BLOCK REDUNDANT CONSTANT MODULUS EQUALIZATION FOR FIR CHANNEL–IRRESPECTIVE BLIND IDENTIFIABILITY

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ABSTRACT

FIR filterbank precoders offer a unifying framework for several digital modulation and multiplexing schemes used in single- and multi-user transmissions, such as OFDM and CDMA. With minimal reduction in bandwidth efficiency, FIR filterbank transmitters can be designed to allow for perfect (in the absence of noise) equalization of any FIR channel with FIR zero-forcing (ZF) equalizer filterbanks. The key idea in this paper is to apply the constant modulus algorithm (CMA) to block transmission schemes that use redundant precoding. The resulting modulation/multiplexing techniques guarantee perfect equalization of an FIR channel with an FIR filterbank equalizer of a priori known order, without the restrictions of blind adaptive FIR CMA equalizers that rely on multiple antennas and fractional sampling.

1. INTRODUCTION AND SYSTEM MODEL

Over the past twenty years, blind adaptive channel equalization has been widely explored and several schemes have been developed. They are usually designed to minimize cost functions based on higher than second-order statistics of the channel output. Among them, Godard's algorithm [4], has received a lot of attention. The more recent Shalvi-Weinstein algorithm (SWA) [9], has convergence properties similar to CMA [6].

The success and effectiveness of the CMA are contingent upon its ability to identify a ZF equalizer in the presence of noise and channel distortions. With infinite data and equalizer length it should at least converge globally to a stable equilibrium in the noise-free case in order to produce satisfying performance in the noisy case. However, unless the (generally non–causal) equalizer has infinite length in a symbol-spaced channel, the convergence of CMA cannot be always assured. Extraneous local minima exist even for a noiseless channel. Moreover, it has been proved that CMA does not converge when a symbol-sampled channel has zeros on (or even close) to the unit circle [2]. To overcome the drawback of symbol-spaced CMA (S-CMA), fractionally-spaced CMA (FS-CMA) or multichannel reception have been proposed (see e.g. [6]). Although [6] addresses the problem of channel zeros on (or close to) the unit circle, coprimeness among the multiple channels is necessary to guarantee FIR channel identifiability. When the multiple channels have common zeros, not only the convergence will be slower, but also the global convergence itself can not be assured.

Recently, it was shown that through redundant filterbank precoding the existence of FIR equalizers for any FIR channel can be guaranteed under some sufficient conditions on the transmit filterbank *only* [7]. Moreover, redundant filterbank precoders offer a unifying discrete-time model which encompasses a wide range of digital modulation and coding schemes [7], such as orthogonal frequency-division multiplexing (OFDM) and discrete multitone (DMT) [1], fractional sampling (FS), (de-)interleaving, as well as multiuser transmissions such as TDMA, FDMA, CDMA, and the most recent discrete wavelet multiple access (DWMA) schemes.

The idea of this paper is to exploit the diversity provided by block transmission schemes to devise self recovering filterbank (FB) transceivers that use a block CMA cost function to estimate the equalizer blindly. The key point is that FB-CMA does not suffer from the lack of identifiability of FS-CMA and hence it leads to a robust blind equalization scheme. It is important to remark that for the CDMA transmission, the substance of our problem is not different from blind MUI interference cancellation. Although we restrict our attention to the downlink case, the main advantage of our approach over other blind decorrelating receivers [5], [10], is that we do not require knowledge of the user of interest signature code, nor we assume symbol whiteness. Simulations will illustrate the performance of the proposed technique.

