A COMBINED STOCHASTIC/DECISION-AIDED SNR ESTIMATOR

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ABSTRACT

A problem of a blind stochastic SNR estimator and a principle of SNR estimation combining a blind estimator and decision-aided estimator are presented. Signal and noise can be detected from the received signal by a decision depending on a decision threshold that is made available by using the second and fourth-order moments of the received signal. Subsequently, the SNR is computed from the detected signal and the detected noise. The bias of SNR estimation (based on detection errors) is removed by applying analytically bias correction.

1. INTRODUCTION

The *signal-to-noise ratio (SNR)* at the receiver's side of a communication link is an important measure for the quality of the communication channel. The SNR is obviously influenced by two unknown power measures, the received *signal* power S of the signal emitted by the sender and attenuated by the channel, and the received noise power N due to thermal noise, crosstalk from neighboured channels, etc. In general, both are unknown and added to the *total received power* $S+N$ of the received superposition of signal and noise, which can actually be observed.
The task of an SNR estimator is to analyze the superimposed received signal and to *determine* which part of the power is due to the transmitted signal and which part is due to noise. We want to distinguish two basically different approaches:

Data-aided Estimation If digital data are transmitted, the receiver tries to reconstruct the transmitted signal by means of the decoded data, and interpret the difference between (an appropriately scaled version of) the received signal and the reconstructed signal as noise.
The major drawback of this method is, of course, that the decoded data become less reliable with decreasing SNR, and thus the estimate exhibits a random error due to decoding errors. This effect is obviously especially serious in a low SNR environment.

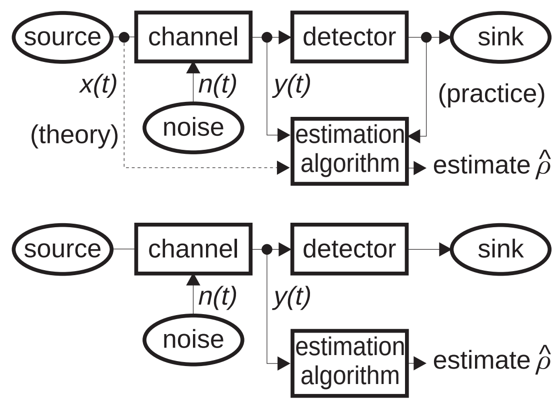

Figure 1: Data-aided SNR estimation (top) and blind SNR estimation (bottom). The data-aided SNR estimation relies on the data *sent* (theory), but actually uses the data *detected* (practice).

Blind Estimation methods do not use decoded data. Instead, they use data-independent properties of the received signal that bear information about the data and the noise components. Due to the abstraction from the actual data being transmitted these properties usually are of statistical nature, such as higher order moments or cumulants.

Reverting the above argument, one advantage of blind estimators is the robustness against erroneously decoded data, making them especially well-suited for low-SNR channels. Another useful property of blind estimators is that they work for digital messages (data) *and* analog messages (e.g. analogously transmitted audio), whereas there's no way to obtain a 'decoded' version of an analog signal for a data-aided estimator. Figure 1 illustrates the differences between both approaches.

2. STATEMENT OF THE PROBLEM

Signal $x(t)$ and noise $n(t)$ are modelled as stochastic processes with probability density functions (p.d.f.) $f_x(x)$ and $f_n(n)$, resp. The received signal $y(t)$ is a superposition of signal and noise:

$$y(t) = x(t) + n(t) \qquad (1)$$

We assume that $\mathbf{x}(t)$ and $\mathbf{n}(t)$ are i.i.d. stochastic processes, and mutually independent. This is valid e.g. for Nyquist systems with sampling after a matched filter and additive white Gaussian noise.
We further assume that signal's and noise's p.d.f.s fulfill the symmetry conditions

$$f_x(x) = f_x(-x), \quad f_n(n) = f_n(-n), \qquad (2)$$

and that not both, signal and noise, are Gaussian.
If $\mathbf{x}(t)$ and $\mathbf{y}(t)$ are complex valued stochastic processes, they are described by their joint p.d.f.s of real and imaginary part.
For the rest of the paper, we restrict our analysis to M-PAM signalling schemes. However, extension of the proposed algorithms to complex AM/PM schemes is straight forward. For sake of simplicity, noise is assumed to be Gaussian. A discrete-time AWGN channel is considered.

