A GENERAL RECURSIVE WEIGHTED MEDIAN FILTER STRUCTURE ADMITTING REAL-VALUED WEIGHTS AND THEIR ADAPTIVE OPTIMIZATION

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ABSTRACT

A generalized *recursive weighted median* (RWM) filter structure admitting negative weights is introduced. RWM filters offer a number of advantages over their non-recursive counterparts, including a significant reduction in computational complexity, increased robustness to noise, and the ability to model "resonant" or vibratory behavior. RWM filters also provide advantages over linear IIR filters, offering near perfect "stop-band" characteristics. Unlike linear IIR filters, RWM filters are always stable under the bounded-input bounded-output criterion, regardless of the values taken by the feedback filter weights. A novel "recursive decoupling" adaptive optimization algorithm for the design of RWM filters is also introduced.

1. INTRODUCTION

This paper introduces a class of recursive weighted median filters, admitting real-valued weights, which are analogous to the class of infinite impulse response (IIR) linear filters. Recursive filter structures are particularly important because they can be used to model "resonances" which appear in many natural phenomena such as in speech. In addition, IIR linear filters often lead to reduced computational complexity reduction. Much like IIR linear filters provide these advantages over linear FIR filters, recursive WM filters also exhibit superior characteristics compared to non-recursive WM filters. For instance, an infinitely iterated use of a weighted median filter can often be synthesized by a single pass of a properly designed recursive weighted median filter. Further, recursive WM filters can synthesize non-recursive WM filters of much larger window sizes. In terms of noise attenuation, recursive median smoothers have far superior characteristics than their non-recursive counterparts.

It will also be shown in this paper that RWM filters can provide advantages over linear IIR filters. Notably, "bandpass" and "highpass" RWM filters exhibit near perfect "stop-band" characteristics not attainable with linear IIR filters. Moreover, unlike their IIR filter counterparts, RWM filters are always stable under the bounded-input bounded-output criterion, regardless of the values taken by the feedback filter weights. In the presence of noise, the advantages of RWM filters over IIR filters are even more overwhelming, with RWM filters offering robustness to noise levels that are unacceptable with traditional IIR filters.

In practice, the (real-valued) filter coefficients of the proposed RWM filter structures must be determined in some fashion. This paper presents, for the first time, an optimization method for the design of recursive WM filters. A novel "recursive decoupling" adaptive optimization algorithm for the design and optimization of RWM filter weights is developed under the mean absolute error (MAE) criterion. In this framework, the previous outputs used to compute the RWM filter output are replaced by previous desired outputs. Thus, the recursive WM filter is approximated by a two-input, single output filter that depends on the input samples and on delayed samples of the desired response. This structure avoids the feedback inherent in the recursive operation, therefore, leading to a much simpler derivation of the gradient in the steepest descent algorithm used to update the filter coefficients. A more detailed description of RWM filters and their optimization algorithm can be found in [1].

This work was supported in part by the NATIONAL SCIENCE FOUNDATION under grants MIP-9530923 and CDA-9703088.

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2. RECURSIVE WM FILTERS ADMITTING REAL-VALUED WEIGHTS

In order to define the class of RWM filters, it is best to first recast the similarities between linear FIR filters and weighted median filters. Given an observation set $X_1, \cdots, X_N$, the sample mean $\bar{\beta} = \text{MEAN}(X_1, \cdots, X_N)$ can be generalized to linear FIR filters as

$$\bar{\beta} = \text{MEAN}(W_1 \cdot X_1, \cdots, W_N \cdot X_N) \quad (1)$$

with $W_i \in R$. It will be seen shortly that it is useful to rewrite (1) as

$$\bar{\beta} = \text{MEAN}(|W_1| \cdot sign(W_1)X_1, \cdots, |W_N| \cdot sign(W_N)X_N), \quad (2)$$

where the sign of the weight affects the corresponding input sample and the weighting is constrained to be non-negative. It was shown in [2] that the sample median $\tilde{\beta} = \text{MEDIAN}(X_1, \cdots, X_N)$, which plays an analogous role to the sample mean in location estimation, can be extended to a general weighted median filter structure admitting positive and negative weights as

$$\tilde{\beta} = \text{MEDIAN}(|W_1| \diamond sign(W_1)X_1, \cdots, |W_N| \diamond sign(W_N)X_N), \quad (3)$$

with $W_i \in R$ for $i = 1, 2, \cdots, N$, and where $\diamond$ is the replication operator defined as $W_i \diamond X_i = \overbrace{X_i, X_i, \cdots, X_i}^{W_i \text{ times}}$. Again, the weight signs are uncoupled from the weight magnitudes and are merged with the observation samples. The weight magnitudes play the equivalent role of positive weights in the framework of weighted median smoothers [3]. Although the weights in (3) may seem restricted to integer values, a more general interpretation of the $\diamond$ operator exists, as will be presented shortly.