2. PRELIMINARIES

Suppose that the channel is FIR with order L and impulse response $\boldsymbol{h} := (h(0), \ldots, h(L))^T$. Let us denote by $\boldsymbol{F}_0, \boldsymbol{G}_0$ the $P \times M$ precoder and $M \times P$ decoder matrices and let us assume that $L < P$. Blocking the channel input/output data, in the $P \times 1$ vectors $\boldsymbol{u}(n) := (u(nP), \ldots, u(nP + P - 1))$

and $\boldsymbol{x}(n) := (x(nP), \ldots, x(nP+P-1))$ respectively, it can be shown that the discrete time input-output relationship for a wide class of linear transceivers can be described in an unified vector-matrix formulation [7]. More specifically, denoting by $\boldsymbol{H}_0$ and $\boldsymbol{H}_1$ the $P \times P$ convolution Toeplitz matrices such that

$$\{\boldsymbol{H}_l\}_{i,j} = h(lP+i-j) \qquad l = 0,1; \quad i,j = 0,\ldots,P-1,$$

and indicating by $\boldsymbol{s}(n)$ the $M \times 1$ vector of the information symbols[1], the transmitted block is:

$$\boldsymbol{u}(n) = \boldsymbol{F}_0 \boldsymbol{s}(n), \tag{1}$$

the noise–free received block can be written as

$$\boldsymbol{x}(n) = \boldsymbol{H}_0 \boldsymbol{u}(n) + \boldsymbol{H}_1 \boldsymbol{u}(n-1), \tag{2}$$

and, finally, the equalizer output is given by [7]

$$\begin{aligned} \hat{\boldsymbol{s}}(n) &= \boldsymbol{G}_0 \boldsymbol{x}(n) + \boldsymbol{G}_0 \boldsymbol{v}(n) \qquad (3) \\ &= \boldsymbol{G}_0 \boldsymbol{H}_0 \boldsymbol{F}_0 \boldsymbol{s}(n) + \boldsymbol{G}_0 \boldsymbol{H}_1 \boldsymbol{F}_0 \boldsymbol{s}(n-1) + \boldsymbol{G}_0 \boldsymbol{v}(n) \end{aligned}$$

where $\boldsymbol{v}(n)$ is the AGN noise vector. The M columns of the $P \times M$ precoder matrix $\boldsymbol{F}_0$ characterize the particular transmission scheme; for example, they contain the spreading codes in CDMA or orthogonal frequencies in OFDM. The length P of each code is greater than M and this provides a diversity gain that manifests itself in different ways: the spreading gain for CDMA and the easy equalization for OFDM, by diagonalizing the channel matrix $\boldsymbol{H}_0$ and then removing inter–symbol interference (ISI). Tall matrix $\boldsymbol{F}_0$ performs a redundant mapping (see eq. (1)). Because for a fixed bandwidth the information rate depends upon the ratio M/P, for a fixed difference $P-M$, the rate reduction can be made arbitrarily small by selecting M sufficiently large. However, larger M's require more complex equalizer and increased decoding delay.

In the following we will assume that $P \geq M+L$; the interblock interference (IBI), represented by the term $\boldsymbol{H}_1 \boldsymbol{F}_0 \boldsymbol{s}(n-1)$ in equation (3), can be canceled using either leading zeros (LZ) at the receiver, i.e.,

$$\boldsymbol{F}_0 := \boldsymbol{F}_{P \times M} \ \ , \ \ \boldsymbol{G}_0 := (\boldsymbol{0}_{M \times L}\, \boldsymbol{G}_{M \times M}), \tag{4}$$

or by appending trailing zeros (TZ) at the transmitter, in the form of guard bits:

$$\boldsymbol{F}_0 := \begin{pmatrix} \boldsymbol{F}_{M \times M} \\ \boldsymbol{0}_{L \times M} \end{pmatrix} \ \ , \ \ \boldsymbol{G}_0 := \boldsymbol{G}_{P \times M}. \tag{5}$$

Adopting a unified notation in both LZ and TZ cases, we arrive at:

$$\hat{\boldsymbol{s}}(n) = \boldsymbol{G}\boldsymbol{H}\boldsymbol{F}\boldsymbol{s}(n) + \boldsymbol{G}\boldsymbol{v}(n), \tag{6}$$

where $\boldsymbol{F}$, $\boldsymbol{G}$ are defined as in (4) or (5); matrix $\boldsymbol{H}$ is respectively $M \times P$ in the LZ case, composed by the last M rows of $\boldsymbol{H}_0$, and $P \times M$ in the TZ case, built with the first M columns of $\boldsymbol{H}_0$.