3. THE CRAMR-RAO BOUND

Given a stochastic process with p.d.f. $f_x(x|\theta)$ dependent on a parameter θ, the Cramr-Rao Bound (CRB) [1] gives us a lower bound for the variance $\mathrm{var}\{\hat{\theta}\}$ of an estimation for θ:

$$\mathrm{var}\{\hat{\theta}\} \geq \frac{1}{n \cdot E\left\{\left(\frac{\partial \ln f_x(x|\theta)}{\partial \theta}\right)^2\right\}} \qquad (3)$$

where $E\{\cdot\}$ denotes the expected value and n is the number of observed samples.
Given an M-PAM transmitter with transmitted power S and p.d.f.

$$f_x(x|S) = \frac{1}{M}\sum_{k=0}^{M-1} \delta\left(x - (2k - M + 1)\sqrt{\frac{3S}{M^2-1}}\right) \qquad (4)$$

and Gaussian noise with variance N we obtain for the received p.d.f.

$$f_y(y|S,N) = \frac{1}{M\sqrt{2\pi N}}\sum_{k=0}^{M-1} e^{-\frac{\left(y-(2k-M+1)\sqrt{\frac{3S}{M^2-1}}\right)^2}{2N}} \qquad (5)$$

Substituting $y' = \frac{1}{\sqrt{S}}y$ and the SNR $\rho = \frac{S}{N}$ in (5) we obtain:

$$f'_y(y'|\rho) = \frac{\sqrt{\rho}}{M\sqrt{2\pi}}\sum_{k=0}^{M-1} e^{-\frac{\rho}{2}\left(y'-(2k-M+1)\sqrt{\frac{3}{M^2-1}}\right)^2} \qquad (6)$$

Figure 2 and 6 also depict numerical results for the equations (3) and (6) for $M \in \{2, 4, 8\}$ and an observation size of $n = 512$. For larger SNR ρ the CRB is almost proportional to ρ^2. For this reason, variances of estimates will be normalized to ρ^2 in the sequel. The normalized CRB shows a prominent maximum at low SNR.

4. THE BLIND STOCHASTIC SNR ESTIMATION

4.1. Algorithm

Using the second and fourth-order moments the SNR can be estimated by observing the noisy signal if only the shapes of signal's and noise's p.d.f.s are known [2]. The estimate $\hat{\rho}$ of the received SNR is computed by [3]:

$$\hat{\rho} = \frac{\hat{\kappa}}{1-\hat{\kappa}} = \frac{\sqrt{\hat{G}_y/G_x}}{1-\sqrt{\hat{G}_y/G_x}} \qquad (7)$$

where

- $\hat{\rho}$ denotes the estimated signal-to-noise ratio S/N,
- $\hat{\kappa}$ is the estimated signal-to-total ratio $S/(S+N)$,
- G_x denotes the transmitted Gauss-unlikeness

$$G_x = m_{x,4}/m_{x,2}^2 - 3,$$

- $\hat{G}_y$ denotes the estimated received Gauss-unlikeness.

Additionally, this algorithm can be seen in [4].

4.2. Analysis

This blind stochastic SNR estimation was simulated with the AWGN channel model, and a measure of efficiency of the SNR estimator with the CRB was used. The CRB for the blind SNR estimation was computed numerically with (3) and (6). Figure 2 shows the results of the simulated blind stochastical SNR estimator for 2-PAM (using 100 simulation runs with 500 packets, each containing 512 samples) compared to the this CRB. The result approachs the theoretical bound for SNR estimation. It turns out that for 2-PAM the blind stochastic estimator is efficient with respect to the CRB.

