Data-Dependent Median Filtering with Multi-Windows for Restoration of Images Corrupted by High Probability Impulsive Noise

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ABUSTRACT

We propose a novel data-dependent median filtering method in order to restore images which are corrupted by 10 ~ 50% probability impulsive noise. Even if the image which is corrupted by high probability impulsive noise, the degree of impulsive noise is not uniform at each local window. Thus, the suitable window length is changed at each the processed point. And if the processed point is not corrupted by impulsive noise, input signal should be not filtered in order to preserve signal details. From these points of view, the proposed filter combines median filters with multi-windows (1×1 window is included) for preserving the signal details and removing impulsive noise perfectly.

1. INTRODUCTION

Median filter is useful for image restoring that corrupted by impulsive noise. A lot of improvement methods about median filter are published [1],[2]. However, these methods can't useful for images that corrupted by high-probability impulsive noise.

The impulsive noise is added independently for each pixel of images. The image that is corrupted by high probability impulsive noise, the degree of impulsive noise is not uniform at each local window. Thus, the suitable window length is changed at each processed point. And if the processed point is not corrupted by impulsive noise, no filtering is best, since filtering destroys the signal details.

From these points of view, we propose a novel data-dependent median filtering method in order to restore images which are corrupted by high probability (i.e., 10 ~ 50%) impulsive noise. The multi-windows median filter which is combines fore median filters with different window length, is proposed. In order to preserve the signal details and remove impulsive noise perfectly, only pixel which corrupted by impulsive noise, is filtered by multi-windows median filter in the proposed method.

2. THE ALGORITHM

2.1 The definition of input images

In this paper, an input image P(i,j) is corrupted by high probability impulsive noise and is given by

$$P(i,j) = \begin{cases} P_0(i,j) : \text{prob. } 1-p_1-p_2 \\ h_1 \quad : \text{prob. } p_1 \\ h_2 \quad : \text{prob. } p_2 \end{cases} \tag{1}$$

where $P_0(i,j)$ is an original image and h_1 is high signal value (e.g., h_1=255), correspond to white impulse noise, h_2 is low signal value (e.g., h_2 =0) , correspond to black impulse noise. p_1 and p_2 are the probability of h_1 impulsive noise and h_2 impulsive noise, respectively. And we assume $p_1 + p_2 > 10\%$.

2.2 Estimation of the number of impulsive noise

Let the P(i,j) is the center pixel of the M × M ($N=M^2$) window. $P_{(k)}(i,j)$ is the k-th smallest data in the window (i.e., $P_{(1)}(i,j) \leqq P_{(2)}(i,j) \leqq \cdots \leqq P_{(N-1)}(i,j) \leqq P_{(N)}(i,j)$). Impulsive noise exists around max. data and min. data in the window. Thus, we can estimate the number of impulsive noise in the window, by using the difference information as follows:

$$I^{(x)}_{max}(i,j) = P_{(N)}(i,j) - P_{(N-x)}(i,j) \tag{2}$$

$$I^{(y)}_{min}(i,j) = P_{(1+y)}(i,j) - P_{(1)}(i,j) \tag{3}$$

$$(x=1,2,\cdots X \quad y=1,2,\cdots Y)$$

We can predict the number of white and black impulsive in the window by using $I^{(x)}_{max}(i,j)$ and $I^{(y)}_{min}(i,j)$, respectively. For example, if one white impulsive noise is exist in the window only $I^{(1)}_{max}(i,j)$ shows large value, and if two white impulsive noises are exist, $I^{(1)}_{max}(i,j)$ shows small value and $I^{(2)}_{max}(i,j)$ shows large value. In order to estimate the number of white and black impulsive noise in the window, we introduce two sets of fuzzy rules RM1(x) and RM2(y) (see Table 1), respectively. Fuzzy sets "Small" and "Large" in the RM1(x) and RM2(y) defined as Fig.1(a).

Table 1 Fuzzy rules of estimation the number of impulsive noise

RM1(X) :	RM2(Y) :
If $I^{(1)}_{max}(i,j)$ is Small and $I^{(2)}_{max}(i,j)$ is Small and . . . . $I^{(X-1)}_{max}(i,j)$ is Small and $I^{(X)}_{max}(i,j)$ is Large	If $I^{(1)}_{min}(i,j)$ is Small and $I^{(2)}_{min}(i,j)$ is Small and . . . . $I^{(Y-1)}_{min}(i,j)$ is Small and $I^{(Y)}_{min}(i,j)$ is Large

The number of impulsive noise in the window is estimated by the following fuzzy rules constructed by RM1(x) and RM2(y).

