REGION NUMBER DETERMINATION IN AUTOMATIC IMAGE SEGMENTATION BASED ON BKYY MODEL SELECTION CRITERION

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ABSTRACT

In the feature space clustering approach to image segmentation, each cluster corresponds to a region in the image. Determination of the appropriate number of clusters (regions) has long been an open problem. In this paper, we apply the Baysian-Kullback Ying Yang (BKYY) Model Selection Criterian on this feature space clustering problem to give an appropriate number of regions in image segmentation. Experimental demonstrations show the results of image segmentation with this automatically determined number of regions.

Keywords: Automatic Image Segmentation, Finite Mixture Model, BKYY Model Selection Criterion, EM algorithm.

1. INTRODUCTION

Image segmentation is an important aspect of computer vision. The goal of it is to partition a given image into some regions corresponding to different objects or the background. Also, it is a basic step for high-level image understanding and interpretation. There are a wide variety of image segmentation techniques [1], among which feature space clustering is one of the most popular methods. Pixels of the same segment can usually be characterized by certain features. These features are quantified into feature variables so that pixels of the same segment essentially have similar values of the feature variables, and pixels of different segments have dissimilar values. Then image segmentation can be performed by clustering the feature space and mapping each point back to the spatial pixel.

Among various clustering techniques like the well known k-mean algorithm, competitive learning, etc, the finite mixture of densities, in particular mixture of Gaussian model, has been widely used in many practical situations. The maximum likelihood approach has been utilized extensively to the fitting of finite mixture models [5, 7]. This approach has attracted considerable interest in the image segmentation field in recent years [2, 3, 4].

In paper [2, 3, 4], the authors use color or grey level feature space, or Gauss-Markov random field in image domain. And by assuming data points are generated from finite mixture distribution, they estimate the probability density using EM algorithm or Generalized EM algorithm with pre-assigned cluster number in feature space. With the learned probability density function, Bayesian pixel classification method was used to produce the image segmentation in paper [4].

In fact, in the feature space clustering method to image segmentation, the number of segment to be yield can be considered as the number of cluster, k, in the feature space. In the clustering methods mentioned above, k has to be specified in advance. If k is correctly selected, good clustering result can be yielded, otherwise, data points cannot be grouped into appropriate clusters and image segmentation cannot be performed appropriately. To determine a reasonable region number is one of difficult things in machine learning. This problem affects the ability to automatically interpret images by a machine, which has been one of the major challenges in computer vision. In the past, most of the work use pre-assigned number of regions or heuristics to determine the number of regions.

Recently, a Bayesian-Kullback scheme, called the YING-YANG Learning Theory and System (BKYY), or YING-YANG machine, has been proposed to act as a general learning scheme for unifying the existing major unsupervised and supervised learning schemes[8, 9]. The theory provided a model selection criterion for determining an appropriate cluster number in clustering problem [10]. The experimental demonstrations [10, 11] show it work well in seleting cluster number.

In this work, we apply BKYY model selection criterion to determine region number to perform automatic image segemtation. The algorithm we used in clustering is the EM algorithm on finite mixture model as done in [4].

2. BACKGROUND

2.1. Clustering using Finite Mixture Model

The data points to be clustered are assumed to be samples from a mixture of k Gaussian densities:

$$P(\mathbf{x}, \Theta) = \sum_{y=1}^{k} \alpha_y G(\mathbf{x}, \mathbf{m}_y, \Sigma_y),$$
$$\text{with} \quad \alpha_y \geq 0, \text{ and } \sum_{y=1}^{k} \alpha_y = 1 \tag{1}$$

where

$$G(\mathbf{x}, \mathbf{m}_y, \Sigma_y) = \frac{\exp[-\frac{1}{2}(\mathbf{x}-\mathbf{m}_y)^T \Sigma_y^{-1} (\mathbf{x}-\mathbf{m}_y)]}{(2\pi)^{d/2} |\Sigma_y|^{\frac{1}{2}}} \tag{2}$$

is the multivariate Gaussian density function, $\mathbf{x}$ is the random vector, d is the dimension of vector $\mathbf{x}$, and the set