4.3. Problem of Algorithm

The blind stochastic estimator has severe disadvantages when it is used for bandwidth-efficient modulation schemes. The estimation remains stable as long as $\hat{\kappa}$ is equal or greater than 0 but smaller than 1. Estimation for M-PAM, $M > 2$, can obtain values for $\hat{\kappa}$ close to the pole in (7),

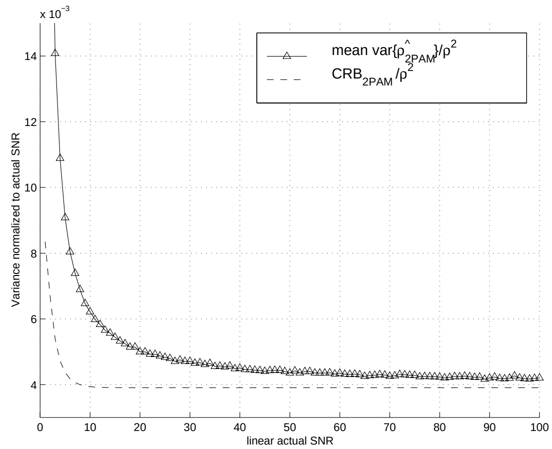

Figure 2: Mean normalized variance $\mathrm{var}\{\hat{\rho}\}/\rho^2$ of the simulated blind stochastical SNR estimator for 2-PAM and CRB of $\mathrm{var}\{\hat{\rho}\}/\rho^2$

causing instability and exploding variance of the estimated $\hat{\rho}$. However, it has to be noted that while $\mathrm{var}\{\hat{\rho}\}$ explodes due to the pole, $\mathrm{var}\{\hat{\kappa}\}$ remains stable.

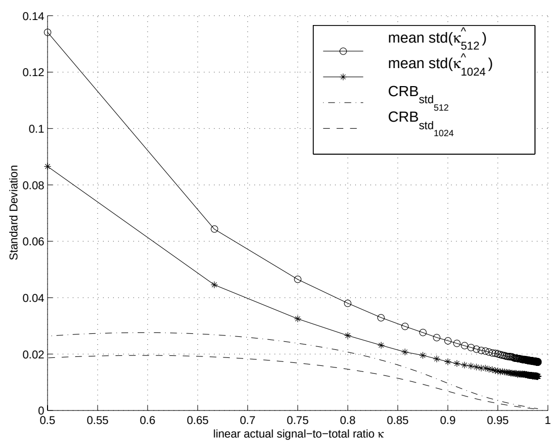

Figure 3: Mean standard deviation of the simulated estimated signal-to-total ratio using blind stochastical SNR estimator with 512 and 1024 samples for 4-PAM and their corresponding CRBs of $\mathrm{std}\{\hat{\kappa}\}$

In fig. 3 the mean standard deviation of simulated estimated signal-to-total ratio using blind stochastical SNR estimator (using 100 simulation runs with 500 packets, each containing 512 and 1024 samples) for 4-PAM are compared to the corresponding CRBs. The CRB of 1024 samples is lower than the CRB of 512 samples. If the actual SNR ρ and the actual signal-to-total ratio κ increase, the standard deviation of estimated $\hat{\kappa}$ will approach the CRB. For 8-PAM the problem is similar. This SNR estimator is not well-suited for 4 and 8-PAM, but it can be used to estimate the signal-to-total ratio.