Equalization amounts to finding a matrix $\boldsymbol{G}$ that inverts the matrix $\boldsymbol{H}\boldsymbol{F}$ in (6). This is possible for any FIR channel provided that

$$\forall\, \boldsymbol{H} \quad \mathrm{rank}(\boldsymbol{H}\boldsymbol{F}) = M. \tag{7}$$

In CDMA, fading effects of multipath channels are alleviated by spreading the information in frequency through spread–spectrum codes. Since in our framework the user codes determine the columns of the precoder matrix $\boldsymbol{F}_0$ one can infer that this selection of the precoder guarantees that $\mathrm{rank}(\boldsymbol{H}\boldsymbol{F}) = M$ almost surely. Sufficient conditions on the filterbank precoder *only* that allow for channel irrespective FIR equalization are given in [7]. In this paper we will report two special cases, that will be used to guarantee the existence of an equalizer matrix $\boldsymbol{G}$, irrespective of the channel zeros.

Theorem 1 (LZ–ZF equalizer) *For $P \geq M+L$ choose the precoder $\boldsymbol{F}$ to be a full column rank matrix with each column not expressible as linear combination of less than $L+1$ Vandermonde vectors*[2]*. A ZF equalizer then exists for any channel $\boldsymbol{H}$ and is $\boldsymbol{G}_{zf} = (\boldsymbol{H}\boldsymbol{F})^{\dagger}$, where † denotes pseudo-inverse.*

Interestingly, using TZ at the transmitter, we can state the following simple result:

Theorem 2 (TZ–ZF equalizer) *For $P = M+L$ and any full rank matrix $\boldsymbol{F}$, the ZF (minimum mean square error) equalizer of any FIR channel exists and is $\boldsymbol{G}_{zf} = \boldsymbol{F}^{-1}(\boldsymbol{H})^{\dagger}$, where † denotes pseudo-inverse and $\boldsymbol{H}$ is a full column rank matrix.*

Note that Thms. 1 and 2 pose no constraints on the FIR channel zeros. In contrast, FIR-ZF equalizers in [6] do not exist for certain configurations of channel zeros, and more important, performance degrades even when channels have zeros close to those non-invertible configurations. If an upper bound $\bar{L} \geq L$ is only available on the channel order, Thms. 1 and 2 hold true with $\bar{L}$ replacing L.

Similar to FS–CMA that equalizes jointly the output of multiple sub-channels, FB–CMA exploits the diversity provided by the columns $\{\mathbf{f}_m\}_{m=0}^{M-1}$ of the precoder $\boldsymbol{F}_0$. The equivalent mth FIR sub-channel is given by $\boldsymbol{H}\mathbf{f}_m$, and can be inverted through the vector equalizer $\boldsymbol{g}_m^H$ (mth row of $\boldsymbol{G}$) such that

$$\boldsymbol{g}_m^H \boldsymbol{H} \mathbf{f}_\mu = \delta(m-\mu), \quad m,\mu \in [0, M-1] \tag{8}$$

where ^{H} denotes transposition and conjugation, and $\delta(l)$ stands for Kronecker's delta.

[1] The vector $\boldsymbol{s}(n)$ contains either the blocked symbols of one user, $\{\boldsymbol{s}(n)\}_m = s(nM+m)$, or the mth user data stream, $\{\boldsymbol{s}(n)\}_m = s_m(n)$.

[2] Vandermonde vectors are vectors of the form $\rho = (1, \rho^1, \ldots, \rho^{P-1})$.