RM1(1) and RM2(1) then
{(1,1) impulsive noises in the window}
RM1(1) and RM2(2) then
{(1,2) impulsive noises in the window}
. . . .
RM1(X)andRM2(Y) then
{(X,Y) impulsive noises in the window }

else RM1(1) then
{(1,0) impulsive noise in the window }
. . . .
RM1(X) then
{(X,0) impulsive noises in the window }
RM2(1) then
{(0,1) impulsive noise in the window }
. . . .
RM2(Y) then
{(0,Y) impulsive noises in the window }

else
{(0,0) impulsive noise in the window }

where (x,y)=(the number of white impulsive noise, the number of black impulsive noise). The degree of activation of the IF-THEN rules and Else rules are represented $\mu_{m(x,y)}(i,j)$ and calculated by the method of [3]. $\mu_{m(x,y)}(i,j)$ means the degree of the existence of (x,y) impulsive noise in the window. The number of impulse noise in the window (i.e., $I_m(i,j)$) is calculated by using $\mu_{m(x,y)}(i,j)$ as follows:

$$I_m(i,j) = \frac{\sum_{x=1}^{X}\sum_{y=1}^{Y} I_{m(x,y)}(i,j) + \sum_{x=1}^{X} I_{m(x,0)}(i,j) + \sum_{y=1}^{Y} I_{m(0,y)}(i,j) + I_{m(0,0)}(i,j)}{N \times \left\{ \sum_{x=1}^{X}\sum_{y=1}^{Y} \mu_{m(x,y)}(i,j) + \sum_{x=1}^{X} \mu_{m(x,0)}(i,j) + \sum_{y=1}^{Y} \mu_{m(0,y)}(i,j) + \mu_{m(0,0)}(i,j) \right\}} \quad (4)$$

$I_{m(x,y)}(i,j), I_{m(x,0)}(i,j), I_{m(0,y)}(i,j), I_{m(0,0)}(i,j)$ are given as follows:

$$I_{m(x,y)}(i,j) = \mu_{m(x,y)}(i,j) \cdot (x+y)$$
$$I_{m(x,0)}(i,j) = \mu_{m(x,0)}(i,j) \cdot (x)$$
$$I_{m(0,y)}(i,j) = \mu_{m(0,y)}(i,j) \cdot (y)$$
$$I_{m(0,0)}(i,j) = \mu_{m(0,0)}(i,j) \cdot 0 \quad (5)$$

$I_m(i,j)$ means the ratio of the number of impulsive noise in the window.

2.3 Data-dependent median filtering with multi-windows

The proposed method selects a suitable window length at each processed point according to the number of impulsive noise in the window. And if the processed point is not corrupted by impulsive noise, no filtering is best, in order to preserve signal details. In order to judge whether the processed point corrupted by impulsive noise or not, the proposed method uses the information H(i,j) which is given by

$$H(i,j) = |P(i,j) - med(i,j)| \quad (6)$$

where P(i,j) is processed point and med(i,j) is the median value in the 7×7 local window. If H(i,j) is the large value, the possibility of the processed point corrupted by impulsive noise is high. The proposed method is constructed by fuzzy rules as follows:

If H(i,j) is Small then Y(i,j) is P(i,j)
else Y(i,j) is Med(i,j)

where Y(i,j) is the output of the proposed method and Med(i,j) is the output of the multi-windows median filter which is explained later (see Eq.(8)). The grade of the fuzzy set "Small" (see Fig.1 (b)) of the information H(i,j) is represented by Φ(i,j). The output of the proposed method is calculated as

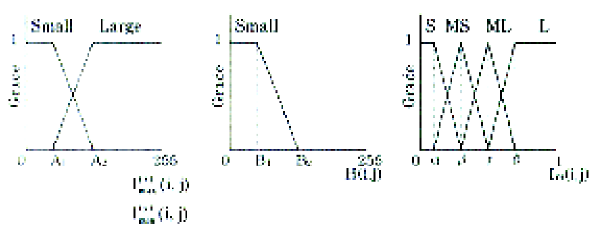
$$Y(i,j) = P(i,j) \times \Phi(i,j) + Med(i,j) \times \{1 - \Phi(i,j)\} \quad (7)$$