5. A COMBINED STOCHASTIC/DECISION-AIDED ESTIMATOR

5.1. Principle of Operation

Revesting the last section, this leads to an estimator using blind stochastical estimation of $\hat{\kappa}$, and then computing a decision-aided estimation for $\hat{\rho}$. The signal-to-total ratio κ is used to adjust the decision threshold d of the decision-aided estimator, for 4-PAM:

$$d = \sqrt{\frac{\kappa \cdot m_{y,2}}{5}} \tag{8}$$

where $m_{y,2}$ denotes second-order moment of the received signal, and 5 is simply the normalized power of 4-PAM.
In the sequel, we use x^* to denote the detected signal, and n^* for the detected noise (i.e. $n^* = y - x^*$). We can find the signal power S_{x^*} from the detected signal x^*, and the noise power N^* from the detected noise n^*. The estimated SNR ρ^* is provided by relation of S_{x^*} and N^*. Due to the detection errors, the estimated ρ^* is biased. However, this bias can easily be removed by applying an analytically computed nonlinear characteristic in the section 5.2. Fig. 4 illustrates the principle of this operation.

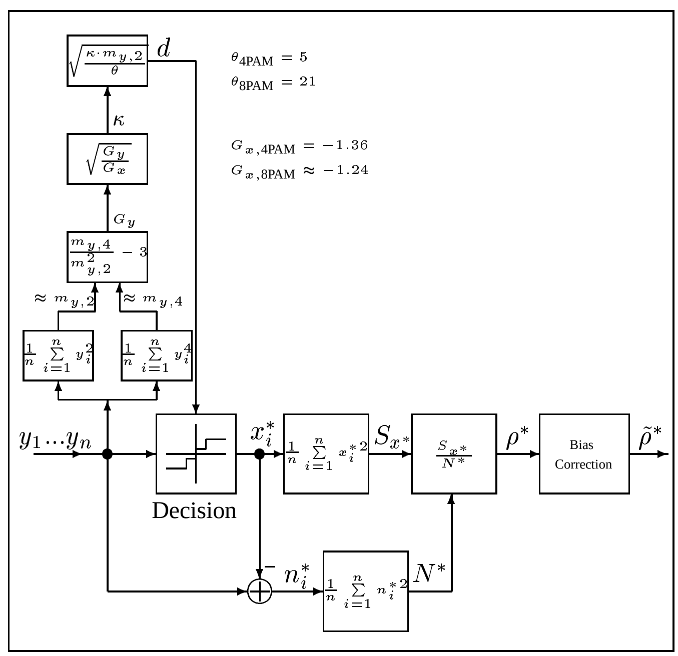

Figure 4: Combined Stochastic/Decision-aided SNR Estimator for 4- and 8-PAM

5.2. Bias Removal

The characteristic of the estimated SNR ρ^* is analytically computed in [5]. The received p.d.f. for 4-PAM can be given by (5), therefore the signal power S_{x^*} for 4-PAM is obtained by using the received p.d.f. and the decision threshold d.

$$\begin{aligned} S_{x^*}(S,N) \;=\;& (-3d)^2 \int_{-\infty}^{-2d} f_y(y)\,dy \;+ \\ &+\;(-1d)^2 \int_{-2d}^{0} f_y(y)\,dy \;+ \\ &+\;(1d)^2 \int_{0}^{2d} f_y(y)\,dy \;+ \\ &+\;(3d)^2 \int_{2d}^{+\infty} f_y(y)\,dy \end{aligned} \tag{9}$$

Additionally, the noise power N^* for 4-PAM is obtained by using the received pdf and the decision threshold d.

$$\begin{aligned} N^*(n^*|S,N) \;=\;& 2\Big[\int_{0}^{+\infty} {n^*}^2 f_y(n^*+3d)\,dn^* \;+ \\ &+\int_{0}^{1d} {n^*}^2 f_y(n^*+1d)\,dn^* \;+ \\ &+\int_{0}^{1d} {n^*}^2 f_y(n^*-1d)\,dn^* \;+ \\ &+\int_{0}^{1d} {n^*}^2 f_y(n^*-3d)\,dn^* \;\Big] \end{aligned} \tag{10}$$

The analytically estimated SNR ρ^* for 4-PAM is now computed as the relation of S_{x^*} in (9) and N^* in (10). The analytically estimated SNR ρ^* for 8-PAM can be computed in the same way as for 4-PAM.
Figure 5 depicts curves of actual SNR ρ for 4- and 8-PAM depending on the analytically estimated SNR ρ^*. The analytically estimated SNR ρ^* for 8-PAM is only greater than the actual SNR ρ, whereas the analytically estimated SNR ρ^* for 4-PAM is equal to the actual SNR ρ, when the actual SNR ρ is greater than 60. These curves are used to remove the bias of the estimated SNR ρ^*.