The filters in (2) and (3) can be thought of as non-recursive filter duals. This duality is next extended to their recursive forms.

The general structure of linear IIR filters is defined by the difference equation

$$Y(n) = \sum_{\ell=1}^{N} A_\ell\, Y(n-\ell) + \sum_{k=0}^{M} B_\ell\, X(n+k), \qquad (4)$$

where the output is formed not only from the input, but also from previously computed outputs. The filter weights consist of two sets: the feedback coefficients $\{A_\ell\}$, and the feed-forward coefficients $\{B_k\}$. In all, $(N+M+1)$ coefficients are needed to define the recursive difference equation of (4).

The generalization of (4) to a RWM filter structure is straightforward. Following a similar approach to that introduced in [2], the summation operation is replaced with the *median* operation, and the *multiplicative* weighting is replaced by weighting through *signed replication*:

$$\begin{aligned} Y(n) \;=\; & \text{MEDIAN}\left(|A_\ell| \diamond \text{sgn}(A_\ell)\, Y(n-\ell)|_{\ell=1}^{N},\right. \\ & \left.|B_k| \diamond \text{sgn}(B_k)\; X(n+k)|_{k=0}^{M}\right) \end{aligned} \qquad (5)$$

Note that if the weights A_ℓ and B_k are constrained to be positive, (5) reduces to the recursive WM smoother previously studied in [4]. The recursive WM filter output for non-integer weights can be determined as follows:

1. Calculate the threshold $T_0 = \frac{1}{2}\left(\sum_{\ell=1}^{N}|A_\ell| + \sum_{k=0}^{M}|B_k|\right)$.
2. Jointly sort the "signed" past output samples $\text{sgn}(A_\ell)Y(n-\ell)$ and the "signed" input observations $\text{sgn}(B_k)X(n+k)$.
3. Sum the magnitudes of the weights corresponding to the sorted "signed" samples, beginning with the maximum and continuing down in order.
4. If $2\,T_0$ is an even number, the output is the average between the signed sample whose weight magnitude causes the sum to become $\geq T_0$ and the next smaller signed sample, otherwise the output is the signed sample whose weight magnitude causes the sum to become $\geq T_0$.

3. STABILITY OF RWM FILTERS

Unlike linear IIR filters, recursive WM filters are garanteed to be stable under the bounded-input bounded-output criterion.

Property 1. Recursive weighted median filters, as defined in (5), are stable under the bounded-input bounded-output criterion, regardless of the values taken by the feedback coefficients $\{A_\ell\}$ for $\ell = 1, 2, \cdots, N$.

Proof: Given a bounded input signal $X(n)$ such that $|X(n)| < M_x$, and initial conditions $Y(-\ell)$, $\ell = 1\cdots N$, denote $M_y = \max(|Y(-N)|, \cdots, |Y(-1)|)$ and $M_{xy} = \max(M_x, M_y)$. $Y(0)$ is the output of a median filter with inputs $[Y(-N)\cdots Y(-1), X(0), X(1)\cdots X(M)]$. Since the output of the RWM operator, as defined in Section 2, is always within the dynamic range of the joint input and previous output data, $Y(0)$ is restricted to the dynamic range of these inputs. But $|Y(-\ell)| < M_{xy}$ for $\ell = 1, 2, \cdots, N$ and $|X(n)| < M_{xy}$ for all n. Hence, we have $|Y(0)| < M_{xy}$. Using this argument recursively for $Y(1), Y(2)\cdots$, with $Y(i)$ depending on $[Y(i-N), \cdots, Y(i-1), X(i)\cdots X(i+M)]$, $i = 1, 2\cdots$, it follows by induction that $|Y(n)| < M_{xy}$ for all n; hence, the output is bounded.