The multi-windows median filter combines several median filters with different window length. It is matter of course that if window length is large high, however, the performance of the preserving details of image is low. Thus, we use multi-windowing method in order to achieve high performance of the noise attenuation and the preserving details of image at same time. The proposed multi-windows median filter use fore window length filters that 5 point cross window, 3 × 3 window, 13 point window, and 5 × 5 window(see Fig.2). The fuzzy rules of the multi-windows median filter are defined by:

If $I_m(i,j)$ is Small (S) then
Med(i,j) is $Med_S(i,j)$
If $I_m(i,j)$ is Medium Small (MS) then
Med(i,j) is $Med_{MS}(i,j)$
If $I_m(i,j)$ is Medium Large (ML) then
Med(i,j) is $Med_{ML}(i,j)$
If $I_m(i,j)$ is Large (L) then
Med(i,j) is $Med_L(i,j)$

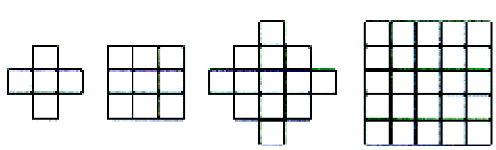
Where $I_m(i,j)$ is the ratio of the number of impulsive noise in the window (Eq.(4)), and $Med_S(i,j)$ is the output of median filter with 5 pints cross window. $Med_{MS}(i,j)$, $Med_{ML}(i,j)$, $Med_L(i,j)$ are the output of median filter with 3×3, 13 points and 5×5 windows, respectively. The grade of the fuzzy sets ("S", "MS", "ML", and "L") are represented $\varepsilon_S, \cdots, \varepsilon_L$. Thus, the output of the multi-windows median filter is :

$$Med(i,j) = \frac{Med_S(i,j)\cdot\varepsilon_S + Med_{MS}(i,j)\cdot\varepsilon_{MS} + Med_{ML}(i,j)\cdot\varepsilon_{ML} + Med_L(i,j)\cdot\varepsilon_L}{\varepsilon_S + \varepsilon_{MS} + \varepsilon_{ML} + \varepsilon_L} \quad (8)$$



(a) Fuzzy sets for $I^{(1)}_{max}(i,j)$ and $I^{(2)}_{max}(i,j)$ (b) Fuzzy sets for H(i,j) (c) Fuzzy sets for $I_m(i,j)$

Fig.1 Fuzzy sets



(a) (b) (c) (d)

Fig.2 Fore filter windows
(a)5 points window
(b)3×3 window

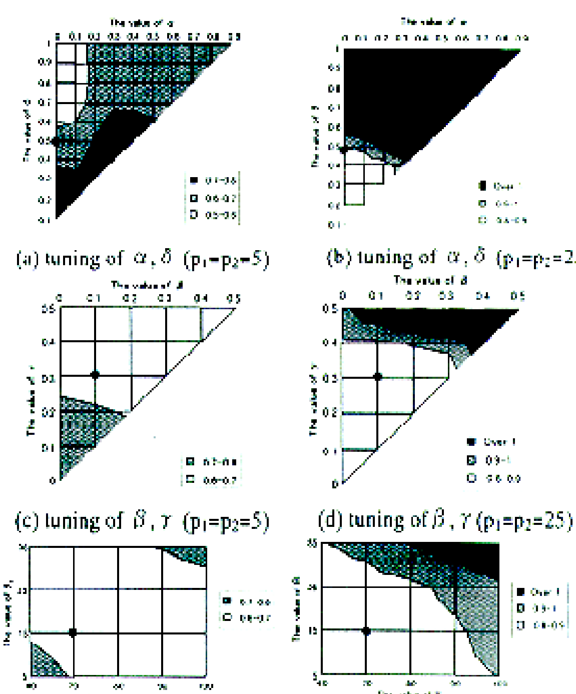
3. EXPERMENTAL RESULTS

3.1 Tuning of fuzzy sets

The proposed method is constructed by three sets of fuzzy rules. We would like to optimize three sets of rules. First, We determine the parameters (A_1,A_2) shown in Fig.1. A_1 and A_2 have already discussed in [4]. According to the results of [4], we determine $(A_1,A_2)=(15,30)$.