3. BLIND SYMBOL RECOVERY WITH FB-CMA

We address here the extension of CMA to the filterbank transceivers. Defining by $\mathbf{e}_m$ the mth canonical vector, the mth row of the equalizing matrix, that we denote as $\boldsymbol{g}_m^H$, satisfies

$$\boldsymbol{g}_m^H \boldsymbol{H}\boldsymbol{F} = \mathbf{e}_m^H, \quad m \in [0, M-1]. \tag{9}$$

The goal is to minimize with respect to $\boldsymbol{g}_m^H$ the CMA cost function

$$J(\boldsymbol{g}_m) = (|\hat{s}_m(n)|^2 - R_2)^2 = (\boldsymbol{g}_m^H \boldsymbol{x}(n)\boldsymbol{x}^H(n)\boldsymbol{g}_m - R_2)^2 \tag{10}$$

where $R_2 := E\{|\hat{s}_m(n)|^4\}/E\{|\hat{s}_m(n)|^2\}$. To derive the gradient of quadratic forms such as $\boldsymbol{z}^H\boldsymbol{A}\boldsymbol{z}$ with respect to the complex column vector $\boldsymbol{z}$ or the row vector $\boldsymbol{z}^H$, we use the definitions

$$\nabla_{\boldsymbol{z}}(\boldsymbol{z}^H\boldsymbol{A}\boldsymbol{z}) = \boldsymbol{z}^H\boldsymbol{A}\,, \quad \nabla_{\boldsymbol{z}^H}(\boldsymbol{z}^H\boldsymbol{A}\boldsymbol{z}) = \boldsymbol{A}\boldsymbol{z} \tag{11}$$

so that the gradient $\nabla_{\boldsymbol{z}}(\boldsymbol{z}^H\boldsymbol{A}\boldsymbol{z})$ with respect to the column vector $\boldsymbol{z}$ is a row vector, while the gradient $\nabla_{\boldsymbol{z}^H}(\boldsymbol{z}^H\boldsymbol{A}\boldsymbol{z})$ with respect to the row vector $\boldsymbol{z}^H$ is a column vector.

Equating to zero the gradient of $J(\hat{\boldsymbol{g}}_m(n))$ with respect to $\boldsymbol{g}_m$, obtained according to (11), we obtain the following gradient update

$$\hat{\boldsymbol{g}}_m^H(n+1) = \hat{\boldsymbol{g}}_m^H(n) - \mu\nabla_{\hat{\boldsymbol{g}}_m(n)} J(\boldsymbol{g}_m), \tag{12}$$

where $\hat{\boldsymbol{g}}_m(n)$ is the estimate of $\boldsymbol{g}_m$ at time n and

$$\nabla_{\hat{\boldsymbol{g}}_m(n)} J(\hat{\boldsymbol{g}}_m(n)) = (|\hat{s}_m(n)|^2 - R_2)\hat{s}_m(n)\boldsymbol{x}^H(n). \tag{13}$$

Thms. 1 and 2 establish the existence of an inverse for *any* FIR channel, which will avoid the ill–convergence cases of CMA or FS-CMA discussed in Section 1. However, the symbol identifiability conditions of Thms. 1 and 2 are necessary but not sufficient for CMA convergence. In other words Thms. 1 and 2 do not imply *blind*-identifiability when the equalizer matrix $\boldsymbol{G}$ is estimated using CMA algorithm. In fact, it is easy to detect that the algorithm based on (12) *only*, is prone to the following shortcomings:

i) convergence to the desired mth row of $\boldsymbol{G}_{zf}$ is not enforced; hence, a permutation between rows or even convergence to the same row of $\boldsymbol{G}$ is possible $\forall\, m$;

ii) unless some constraint is added, the trivial solution $\boldsymbol{g}_m = \mathbf{0}$ is a local minimum;

iii) for every m, each estimate $\hat{\boldsymbol{g}}_m(n)$ is affected by a phase ambiguity. Together with ii), this implies that the CMA can converge to

$$\hat{\boldsymbol{g}}_m^H \boldsymbol{H}\boldsymbol{F} = e^{j\phi_m}\mathbf{e}_\mu^H, \tag{14}$$

with $\mu = 0, \ldots, M-1$. As a consequence, even in the best case where there is no ambiguity in the index m and thus $\hat{\boldsymbol{g}}_m^H \boldsymbol{H}\boldsymbol{F} = e^{j\phi_m}\mathbf{e}_m^H$, the noise free symbol block estimate $\hat{\boldsymbol{s}}_m(n)$ will be the input block $\boldsymbol{s}(n)$ multiplied by the unknown sequence $(e^{j\phi_0}, \ldots, e^{j\phi_{M-1}})$.