4. RECURSIVE WM FILTERS AND THRESHOLD DECOMPOSITION

Threshold decomposition is a powerful theoretical tool used in the analysis and design of RWM filters. For the purpose of this paper, we adapt a threshold decomposition formulation similar to that described in [2].

Consider the real-valued vector $\mathbf{Z} = [Z_1 \cdots Z_L]^T$. Threshold decomposition maps this real-valued vector to an infinite set of binary vectors $\mathbf{z}^q \in \{-1, 1\}^L$, $q \in (-\infty, \infty)$, where

$$\mathbf{z}^q = [sgn(Z_1 - q)\cdots sgn(Z_L - q)]^T = [z_1^q \cdots z_L^q]^T. \qquad (6)$$

In (6), sgn denotes the sign function defined as

$$sgn(Z_i - q) = \begin{cases} +1 & \text{if } Z_i - q \geq 0 \\ -1 & \text{if } Z_i - q < 0. \end{cases}$$

The original vector $\mathbf{Z}$ can be exactly reconstructed from its binary representation through the inverse process [2] as

$$Z_i = \frac{1}{2}\int_{-\infty}^{+\infty} z_i^q\, dq \qquad \text{for } i = 1, \cdots, L. \qquad (7)$$

Thus, a real-valued vector has a unique threshold signal representation, and vice versa: $\mathbf{Z} \overset{T.D.}{\Longleftrightarrow} \{\mathbf{z}^q\}$, where $\overset{T.D.}{\Longleftrightarrow}$ denotes the one-to-one mapping provided by the threshold decomposition operation. Since q can take any real value, the infinite set of binary vectors $\{\mathbf{z}^q\}$ seems redundant in representing the real valued vector $\mathbf{Z}$. Indeed, some of the binary vectors $\{\mathbf{z}^q\}$ are infinitely repeated. For $Z_{(1)} < q \leq Z_{(2)}$, for instance, all the binary vectors $\{\mathbf{z}^q\}$ are identical, where $Z_{(i)}$ denotes the ith order statistic of $[Z_1, \cdots, Z_L]^T$. As shown in [3], threshold signal representation can be simplified based on the fact that there are at most $L+1$ different binary vectors $\{\mathbf{z}^q\}$ for each observation vector $\mathbf{Z}$. Using this fact, (6) reduces to

$$\mathbf{z}^q = \begin{cases} [1\cdots 1]^T & \text{for } -\infty < q \leq Z_{(1)} \\ [z_1^{Z^+_{(i)}} \cdots z_L^{Z^+_{(i)}}]^T & \text{for } Z_{(i)} < q \leq Z_{(i+1)}, \\ & 1 \leq i \leq L-1 \\ [-1, -1\cdots -1]^T & \text{for } Z_{(L)} < q < +\infty \end{cases} \qquad (8)$$

where $Z^+_{(i)}$ denotes a value on the real line approaching $Z_{(i)}$ from the right. Using threshold signal decomposition, the recursive WM operation in (5) can be implemented as

$$\begin{aligned} Y(n) = \text{MEDIAN}(&|A_\ell| \diamond \frac{1}{2}\int_{-\infty}^{+\infty} sgn[sgn(A_\ell)Y(n-\ell) - q]\,|_{\ell=1}^{N}, \\ &|B_k| \diamond \frac{1}{2}\int_{-\infty}^{+\infty} sgn[sgn(B_k)X(n+k) - q]|_{k=0}^{M}). \end{aligned} \qquad (9)$$

At this point, we resort to the weak superposition property of the nonlinear median operator, which states that applying a weighted median operator to a real-valued signal is equivalent to decomposing the real-valued signal using threshold decomposition, appling median operator to each binary signal separately and then adding the binary outputs to obtain the real-valued output. This superposition property allows us to interchange the integral and median operators in (9), leading to

$$Y(n) = \frac{1}{2}\int_{-\infty}^{-\infty} \text{MEDIAN}\left(|A_\ell| \diamond sgn[sgn(A_\ell)Y(n-\ell) - q]|_{\ell=1}^{N},\right.$$
$$\left.|B_k| \diamond sgn[sgn(B_k)X(n+k) - q]|_{k=0}^{M}\right). \qquad (10)$$