From Fig.1, membership functions of $I_m(i,j)$ and H(i,j) are defined by parameters ($\alpha, \beta, \gamma, \delta$) and (B_1,B_2) respectively. We determine the optimal membership function experimentally by using image "Lenna". We evaluate the filtering performance by the MSE(Mean Square Error). We assume the value of impulsive noise as $h_1=255$, $h_2=0$ in Eq.(1)

The adequate membership functions are determined by comparing to the method of the optimal window length median filtering. The performance of both ($p_1=p_2=5\%$) and ($p_1=p_2=25\%$) for various ($\alpha, \beta, \gamma, \delta$) are shown in Fig.3(a)~(d), and for various (B_1,B_2) are shown in Fig.3(e),(f). White region of each figure in Fig.3 shows optimal region for ($\alpha, \beta, \gamma, \delta$) or (B_1,B_2). From these figures, we determine the parameter as $(B_1,B_2)=(15,70)$, ($\alpha, \beta, \gamma, \delta$)=(0.0,0.1,0.3,0.5) which are adequate for both ($p_1=p_2=5\%$) and ($p_1=p_2=25\%$). Thus, the proposed filter with these parameters can restore the images which are corrupted by 10 ~50% probability impulsive noise.



(a) tuning of α, δ ($p_1=p_2=5$) (b) tuning of α, δ ($p_1=p_2=25$)
(c) tuning of β, γ ($p_1=p_2=5$) (d) tuning of β, γ ($p_1=p_2=25$)

3.2 The performance of multi-windows median filter

We discuss the performance of multi-windows median filter. The multi-windows median filter which constructed by four median filters with different window length, can restore images that corrupted by high probability impulsive noise (p_1= p_2=5%～25%). Figure 4 shows the results of fore median filters with single window and the proposed method. We use "Lenna" in this simulation. Here, the output of the "proposed method 1" means Med(i,j) given by Eq.(8). Furthermore, the output of the "proposed method 2" is Y(i,j) defined by Eq.(7). From Fig.4, fore median filters with single window are not adaptive for various probability of impulsive noise. The "proposed method 1" is superior to median filter with optimal window length. The "proposed method 1" shows a good performance, since the "proposed method 1" combines four median filters with different window length adequately.

We would like to analyze the mechanism of the multi-windows median filter with fixed parameters. Figure 5 shows the ratio of each window is utilized for filtering. We can see that multi-windows median filter uses together fore window agree with the degree of impulsive noise in the window.

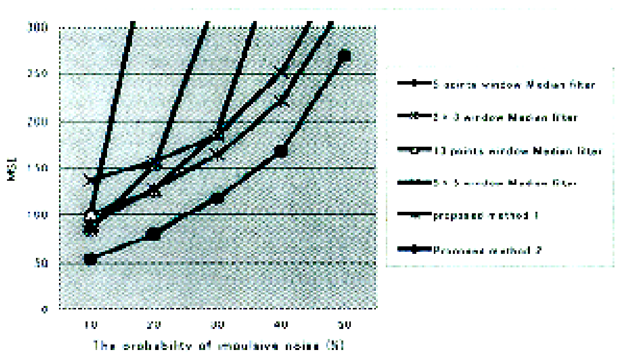


Fig.4 Filtering results for various probability of impulsive noise

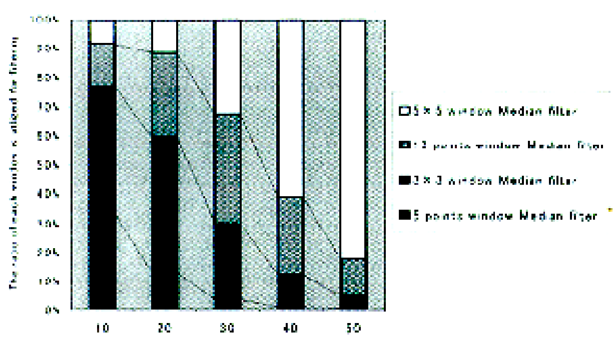


Fig.5 The frequency of each window's utility of multi-windows median filter

The "proposed method 2" is better than the "proposed method 1" for all condition of impulse noise. In the case of the "proposed method 2", if the processed point is judged to be not corrupted by impulse noise, the output is same as the input (i.e., Y(i,j)=P(i,j)=P_0(i,j)). Thus, the "proposed method 2" has a good property of details preserving. The proposed method is useful for restoration images which are corrupted by high probability impulsive noise.