To address i) – iii) and improve the convergence rate of CMA, our FB-CMA enforces the structure of the ZF equalizer estimate $\hat{\boldsymbol{G}}(n)$ by minimizing the combined cost function with relative weight α:

$$\mathcal{J}(\boldsymbol{G}) = \sum_{m=0}^{M-1} J(\boldsymbol{g}_m) + \alpha\Gamma(\boldsymbol{G}) \tag{15}$$

where

$$\Gamma(\boldsymbol{G}) := \mathrm{tr}([\boldsymbol{G}\boldsymbol{H}\boldsymbol{F} - \boldsymbol{I}]^H[\boldsymbol{G}\boldsymbol{H}\boldsymbol{F} - \boldsymbol{I}]), \tag{16}$$

and $\mathrm{tr}(\boldsymbol{A})$ indicates the trace of the matrix $\boldsymbol{A}$. The $\mathrm{tr}(\boldsymbol{A}\boldsymbol{B}^H)$ defines an inner product in the space of matrices and thus $\Gamma(\boldsymbol{G})$ is zero if and only if $\boldsymbol{G}\boldsymbol{H}\boldsymbol{F} = \boldsymbol{I}$; this in turn imposes (9) and thus enforces order to all rows of $\boldsymbol{G}$ and $\phi_m = \phi_0$, $\forall m$.

The minimization of $\mathcal{J}(\boldsymbol{G})$ can be also performed adaptively through a stochastic gradient approach. To derive the gradient of $\mathcal{J}(\boldsymbol{G})$ with respect to $\boldsymbol{G}$ it is convenient to express $J(\boldsymbol{g}_m)$ as a function of the matrix $\boldsymbol{G}$, using the substitution $\boldsymbol{g}_m^H = \mathbf{e}_m^H\boldsymbol{G}$ as follows [c.f. (10)]

$$J(\boldsymbol{g}_m) = (\mathbf{e}_m^H\boldsymbol{G}\boldsymbol{x}(n)\boldsymbol{x}^H(n)\boldsymbol{G}^H\mathbf{e}_m - R_2)^2. \tag{17}$$

Thus, the gradient of $J(\boldsymbol{g}_m)$ with respect to $\boldsymbol{G}^H$ is the matrix:

$$\begin{aligned} \nabla_{\boldsymbol{G}^H} J(\boldsymbol{g}_m) &= 2(\hat{s}_m(n) - R_2)\mathbf{e}_m\mathbf{e}_m^H\boldsymbol{G}\boldsymbol{x}(n)\boldsymbol{x}^H(n) \\ &= 2(\hat{s}_m(n) - R_2)\hat{s}_m(n)\mathbf{e}_m\boldsymbol{x}^H(n) \\ &\equiv \mathbf{e}_m\nabla_{\boldsymbol{g}_m} J(\boldsymbol{g}_m). \end{aligned} \tag{18}$$

The gradient with respect to $\boldsymbol{G}^H$ of $\mathcal{J}(\boldsymbol{G})$ is thus

$$\nabla_{\boldsymbol{G}^H}\mathcal{J}(\boldsymbol{G}) = \sum_{m=0}^{M-1}\mathbf{e}_m\nabla_{\boldsymbol{g}_m} J(\boldsymbol{g}_m) + \alpha\nabla_{\boldsymbol{G}^H}\Gamma(\boldsymbol{G}), \tag{19}$$

where $\nabla_{\boldsymbol{G}^H}\Gamma(\boldsymbol{G})$ is given by

$$\nabla_{\boldsymbol{G}^H}\Gamma(\boldsymbol{G}) = (\boldsymbol{G}\boldsymbol{H}\boldsymbol{F} - \boldsymbol{I})\boldsymbol{F}^H\boldsymbol{H}^H. \tag{20}$$