To simplify the above expression, let $\{\mathbf{s}_Y^q\}$ and $\{\mathbf{s}_X^q\}$ denote the threshold decomposition of the signed past output samples and the signed input samples respectively, i.e. $\mathbf{S}_Y(n) \overset{T.D.}{\Longleftrightarrow} \mathbf{s}_Y^q(n)$, $\mathbf{S}_X(n) \overset{T.D.}{\Longleftrightarrow} \mathbf{s}_X^q(n)$, where $\mathbf{S}_Y(n) = [sgn(A_1)Y(n-1)\cdots sgn(A_N)Y(n-N)]^T$ and $\mathbf{S}_Y(n) = [sgn(B_0)X(n)\cdots sgn(B_M)X(n+M)]^T$. With this notation and following a similar approach to that presented in [2], it can be shown that (10) reduces to

$$Y(n) = \frac{1}{2}\int_{-\infty}^{-\infty} sgn\left(\mathbf{A}_a^T\mathbf{s}_Y^q(n) + \mathbf{B}_a^T\mathbf{s}_X^q(n)\right)dq \qquad (11)$$

where $\mathbf{A}_a$ is the vector whose elements are the magnitudes of the feedback coefficients: $\mathbf{A}_a = [|A_1|, |A_2|, \cdots, |A_N|]^T$, and $\mathbf{B}_a$ is the vector whose elements are the magnitudes of the feed-forward coefficients: $\mathbf{B}_a = [|B_0|, |B_1|, \cdots, |B_M|]^T$. Note in (11) that the filter's output depends on the signed past outputs, the signed input observations, and the feedback and feed-forward coefficients.

5. ADAPTIVE RWM FILTERING ALGORITHM

In general, the coefficients of the recursive WM filter have to be designed in some optimal fashion. In this section, we develop the first adaptive optimization algorithm for the design of recursive WM filters under the MAE criterion. Threshold decomposition developed in the last section is used to find adaptive solution for the optimal weights.

Consider an observed process $\{X(n)\}$ that is statistically related to a desired process $\{D(n)\}$. Further, assume that both processes are jointly stationary. Under the MAE criterion the goal is to determine the weights $\{A_\ell\}|_{\ell=1}^{M}$ and $\{B_k\}|_{k=0}^{M}$ so as to minimize the cost function

$$J(A_1\cdots A_N, B_0\cdots B_M) = J(\mathbf{A},\mathbf{B}) = E\{|D(n) - Y(n)|\} \qquad (12)$$

where $E\{\cdot\}$ denotes statistical expectation and $Y(n)$ is the output of the recursive WM filter given by (5). To form an iterative optimization algorithm, the steepest descent algorithm is used, in which the gradient of the cost function (∇J) has to be computed to update the filter weights. Due to the feedback operation inherent in the recursive WM filter, however, the computation of ∇J becomes intractable.

To overcome this problem, the optimization framework referred to as *Equation Error Formulation* is used [5]. Equation Error Formulation is used in the design of linear IIR filters and is based on the fact that ideally the filter's output is close to the desired response. Hence, the previous outputs $\{Y(n-\ell)\}|_{\ell=1}^{N}$ in (5) are replaced with the previous desired outputs $\{D(n-\ell)\}|_{\ell=1}^{N}$ to obtain a two-input, single-output filter that depends on the input samples $\{X(n+k)\}|_{k=0}^{M}$ and on the delayed samples of the desired response $\{D(n-\ell)\}|_{\ell=1}^{N}$:

$$\hat{Y}(n) = \text{MEDIAN}\left(|A_\ell| \diamond \text{sgn}(A_\ell)\, D(n-\ell)|_{\ell=1}^{N},\right.$$
$$\left.|B_k| \diamond \text{sgn}(B_k)\, X(n+k)|_{k=0}^{M}\right). \qquad (13)$$

The approximation leads to an output $\hat{Y}(n)$ that does not depend on delayed output samples and, therefore, the filter no longer introduces feedback, reducing the RWM filter to a two-input, single output non-recursive system. This "recursive decoupling" optimization approach provides the key to a gradient-based optimization algorithm for recursive WM filters.

According to the approximate filtering structure, the cost function to be minimized is $\hat{J}(\mathbf{A},\mathbf{B}) = E\{|D(n) - \hat{Y}(n)|\}$, where $\hat{Y}(n)$ is the non-recursive filter output of (13). Since $D(n)$ and $X(n)$ are not functions of the feedback coefficients, the derivative of $\hat{J}(\mathbf{A},\mathbf{B})$ with respect to the filter weights is non-recursive and its computation is straightforward.

