obtained via a non-linear technique. The technique relies on the Hilbert transform of the input received signal.

4. EVALUATING A SIGNAL AT FRACTIONAL DELAYS USING THE HILBERT TRANSFORM

Let the sequence x(k), $k \in \Im$ has been obtained by uniformly sampling a real signal $x(t)|_{t=kT}$ at sampling intervals of *T*. Consider the problem of estimating the signal value x(t) using the sequence x(k), at some time *t* given by $t = kT + \varepsilon T$ where $0 \le \varepsilon \le 1$. Suppose z(k) is the analytic signal associated with x(k), i.e.

$$z(k) = x(k) + jH\{x(k)\} ,$$
 (8)

where H[.] denotes the Hilbert transform (H.T.) of a signal. Using equation (8) the amplitude and phase of the signal can also be obtained as,

$$A(k) = |z(k)|$$
; $\phi(k) = \arg(z(k))$. (9)

The following points are noted:

- 1. In most of the applications in sonar the Hilbert transform of the input signal is available at the receiver without the need of additional processing. This is because of the quadrature demodulation at the receiver.
- The functions A(k) and φ(k) are both slow varying in most of the signals used in active sonar. This is because the matched filtering operation in equation (2) results in slow varying signals for ρ(τ, α).

As, A(k) and $\phi(k)$ are slow varying it is possible to linearly interpolate these signals to obtain an estimate of the analytic signal at time $t = kT + \varepsilon T$. This is obtained using the amplitude and phase of $z(k + \varepsilon)$ which is derived using the following relations:

$$|z(k+\varepsilon)| = \varepsilon A(k+1) + (1-\varepsilon)A(k) ;$$

$$\arg(z(k+\varepsilon)) = \varepsilon \phi(k+1) + (1-\varepsilon)\phi(k) .$$
(10)

 $x(kT + \varepsilon T)$ then results as the real part of $z(k + \varepsilon)$.

Table 1 shows the results of an experiment performed to determine the accuracy of the proposed Hilbert transform interpolation technique. The following Linear Frequency modulated signal having a Gaussian shaped envelope was used in the experiment.

$$x(t) = e^{-10t^2} \cos(2\pi f_c t + \pi \beta t^2) , \qquad (11)$$

with $f_c = 0.24$ and $\beta = 0.12$. The signal duration was selected as -1 < t < +1 seconds. At first a sequence x(k)was obtained by sampling the signal in equation (11) by a *IHz* sampling frequency. Note that the signal in equation (11) occupies the full Nyquist bandwidth $\pm 0.5Hz$. Suppose another sequence is defined as the values obtained by sampling the signal x(t) at a frequency of d*Hz*, i.e.

$$x_d(k) = x(t)\Big|_{t-d}$$
 (12)

For $d \in \Re$ we can estimate the sequence $x_d(k)$ from the sequence x(k) using fractional delay estimation. The exact value of $x_d(k)$ can also be derived using equation (11). Therefore, it is possible to calculate estimation error, and thus evaluate various fractional delay sampling algorithms using simulation. Table 1 shows the performance results of the Hilbert transform interpolation in calculating the fractionally delayed sample values. For comparison purposes, results from a Linear Interpolation algorithm is also shown in Table 1. Results from Table 1 demonstrate that the fractionally sampled sequence values

could be accurately estimated (within an error of 10^{-5}) using the described Hilbert transform interpolation technique, over a large range of the value *d*.

Value of	Error from	Error from
d	H.T.	Linear
	Interpolation	Interpolation
0.092	7.2403×10^{-6}	4.3975
0.320	7.2402×10^{-6}	4.3975
0.900	7.2413×10 ⁻⁶	4.3989
1.100	7.2370×10^{-6}	4.3931
3.900	10.454×10^{-6}	3.9211
13.700	11.990×10^{-6}	4.9335

TABLE 1 : Comparison of Hilbert Transform
Interpolation with Linear Interpolation.

5. GAUSSIAN ENVELOPED LINEAR FREQUENCY MODULATED SIGNALS

The performance of the proposed digital beamforming method, which uses Hilbert transform interpolation technique, is demonstrated using Gaussian LFM (G-LFM) signals. GLFM signals have been considered here as it has been shown in [3, 4] that they can provide desirable directivity patterns for target bearing estimation. Furthermore, exact closed form expressions for G-LFM signal beam and range-bearing resolution patterns have been derived in [5] which could be used for verifying the performance of the proposed technique.

The assumed G-LFM sonar transmission signals is given by,

$$a(t) = e^{-\gamma t^{2}} \cos(2\pi f_{c} t + \pi \beta t^{2}) , \qquad (13)$$

where f_c is the centre frequency and β is the frequency sweep rate. The parameter γ controls the Gaussian shaped amplitude of the transmitted signal. The autocorrelation of the G-LFM signal in equation (13) is given by

$$R(\tau) = E_0 e^{-\xi \tau^2 / f_c^2} \cos(2\pi f_c \tau) \quad ; \quad \xi = \frac{\pi^2 + \beta^2}{4\pi f_c^2} , \quad (14)$$

where E_0 is the energy of the transmitted signal. Using the result of equation (14) it is possible to derive theoretical expressions for the beam pattern and range – bearing resolution pattern of G-LFM signals [5].