Therefore, in force of (18) and (19), the nth adaptation step consists of

$$\begin{aligned} \hat{\boldsymbol{G}}(n+1) &= \hat{\boldsymbol{G}}(n) - \mu \begin{pmatrix} \nabla_{\boldsymbol{g}_0(n)} J(\hat{\boldsymbol{g}}_0(n)) \\ \vdots \\ \nabla_{\boldsymbol{g}_{M-1}(n)} J(\hat{\boldsymbol{g}}_{M-1}(n)) \end{pmatrix} \\ &- \lambda\,[\hat{\boldsymbol{G}}(n)\hat{\boldsymbol{H}}(n)\boldsymbol{F} - \boldsymbol{I}]\boldsymbol{F}^H\hat{\boldsymbol{H}}^H(n), \end{aligned} \tag{21}$$

where μ is the step size and $\lambda = \mu\alpha$.

The last term corresponding to $\nabla_{\boldsymbol{G}^H}\Gamma(\boldsymbol{G})$, requires the channel matrix estimate $\hat{\boldsymbol{H}}(n)$. Since $\boldsymbol{H}$ is a Toeplitz matrix defined by the channel vector $\boldsymbol{h}$, it is straightforward to verify that $\boldsymbol{g}_m^H\boldsymbol{H} \equiv \boldsymbol{h}^T\boldsymbol{\mathcal{G}}_m$ where $\boldsymbol{\mathcal{G}}_m$ is an appropriate Hankel matrix, built with the elements of $\{\boldsymbol{g}_m^H\}_k = g_m(k)$. The structure of the Hankel matrix depends on whether LZ reception or TZ transmission is adopted. In particular, $\boldsymbol{\mathcal{G}}_m$ in the LZ case is $(L+1)\times P$ and is the Hankel matrix with first column $(0,\ldots,0,g_m(0))^T$ and last row $(g_m(0),\ldots,$ $g_m(M-1),0,\ldots,0)$, while in the TZ case $\boldsymbol{\mathcal{G}}_m$ is $(L+1)\times M$ and is the Hankel matrix with first column $(g_m(0),\ldots,$ $g_m(L))^T$ and last row $(g_m(L),\ldots,g_m(P-1))$. Exploiting the equivalence $\boldsymbol{g}_m^H\boldsymbol{H} \equiv \boldsymbol{h}^T\boldsymbol{\mathcal{G}}_m$, we can derive from (9) an estimate of the channel impulse response, $\hat{\boldsymbol{h}}_m(n)$, based on $\hat{\boldsymbol{g}}_m(n)$. Given $\hat{\boldsymbol{g}}_m(n)$, for each $m = 1,\ldots,M$, we solve the linear system of equations

$$\begin{aligned} \hat{\boldsymbol{g}}_m^H(n)\boldsymbol{H}\boldsymbol{F} &= \boldsymbol{h}^T\hat{\boldsymbol{\mathcal{G}}}_m(n)\boldsymbol{F} \approx \mathbf{e}_m^H \qquad (22) \\ &\Rightarrow \quad \hat{\boldsymbol{h}}_m^T(n) = \mathbf{e}_m^H(\hat{\boldsymbol{\mathcal{G}}}_m(n)\boldsymbol{F})^{\dagger}. \end{aligned}$$

The complexity of (22) is moderate, since requires the inversion of an $(L+1)\times(L+1)$ matrix and L is usually small and $\ll M$. Not all $\hat{\boldsymbol{h}}_m(n)$ estimates will be equally reliable, but the CMA cost function $J(\hat{\boldsymbol{g}}_m(n))$ will be exploited to weight each estimate appropriately. The simple strategy that we propose is to estimate $\boldsymbol{h}$ as:

$$\hat{\boldsymbol{h}}(n) = \frac{1}{||\boldsymbol{h}_{m_0}||}\hat{\boldsymbol{h}}_{m_0}(n) \qquad (23)$$

$$m_0 = \arg\min_{m_0}\left[\frac{1}{n}\sum_{i=1}^{n} J(\hat{\boldsymbol{g}}_m(i))\right]. \qquad (24)$$