The adaptive optimization algorithm is derived as follows. Define the vector $\mathbf{S}(n) = [\mathbf{S}_D^T(n),\ \mathbf{S}_X^T(n)]^T$ as that containing the signed samples in the sliding window of the two-input, single-output non-recursive filter of (13) at time n, where $\mathbf{S}_D(n) = [sgn(A_1)\ D(n-1)\cdots sgn(A_N)D(n-N)]^T$ and $\mathbf{S}_X(n) = [sgn(B_0)X(n)\ \cdots sgn(B_M)X(n+M)]^T$. With this notation and using threshold decomposition, $\hat{J}(\mathbf{A},\mathbf{B})$ becomes

$$\hat{J}(\mathbf{A},\mathbf{B}) = \frac{1}{2}E\left[\left|\int_{-\infty}^{+\infty} e(n)dq\right|\right] \qquad (14)$$

where $e^q(n) = sgn(D(n)-q) - sgn\left(\mathbf{A}_a^T\mathbf{s}_D^q(n) + \mathbf{B}_a^T\mathbf{s}_X^q(n)\right)$, with $\{\mathbf{s}_D^q(n)\}$ as the corresponding threshold decomposition of the vector $\mathbf{S}_D(n)$. Note that for a fixed n, the integral operator acts on a strictly negative function or a strictly positive function. Therefore, the absolute value and integral operators in (15) can be interchanged. Moreover, it can be shown that $e^q(n)$ takes on values in the set $\{-2, 0, 2\}$; therefore, the absolute value operator can be replaced by a properly scaled second power operator:

$$\hat{J}(\mathbf{A},\mathbf{B}) = \frac{1}{4}\int_{-\infty}^{+\infty} E\left[(e^q(n))^2\right]dq, \qquad (15)$$

where we have used the linear property of the expectation operator. Taking derivatives of the above expression with respect to the filter coefficients A_ℓ and B_k yields, respectively,

$$\frac{\partial\hat{J}}{\partial A_\ell}(\mathbf{A},\mathbf{B}) = \frac{1}{2}\int_{-\infty}^{+\infty} E\left[e^q(n)\frac{\partial}{\partial A_\ell}sgn(\mathbf{A}_a^T\mathbf{s}_D^q + \mathbf{B}_a^T\mathbf{s}_X^q)\right];$$
$$(16)$$
$$\frac{\partial\hat{J}}{\partial B_k}(\mathbf{A},\mathbf{B}) = -\frac{1}{2}\int_{-\infty}^{+\infty} E\left[e^q(n)\frac{\partial}{\partial B_k}sgn(\mathbf{A}_a^T\mathbf{s}_D^q + \mathbf{B}_a^T\mathbf{s}_X^q)\right].$$

Since the sgn function has a discontinuity at the origin, it introduces the $dirac$ function in its derivative which is not convenient for further analysis. In order to overcome this difficulty, the sgn function is approximated by the differentiable hyperbolic tangent function $sgn(x) \approx tanh(x)$ whose derivative is $\frac{\partial}{\partial x}tanh(x) = sech^2(x)$. Using this approximation in (16) and after some simplifications, it follows that

$$\frac{\partial\hat{J}}{\partial A_\ell} \approx -\frac{1}{2}\int_{-\infty}^{+\infty} E\left[e^q(n)sech^2(\mathbf{A}_a^T\mathbf{s}_D^q + \mathbf{B}_a^T\mathbf{s}_X^q)\ sgn(A_\ell)s_{D_\ell}^q\right]dq;$$
$$(17)$$
$$\frac{\partial\hat{J}}{\partial B_k} \approx -\frac{1}{2}\int_{-\infty}^{+\infty} E\left[e^q(n)sech^2(\mathbf{A}_a^T\mathbf{s}_D^q + \mathbf{B}_a^T\mathbf{s}_X^q)\ sgn(B_k)s_{X_k}^q\right]dq,$$