3.3 Compare with another methods

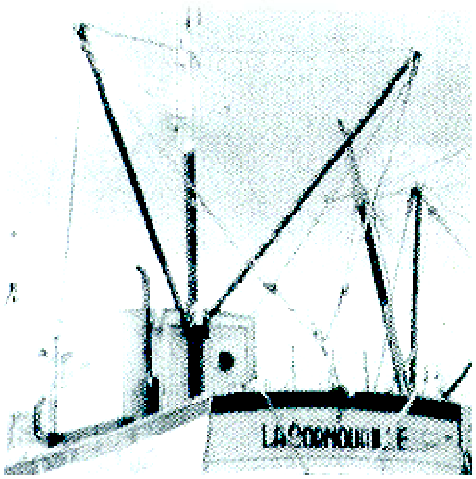
Computer simulations have been carried out to compare the performance of the proposed method defined by Eq.(7), with the method of "the optimal repeating operation of 3×3 median filter". "Boat", "Lighthouse" and "Building" are also used in the simulations. The parameters of the membership functions of the proposed method are set according to the results of 3.1.

The original image corrupted by impulsive noise with (p_1,p_2)=(20%,20%) and (h_1,h_2)=(255,0) is used for simulation (see Fig.6(b)). Visual quality can be observed in Fig.6(c),(d). It is clear that the proposed method (Fig.6(c)) preserves the image details (i.e. letters on the stern and lines from the mast). The MSE of the proposed method and the comparison method (i.e., optimal repeating operation of 3×3 median filter and single operation of 5×5 median filter) are shown in Table 2. From these results, the proposed method shows best results for all images.

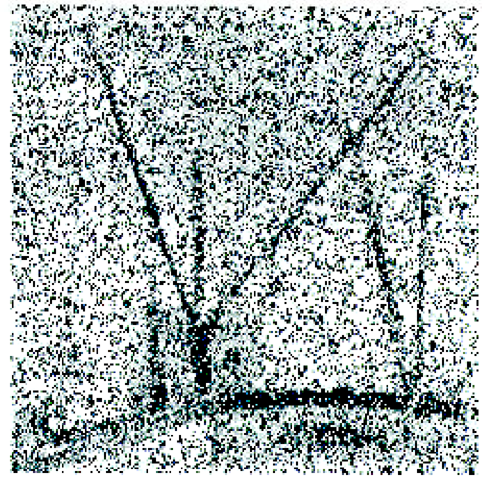
Table 2 The results of filtering
(Impulsive noise p_1=p_2=20%)

Image	Repeating 3×3 median filtering	5×5 median filtering	Proposed method
Lenna	191.85	251.21	168.19
Boat	172.15	224.72	127.38
Light-house	482.66	592.83	455.05
Building	204.10	281.53	196.795

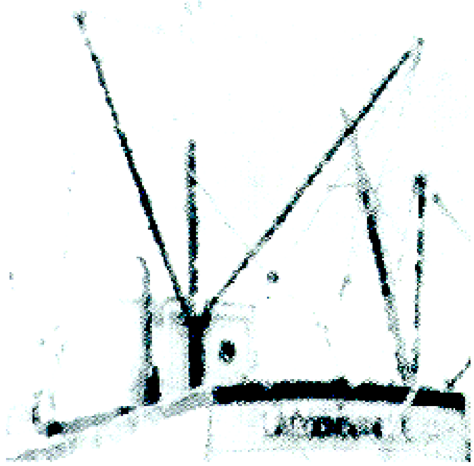
Finally, we study the performance of the proposed filter for another impulsive noise models. Table 3 shows the results for models of (h_1,h_2) = (245,10), (235,20) and (225,30). Where both p_1 and p_2 are fixed as 20%. From Table 3, the proposed method shows high performance for all noise models because of the robustness of fuzzy systems.



(a) Boat



(b) Boat (40%)



(c) Proposed method

(d) Repeating operation of 3×3 median filtering

Fig.6 The results of filtering

Table 3 The results of filtering
(Impulsive noise $p_1=p_2=20\%$)

Image (h_1,h_2)	Repeating 3×3 median filtering	5×5 median filtering	Proposed Method
Lenna (245,10)	199.30	271.61	182.52
Lenna (235,20)	200.74	266.69	192.62
Lenna (225,30)	202.83	270.27	200.21

4. CONCLUSIONS

In this paper, a novel median filter with multi-windows is proposed, for the purpose of restore the image which corrupted by high probability impulsive noise. The tuning of fuzzy sets are relative easy, because of the robustness of fuzzy

is cleared by a lot of experimental results.

5. REFERENCES

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