$\Theta = \{\alpha_y, \mathbf{m}_y, \Sigma_y\}_{y=1}^{k}$ are parameters of the finite mixture model with the conventional notation α_y being the mixing weights, $\mathbf{m}_y$ and Σ_y being the mean vector and covariance of the yth component. These parameters are to be estimated by maximum likelihood learning (MLE) with ordinary EM algorithm [6, 7]:

E-step:

$$P(y|\mathbf{x}_i) = \frac{\alpha_y G(\mathbf{x}_i, \mathbf{m}_y, \mathbf{\Sigma}_y)}{\sum_{y=1}^{k} \alpha_y G(\mathbf{x}_i, \mathbf{m}_y, \mathbf{\Sigma}_y)},$$
$$\text{with } y = 1, ..., k, \tag{3}$$

M-step:

$$\alpha_y^{new} = \frac{1}{N} \sum_{i=1}^{N} \frac{\alpha_y G(\mathbf{x}_i, \mathbf{m}_j, \mathbf{\Sigma}_j)}{\sum_{y=1}^{k} \alpha_y G(\mathbf{x}_i, \mathbf{m}_y, \mathbf{\Sigma}_y)} = \frac{1}{N} \sum_{i=1}^{N} P(y|\mathbf{x}_i) \tag{4}$$

$$\mathbf{m}_y^{new} = \frac{\sum_{i=1}^{N} P(y|\mathbf{x}_i)\mathbf{x}_i}{\sum_{i=1}^{N} P(y|\mathbf{x}_i)} \tag{5}$$

$$\mathbf{\Sigma}_y^{new} = \frac{\sum_{i=1}^{N} P(y|\mathbf{x}_i)[(\mathbf{x}_i - \mathbf{m}_y^{old})(\mathbf{x}_i - \mathbf{m}_y^{old})^T]}{\sum_{i=1}^{N} P(y|\mathbf{x}_i)}. \tag{6}$$

where N is the total number of data points. The two step are iterated until convergence.

Each component of the finite mixture is regarded as one cluster. After learning of the parameter, the posterior probability $p(y|\mathbf{x}_i)$ represent the probability that data point $\mathbf{x}_i$ belongs to cluster y. Now we use Bayesian decision $y* = \arg\max_y p(y|\mathbf{x}_i)$ to classify $\mathbf{x}_i$ into cluster $y*$

2.2. BKYY Model Selection Criterion

As stated in [8, 9], the above finite mixture model is a special case of the BKYY system and the EM algorithm can be rederived based on the minimization of the cost function (namely, the Kullback divergence between the two Baysian representation, of the joint density of $\mathbf{x}$ and y) used in the theory. The number of cluster, k, is actually a structural scale parameter of the BKYY system and can be selected according to the Model Selection Criterion [9, 10], which can be summarized as follows.

According to [10] (Section 3), we can write the cost function as a function of k

$$J_2(k) = \frac{1}{2} \sum_{y=1}^{k} \alpha_y^* \ln |\Sigma_y^*| - \sum_{y=1}^{k} \alpha_y^* \ln \alpha_y^*. \tag{7}$$

where the asterisked parameters α_y^* and Σ_y^* denotes the resulting parameters after maximum likelihood learning of the system with the assigned k. The selection criterion of k is the one that minimize $J_2(k)$, i.e. the appropriate $k^* = \arg\min_k J_2(k)$.

In practice, we usually start with $k = 1$, estimating the best parameter Θ^* by EM algorithm, and compute $J_2(k)$. Then we proceed to $k \to k+1$, computing $J_2(k = 2)$ and so on. After getting a series of $J_2(k)$, we choose the minimal one and regard the corresponding k as an appropriate cluster number.