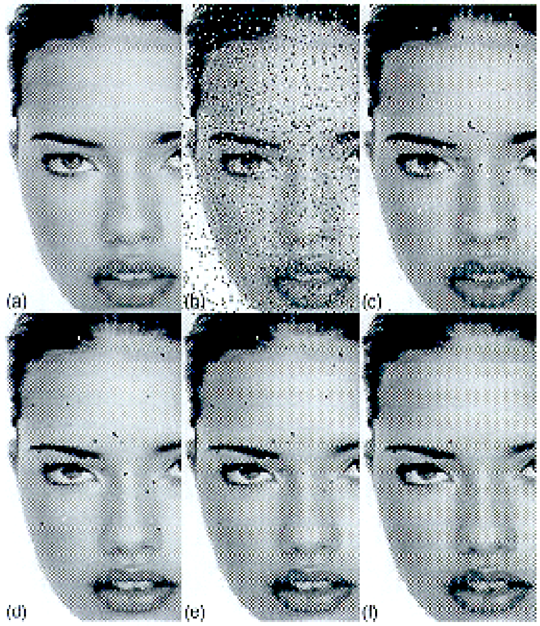

Figure 1: (a) Original "portrait" image, (b) image with salt and pepper noise, (c) non-recursive center WM filter, (d) optimal non-recursive WM filter, (e) recursive center WM filter, (f) optimal RWM smoother.

where $s^q_{D_\ell}$ and $s^q_{X_k}$ are the ℓth and kth components of the vectors $\mathbf{s}^q_D$ and $\mathbf{s}^q_X$ respectively. Assuming that the desired output $D(n)$ is one of the signed samples, say $S_{(m)}$, and that the actual output $\hat{Y}(n)$ is $S_{(j)}$, $e^q(n) = 0$ for $q \in \{(-\infty, \min(S_{(m)}, S_{(j)})] \cup (\max(S_{(m)}, S_{(j)}), +\infty)\}$, thus $\frac{\partial \hat{J}}{\partial A_\ell}$ reduces to

$$\frac{\partial \hat{J}}{\partial A_\ell} \approx -\frac{1}{2} \int_{\min(S_{(m)},S_{(j)})}^{\max(S_{(m)},S_{(j)})} e^q(n) sech^2 \left(\mathbf{A}_a^T \mathbf{s}_D^q + \mathbf{B}_a^T \mathbf{s}_X^q \right) sgn(A_\ell) s^q_{D_\ell} dq, \tag{18}$$

where the instantaneous estimate for the gradient is used. Evaluating the above integral and using (8) leads to $\frac{\partial \hat{J}}{\partial A_\ell} \approx$

$$-\frac{1}{2} sgn(A_\ell) \sum_{i=\min(m,j)}^{\max(m,j)-1} (S_{(i+1)} - S_{(i)}) e^{S^+_{(i)}}(n) sech^2(\mathbf{A}_a^T \mathbf{s}_D^{S^+_{(i)}} + \mathbf{B}_a^T \mathbf{s}_X^{S^+_{(i)}}) s_{D_\ell}^{S^+_{(i)}} \tag{19}$$

for $\ell = 1 \cdots N$. Similar simplifications can be made to $\frac{\partial \hat{J}}{\partial B_k}$ leading to $\frac{\partial \hat{J}}{\partial B_k} \approx$

$$-\frac{1}{2} sgn(B_k) \sum_{i=\min(m,j)}^{\max(m,j)-1} (S_{(i+1)} - S_{(i)}) e^{S^+_{(i)}}(n) sech^2(\mathbf{A}_a^T \mathbf{s}_D^{S^+_{(i)}} + \mathbf{B}_a^T \mathbf{s}_X^{S^+_{(i)}}) s_{X_k}^{S^+_{(i)}} \tag{20}$$

for $k = 0, 1, \cdots, M$.

Using the gradient, the optimal coefficients can be found through the steepest descent recursive update: $A(n+1) = A(n) + 2\mu[-\frac{\partial \hat{J}}{\partial A_\ell}]$ and $B(n+1) = B(n) + 2\mu[-\frac{\partial \hat{J}}{\partial B_k}]$.

This adaptive optimization algorithm is also suitable for the design of recursive WM smoothers which do not admit negative weight values. Upon closer examination, it turns out that the constraint of having non-negative weights can be accomplished by a projection operator that maps the updated weights to zero whenever they are negatives.