From $\hat{\boldsymbol{h}}(n)$ we can built $\hat{\boldsymbol{H}}(n)$ needed for the gradient in (21). Notice that the channel normalization in (23) avoids the possibility of converging to the trivial solution $\boldsymbol{g}_m = \mathbf{0}$, and this, together with the minimization of $\Gamma(\boldsymbol{G}(n))$, addresses points i), ii) and iii) raised before. To improve our solution of the problem discussed in point iii), we force to zero the cost function $\Gamma(\boldsymbol{G})$ in the Nth and last iteration of the learning period of FB-CMA, by equating the equalizer estimate to $\hat{\boldsymbol{G}}_{zf}(N) = (\hat{\boldsymbol{H}}(N)\boldsymbol{F})^{\dagger}$. If the equalizer has been correctly estimated, also the channel estimate will be accurate. Thus, the equalized vector $\hat{\boldsymbol{s}}(n)$ will be affected only by a complex scale ambiguity with respect to the input vector $\boldsymbol{s}(n)$, inherently present in all blind equalization methods.

4. NUMERICAL RESULTS

In this section we will present some numerical results that underline the main features of FB-CMA. To avoid dependence of the method's performance on the channel gain, in our simulations we implemented two modifications in eq. (21): i) we normalize the step size using $\mu/\sqrt{\boldsymbol{x}^H\boldsymbol{x}}$ rather than μ; ii) when the cost function goes below 0.05 we do not normalize the channel estimate as in (22), but we simply set $\hat{\boldsymbol{h}}(n) = \hat{\boldsymbol{h}}_{m_0}(n)$. Furthermore, we adopt the SNR definition $SNR := \mathrm{tr}(\boldsymbol{F}^H\boldsymbol{F})/\sigma_v^2$, where σ_v^2 is the noise variance.

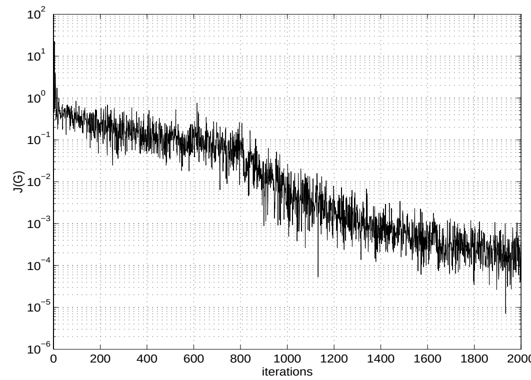

Figure 1: CMA cost fuction vs. number of iterations.

Example 1 *(CMA for TZ-TDMA)* An interesting special TZ-precoder is $\boldsymbol{F} = \boldsymbol{I}$, corresponding to a conventional TDMA transmission scheme where TZ are appended to consecutive blocks of data. Contrary to conventional CMA, even this simple modification is sufficient to meet the conditions of Thm. 2 and guarantee the existence of a ZF equalizer of predetermined finite order for any channel. In our simulation we considered the third order channel with impulse response $\boldsymbol{h} = (1,1,-1,-1)^T$, that has a double zero in -1 and one zero in 1, a case where conventional S–CMA will experience ill convergence. Notice that downsampling by two this channel impulse response we obtain two identical first order channels $\boldsymbol{h}_1$ and $\boldsymbol{h}_2$, with one common zero located at 1, a very difficult case also for FS-CMA. Fig. 1 shows the learning curve of our FB-CMA for $M = 6$ and $P = M + L = 9$ at an SNR of 20 dB, for QPSK symbols. The scattering diagram at the last iteration is shown in Fig. 2.The step size is $\mu = \lambda = 0.05$. From the figures we observe the good convergence properties of the algorithm with relatively short data records. In Fig. 3, we show the ISI after equalization for block sizes $M = (4,6,8)$. We define ISI as