3. APPLICATION TO IMAGE SEGMENTATION AND EXPERIMENTAL DEMONSTRATION

As a first trial, we slecte the 3-dimensional RGB color vector of the pixels as feature variables. Features on spatial relations of the pixels should be added in further work. For a $m \times n$ pixel image, the $N = mn$ RGB vectors of the pixels are inputed as data points in the feature space for clustering. Then the parameter learning for each k and the determination of k^* according to the model selection criterion mentioned in section 2.2 is run. After k^* is determined, each pixel is classified into the one of the k^* clusters, i.e., segments, according to their posterior probability as mentioned in section 2.1. In this way, image segmentation with auto-determination of segment number is performed.

In our experiments, we tried two commonly used images - the “house” image and “sailboat” image. Both images are of 128 $\times$ 128 pixels. The image segmentation results are shown in Figures 1 and 2 respectively, though only grey-scaled rather color pictures are printed out.

From the J_2 - k curve in Fig. 1(c), we can see that there are two local minima. One is at $k = 3$ and the other is at $k = 9$. Intuitively speaking, this illustrates that 3 is an appropriate number of segments for rough segmentation and 9 is another appropriate number of segments which gives more refined segments. Sub-figure 1(d)-(h) show the resulting segmentation with $k = 2, 3, 5, 9, 12$ for comparison. For the 2-region segmentation, we can see that the roof is mixed with the wall. 3-region segmentation can represent the main structure of the house image. While segmentation with more regions increases more details of the image, there is hardly significant increase in quality of segmentation when increasing k from 9 to 12, and 9 can more or less be regarded as an appropriate segment number.

In Fig. 2, results of 5, 8 and 11-region segmented images are shown. $k = 8$ is determined to be an appropriate segment number by the model selection criterion. The 8-region segmentation can reconstruct the original "sailboat" image well, while the sky and cloud are mixed in the 5-region segmentation and the 11-region segmentation does not show significant increase in segmentation quality.

4. CONCLUSION

While most previous works on feature space clustering for image segmentation need manually specifying the number of segments in *a priori*, we apply the model selection criterion of BKYY theory to this approach and obtain a method for automatic determination of an appropriate number of segmentation. In other words, it is possible that the BKYY model selection criterion give reasonable insight on the structure of the presented image.

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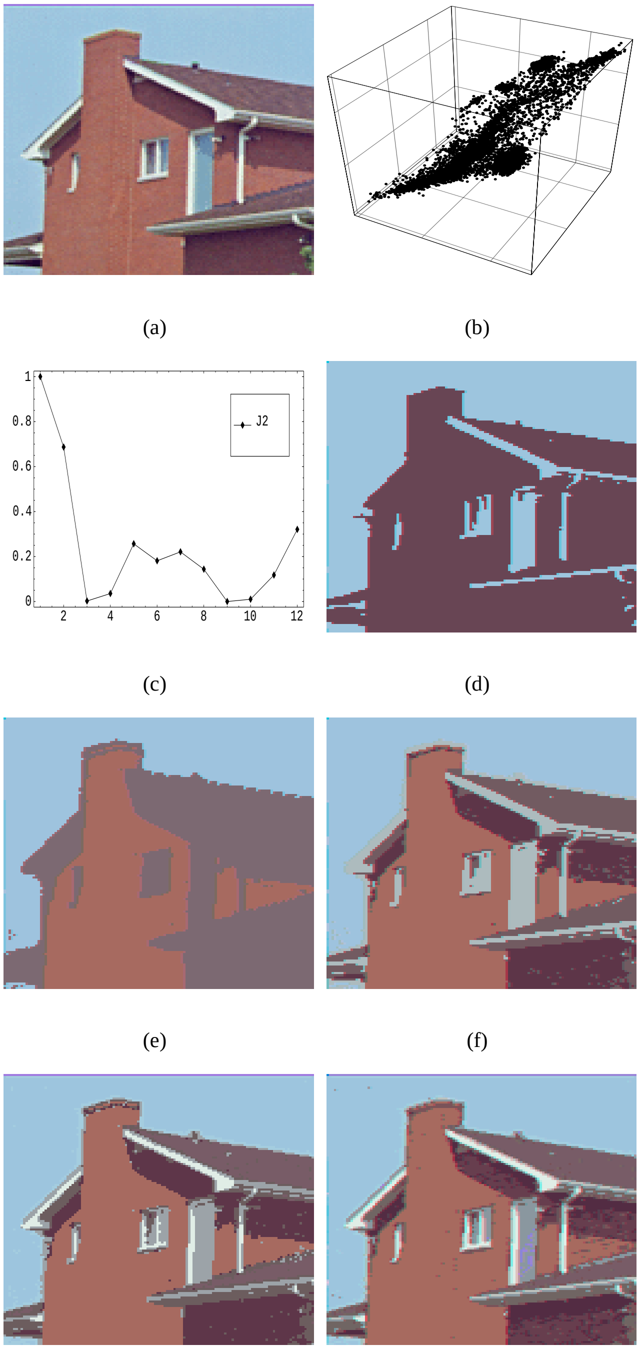