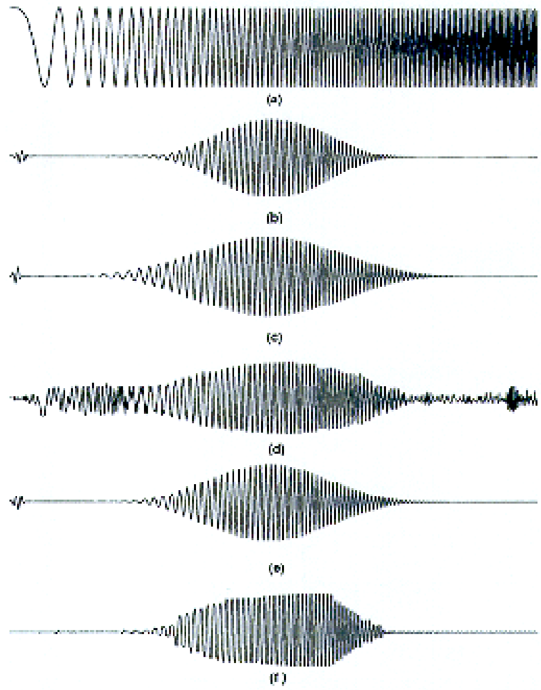

Figure 2: (a) Input test signal, (b) desired signal, (c) linear FIR filter output, (d) non-recursive WM filter output, (e) linear IIR filter output, (f) recursive WM filter output.

6. APPLICATIONS OF RECURSIVE WM FILTERS

6.1. Image Denoising

Figures 1(a) and 1(b) show the original "portrait" image and the noisy image used in the simulations. First, the noisy image is filtered by a 3×3 recursive center WM filter[1] and by a 3×3 non-recursive center WM filter with the same set of weights. Figures 1(c) and 1(e) show the respective filter outputs with a center weight $W_c = 5$. Figures 1(d) and 1(f) show the output of the optimal RWM filter and the output of the optimal non-recursive WM filter respectively. As can be seen by comparing the images, the recursive WM filter outperforms non-recursive WM filter.

6.2. Design of a Band Pass RWM Filter

The application at hand is the design of a 62-tap bandpass RWM filter with passband $0.075 \leq \omega \leq 0.125$ (normalized frequency with Nyquist = 1). We use white Gaussian noise with zero mean and variance equal to one as the input training signal. The desired signal is provided by the output of a large FIR filter (122-tap linear FIR filter) designed by Matlab's fir1 function. The 31 feedback filter coefficients were initialized to small random numbers (in the order of 10^{-3}), whereas the feed-forward filter coefficients were initialized to the values designed by Matlab's fir1 with 31 taps and the same passband of interest. The step size used in the adaptive optimization was 10^{-3}.

[1] Center WM operation refers here to the median based filtering operation where all the samples in the window are weighted by 1 except the center sample that is weighted by $W_c > 1$.

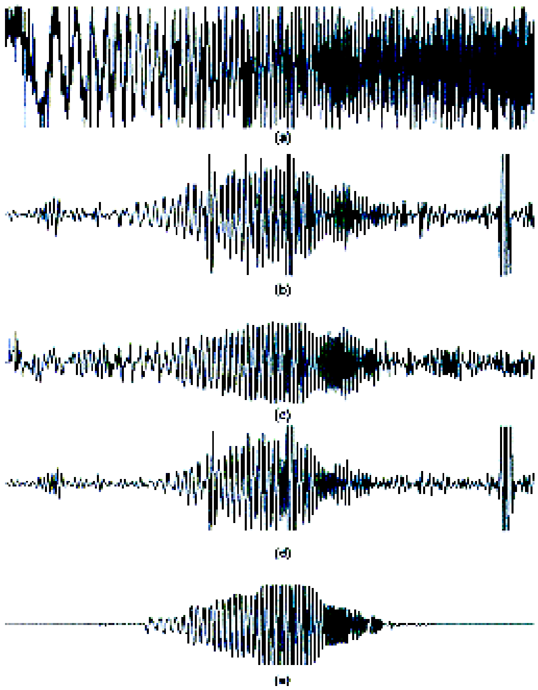

Figure 3: (a) Chirp test signal in stable noise, (b) linear FIR filter output, (c) non-recursive WM filter output, (d) linear IIR filter output, (e) recursive WM filter output.

Figure 2(a) depicts a linear swept-frequency signal spanning instantaneous frequencies from 0 to 0.4, used as a test signal. Figure 2(b) shows this chirp signal filtered by the 122-tap linear FIR filter. Figure 2(c) shows the output of a 62-tap linear FIR filter used here for comparison purposes. The adaptive optimization algorithm described in [2] was used to optimize a 62-tap non-recursive WM filter admitting negative weights. The filtered signal obtained with the optimized weights is shown in Fig. 2(d). Matlab's **yule-walk** function was used to design a 62-tap linear IIR filter. Figure 2(e) depicts the linear IIR filter's output. Figure 2(f) shows the filtered signal of the optimal RWM filter.