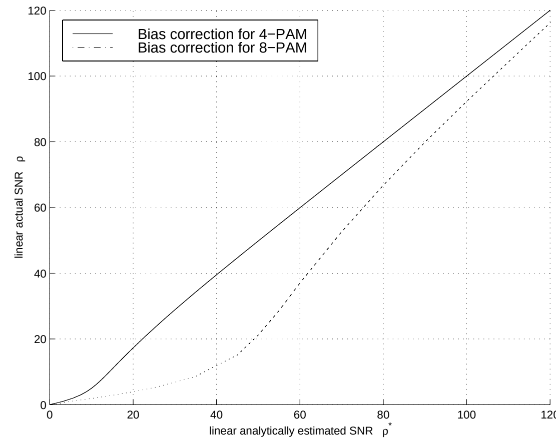

Figure 5: Bias correction to correcting the estimated SNR ρ^* for 4- and 8-PAM

5.3. Results

Figure 6 shows the results of the simulated combined stochastic/decision-aided SNR estimator for 4- and 8-PAM (using 100 simulation runs with 500 packets, each containing 512 samples) with bias correction compared to the CRB. Both variances $\mathrm{var}\{\tilde{\rho}^*\}/\rho^2$ of simulation remain stable, and they have similar characteristics to the CRBs. The $\mathrm{var}\{\tilde{\rho}^*\}/\rho^2$ of 4-PAM is closer its CRB than the $\mathrm{var}\{\tilde{\rho}^*\}/\rho^2$ of 8-PAM. Figure 6 also clarifies that the combined stochastic/decision-aided SNR estimator has nearly the same efficiency for 4-PAM as for 8-PAM with respect to the CRB.

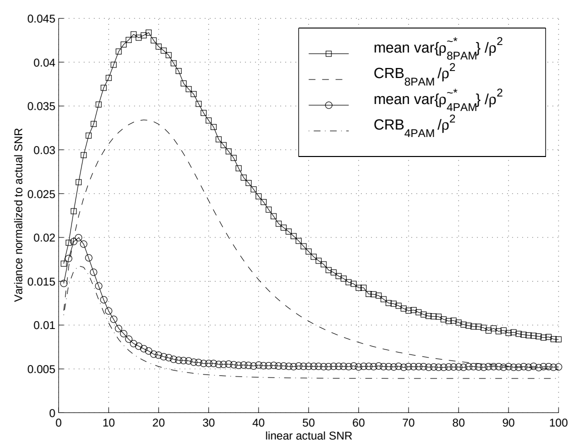

Figure 6: Mean normalized variances $\mathrm{var}\{\tilde{\rho}^*\}/\rho^2$ of the simulated combined stochastic/decision-aided SNR estimator for 4- and 8-PAM and their corresponding CRBs of $\mathrm{var}\{\hat{\rho}\}/\rho^2$

6. CONCLUSION

The blind stochastic SNR estimator has a good efficiency in the sense of the CRB for 2-PAM. However, it is not very well-suited for 4-PAM and higher PAM or QAM modulation schemes, because estimation for M-PAM, $M > 2$, at high actual SNR can obtain values for $\hat{\kappa}$ close to the pole, causing instability and exploding variance of the estimated $\hat{\rho}$. If detected data are available or can be made available, the combined stochastic/decision-aided SNR estimator solves this problem in a near-optimum manner.

7. REFERENCES

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