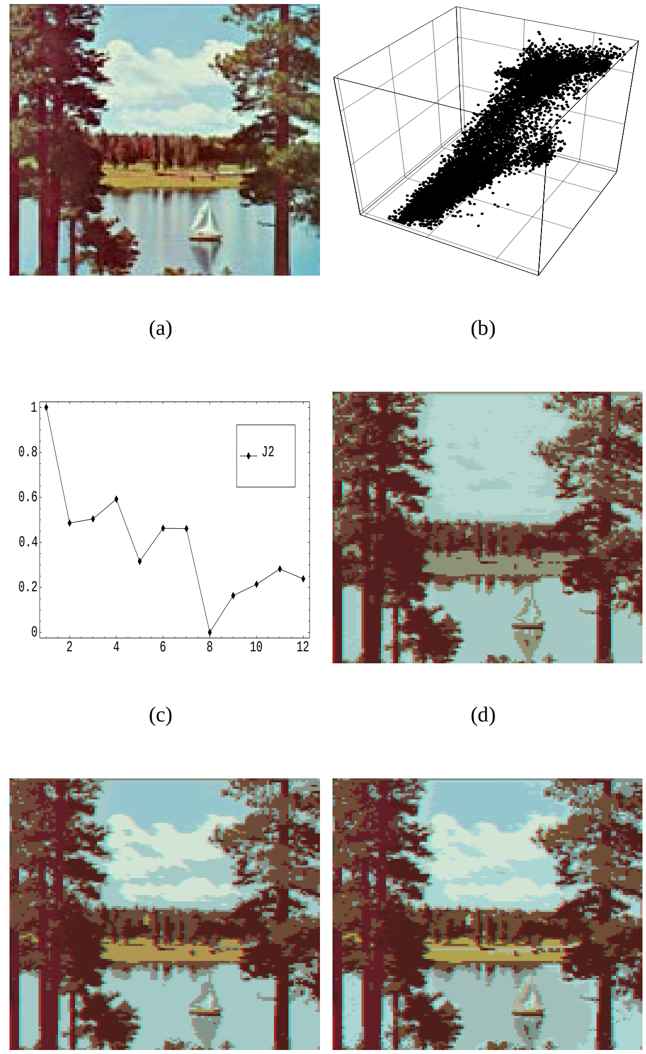
(a) (b)

(c) (d)

(e) (f)

(g) (h)

Figure 1: “House” image. (a) original image, 128×128 pixels; (b) feature space data distribution; (c) $J_2 - k$ curve; (d) 2-region segmented image. (e) 3-region segmented image. (f) 5-region segmented image. (g) 9-region segmented image. (h) 12-region segmented image. Each region is represented by its mean vector color. 3-region and 9-region segmentation is determined by BKYY criterion.



(a) (b)

(c) (d)

(e) (f)

Figure 2: “Sailboat” image. (a) original image, 128×128 pixels; (b) feature space data distribution; (c) $J_2 - k$ curve; (d) 5-region segmented image; (e) 8-region segmented image; (f) 11-region segmented image. Each region is represented by its mean vector color. BKYY criterion selected region number is 8.