Consider the following G-LFM signal simulation example. The signal parameters were selected as $f_c = 200Hz$, $\beta = 112Hz/s$, $\gamma = \pi/2$ with a sampling frequency of 1000 Hz. No noise was used in the simulation. Figure 1 shows the matched filter output of the received signal at one sensor. Note that the matched filter output has a smooth amplitude which justifies the use of the Hilbert transform interpolation technique. In the simulations the signal source was situated at an angle of 30^{0} from the array axis. The beam pattern resulting from a linear array of 38 elements having uniform weights is shown in Figure 2. The beam pattern was obtained from fractional delaying the input using the H.T. interpolation. Theoretical beam pattern obtained from the results of reference [5] is also shown in Figure 2. Figure 3 shows a contour plot of the range - bearing resolution pattern of the received signal obtained from H.T. interpolation. The contour plot of Figure 3 agrees well with the theoretical results provided in reference [5].



FIGURE 1: Matched Filter Output at a Single Sensor (No noise)

Figures 4 to 6 are obtained using simulations with noisy signals at the sensor inputs. For the simulations the signal to ratio (*SNR*) was defined as

$$SNR = \frac{\left|S_{0}\right|^{2}}{\alpha_{0}\eta} \times \int_{-\infty}^{\infty} a^{2}(t)dt \quad , \tag{15}$$

where η is the noise power. In Figures 4-6 the *SNR* used in the simulations are noted in *dB* values. Figures 4 and 5 show the beam pattern obtained from both H.T. interpolation as well as from theoretical expressions, for *SNR* given by *SNR* = 0*dB* and *SNR* = -30dB. The beam patterns in Figures 4 and 5 demonstrate the usefulness of the H.T. interpolation technique even under very low *SNR* conditions. The matched filter output in Figure 6 shows the severity of the noise at *SNR* = -30dB, where the received signal was completely buried in the noise. However, as seen by figure 5, the H.T. interpolation technique of digital beamfoming performs very satisfactorily, even in the presence of severe noise.



FIGURE 2: Directivity Patterns (dashed) Theoretical Pattern from Reference [5] (solid) Pattern Derived from the Calculation of Fractional Time Delays using H.T. Interpolation (No noise).



FIGURE 3: Range – Bearing Resolution Pattern Obtained from the Calculation of Fractional Time Delays using H.T. Interpolation (No noise).



FIGURE 4: Directivity Patterns (dashed) Theoretical Pattern from Reference [5] (solid) Pattern Derived from the Calculation of Fractional Time Delays using H.T. Interpolation (SNR = 0dB).



FIGURE 5: Directivity Patterns (dashed) Theoretical Pattern from Reference [5] (solid) Pattern Derived from the Calculation of Fractional Time Delays using H.T. Interpolation (SNR = -30dB).

6. CONCLUSIONS

A technique for digital beamforming has been proposed in the paper. The technique works via a non-linear signal interpolation derived through the Hilbert transform. The proposed technique is very efficient yet accurate and is especially suitable for the application of beamforming using wideband signals. Results from the proposed method of digital beamforming is presented in the paper using Gaussian LFM signals. The simulation has shown that even in the presence of very severe noise the proposed technique could be efficiently used for wideband digital beamforming.



FIGURE 6: Matched Filter Output at a Single Sensor ($SNR = -30 \ dB$)

7. REFERENCES

- Weiss, L.G., "Wavelets and Wideband Correlation Processing," *IEEE Signal Processing Magazine*, pp. 13-32, January 1994.
- [2] Van Veen, B. D., and Buckley, K. M., "Beamforming: A Versatile Approach to Spatial Filtering," *IEEE Signal Processing Magazine*, pp. 4-24, April 1988.
- [3] Abeysekera, S. S. and Leung, Y. H. "Use of Gaussian LFM Signals for Active Sonar Processing," Technical Report No. SPL-TR-003, p. 51, Australian Telecommunications Research Institute, Curtin University of Technology, Perth, Australia, July 1996.
- [4] Abeysekera, S. S. and Leung, Y. H. "Design of an Optimum Wideband Active Sonar Array with Robustness," *International Conference on Acoustics, Speech and Signal Processing - ICASSP '97*, pp. 523 - 526, Munich, Germany, 1997.
- [5] Abeysekera, S. S. and Leung, Y. H. "Directivity Pattern of a Linear Active Sonar Array Resulting from Gaussian LFM Signals," Fifth International Symposium on *Digital Signal Processing for Communication Systems - DSPCS* '99, pp. 189 - 196, Perth, Australia, February 1999.