Comparing the different filtered signals in Fig. 2, it can be seen that recursive filtering operations perform much better than their non-recursive counterparts. In particular, the RWM filter has a significantly better performance than a non-recursive WM filter having the same number of coefficients.

In order to test the robustness of the designed filters, the test signal was contaminated with additive impulsive noise. Figure 3 depicts the chirp test signal with added α-stable noise and the outputs of the different filters. Both linear FIR and linear IIR filters are severely affected by the noise component, whereas the non-recursive and recursive WM filter outputs remain practically unaltered. To better evaluate the frequency response of the various filters, the power spectral densities of the filter outputs are shown in Fig. 4.

7. CONCLUSIONS

In this paper, two important contributions were presented. First, a recursive WM filter admitting negative weights was introduced. This new filtering framework is useful in applications that require a robust band-pass or high-pass characteristic, together with a near perfect "stop-band". In the presence of impulsive noise, the performance of the RWM filter is significantly better than that of the linear IIR filter. The second contribution of this paper is the introduction, for the first time, of an adaptive optimization algorithm for the design of recursive WM filters. This "recursive-decoupling" algorithm uses the threshold decomposition representation to find an adaptive expression for the update of the filter coefficients. This optimization algorithm is equally suited to the optimization of recursive WM smoothers having non-negative weights.

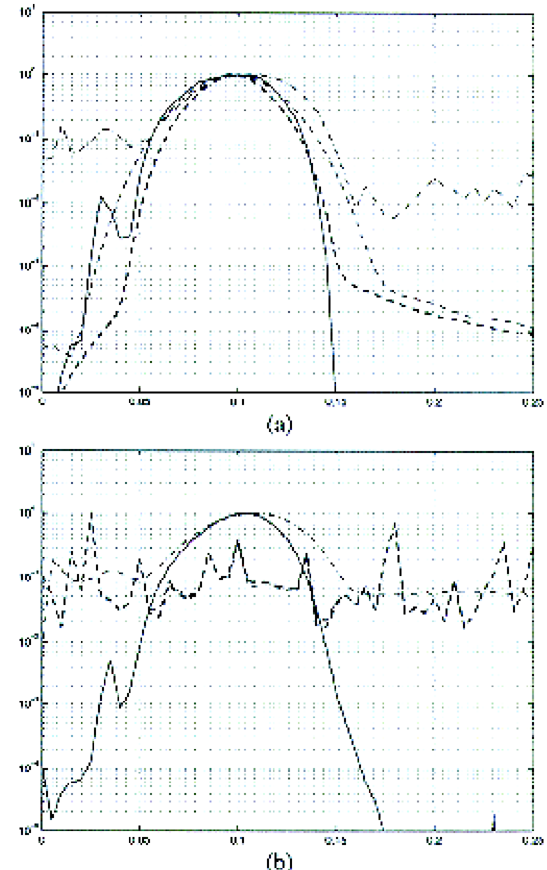

Figure 4: Frequency response to (a) a noiseless sinusoidal signal and (b) a noisy sinusoidal signal; (—) RWM, (— · — · —) non-recursive WM filter, (- - -) linear FIR filter, and (- - -) linear IIR filter.

8. REFERENCES

[1] G. R. Arce and J. L. Paredes, "Recursive Weighted Median Filters Admitting Negative Weights and Their Optimization," *IEEE Transactions on Signal Processing*. Submitted.

[2] G. R. Arce, "A general weighted median filter structure admitting negative weights," *IEEE Transactions on Signal Processing*, vol. SP 46, Dec. 1998.

[3] L. Yin and Y. Neuvo, "Fast adaptation and performance characteristics of fir–wos hybrid filters," *IEEE Transactions on Signal Processing*, vol. 42, July 1994.

[4] Y. Han, I. Song, and Y. Park, "Some root properties of recursive weighted median filters," *Signal Processing*, vol. 25, 1991.

[5] J. J. Shynk, "Adaptive IIR filtering," *IEEE ASSP Magazine*, vol. 6, pp. 4–21, April 1989.