$$ISI = \frac{||\{\boldsymbol{G}\boldsymbol{H}\boldsymbol{F}\}_{j\neq i}||}{||\boldsymbol{G}\boldsymbol{H}\boldsymbol{F}||} \qquad (25)$$

where $||\ ||$ indicates Frobenious norm. Increasing the block size increases the transmission efficiency but, as evidenced by the curves in Fig. 3, the trade off is slower convergence. In this case it suffices to increase M as much as 8 to experience severe degradation in performance. However, even for bigger size blocks, the average performance over Rayleigh

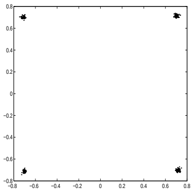

Figure 2: $\hat{\boldsymbol{s}}_m(N)$.

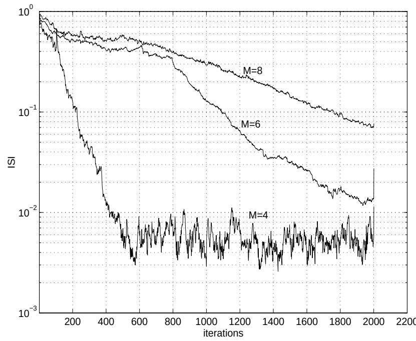

Figure 3: $M = (4, 6, 8)$; ISI vs. number of iterations.

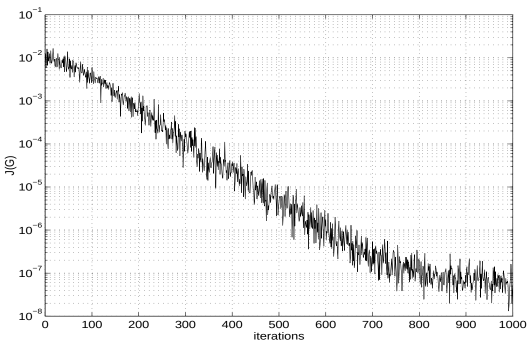

Figure 4: CMA cost fuction vs. number of iterations.

fading is much better, as will be evident from the next example.

Example 2 *(FB-CMA for downlink CDMA)* In this example we show the performance of the FB-CMA method for a TZ precoder built with Walsh-Hadamard codes. These codes have been selected as spreading codes for next generation cellular system standard (UMTS). Our adaptive algorithm recovers the multiplexed data transmitted by the base station and corrupted by an unknown multipath channel. We simulated a system using QPSK symbols, with $M = 12$ and $L = 2$. The channel is generated as an FIR filter with two Gaussian complex random taps with variance 1. TZ's are appended to each symbol so that the symbol duration is $P = M + L$. The step sizes are $\mu = \lambda = 0.05$ and the $SNR = 30\ dB$. Fig. 4 shows the curve of the CMA cost function $J(\boldsymbol{G})$, averaged over 40 Rayleigh channels, versus the iteration number and Fig. 5 shows the corresponding average ISI after equalization. Although, compared to existing blind adaptive multiuser detection methods, the performance is extremely encouraging, there is a limitation in the application of our method to a CDMA scenario: knowledge of the precoder matrix required by our FB-CMA, implies the knowledge of all possible users' codes by every user (there is no need to know whether they are active or not).

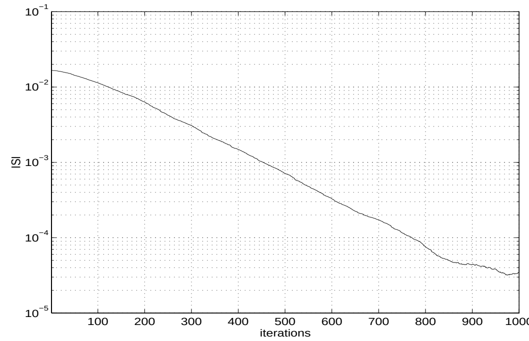

Figure 5: ISI vs. number of iterations.

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