Color Texture Morphology

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The paper presents a computational model to extract local symmetry axes corresponding to the areas of homogeneous texture in an image. The approach is to construct an energy functional whose minimizer is a special distance function which attains its maxima at the boundaries between different texture regions and decays away from texture boundaries. Local symmetry axes are extracted from this distance function. The method extends to color texture images. Texture homogeneity is measured using the filter responses of a set of Gabor filters which are derived from dilation and rotation of a single Gabor function.

Keywords: nonlinear PDEs, Gabor wavelets, local symmetry axis, coupled equations

1 Introduction

The paper presents a computational model to extract local shape symmetries corresponding to the areas of uniform texture in a given image. The method extends to color texture images. Local texture properties are derived based on the power responses of a set of Gabor filters. These filters form a continuous family parametrized by two variables. Just like wavelet bases, they can be derived from dilation and rotation of a single filter. The representation of an image using the power responses of Gabor filters are called Gabor-Wavelet representation. Evidence suggests that the simple cells in the visual cortex of the mammalian brain can be modeled by Gabor functions. The motivation for using Gabor filters in texture representation is discussed in the references [1, 2].

Gabor Wavelet representation is widely used in segmenting images based on textural properties of object surfaces [3, 4, 5]. Typically, these approaches form an auxiliary image that is based on the power responses and then they use the gray scale image analysis techniques to analyze the image. In [5] Lee et.al directly search for texture boundaries by minimizing an energy functional.

The approach proposed in this paper differs from others in the following way. We compute the shape descriptors represented in the form of local symmetry axes directly from the image without determining the texture boundaries. This is achieved by constructing an energy functional whose minimizer is a special distance function called the edge strength function. The edge strength function attains its maximum value at the boundaries between different texture regions and decays exponentially away from texture boundaries. Power responses of a set of self similar Gabor filters are used as a measure of texture homogeneity.

The paper is organized as follows. In Section 2 Gabor Wavelet representation of texture is explained. In Section 3 the computation of the edge strength function hence the computation of local symmetry axis is given under the assumption that the image is piecewise constant and corrupted with random noise. The computation of the edge strength function is extended to texture images in Section 4.

2 Gabor Wavelet Representation of Texture

Gabor filter is a complex exponential that is modulated by an elongated Gaussian function:

$$h(x,y) = \frac{1}{4\pi\sigma^2} e^{\left(\frac{4x^2+y^2}{-8\sigma^2}\right)} e^{ix} \quad (1)$$

The filter is band pass in x direction and low pass in y direction. A set of self similar Gabor filters $G(\vec{x}, \sigma, \Theta)$ where σ is the radial frequency and Θ is the angular orientation of the filter, may be generated by rotation and dilation of the preceding single Gabor filter $h(x,y)$. Filter response $P(\vec{x}, \sigma, \Theta)$ of each filter is the convolution of the filter with the image. The set of total 24 filters at 8 orientations and 3 different radial frequencies has been successfully employed to represent various textures [5].

3 Symmetry Axis: Piecewise Constant Case

In [6, 7], it was shown that a function called the edge strength function can be used to extract local symmetry axis and the shape skeletons. The edge strength function $v(\vec{x})$ is computed from a gray scale image $g(\vec{x})$.

To be more specific, consider the following functional

$$\begin{aligned} E(u,v) &= \int_R \beta \left|u(\vec{x}) - g(\vec{x})\right|^2 d\vec{x} \\ &+ \int_R \left\{(1-v(\vec{x}))^2 \alpha \|\nabla u\|^2\right\} d\vec{x} \\ &+ \int_R \frac{\rho}{2}\|\nabla v\|^2 + \frac{v(\vec{x})^2}{2\rho} d\vec{x} \qquad (2) \end{aligned}$$

where $u(\vec{x})$ is the approximate image and α, β and ρ are the parameters. By minimizing the preceding functional we seek for an approximate image $u(\vec{x})$ that best represent the image and the edge strength function $v(\vec{x})$ that varies between 0 and 1. The edge strength function $v(\vec{x})$ may be interpreted as the probability of the existence of an edge at a given image location $\vec{x} = (x, y)$. Key point is that the third term of the preceding functional approaches to the length of the segmentation loci as $\rho \to \infty$. Thus, $v(\vec{x})$ is the blurred version of the true object boundaries (discontinuity loci of the image) with a blurring radius equal to ρ.

The crucial result presented in [6, 7] is that the successive level curves of $v(\vec{x})$ mimic the behavior of the fronts propagating with a speed proportional to curvature. Thus, the curvature of the level curves are inversely proportional to the gradient of $v(\vec{x})$ along the level curves. Hence the curvature maxima of the level curves can be tracked by tracking the minima of the gradient. Figure 1 depicts the level curves of $v(\vec{x})$ superimposed on the original gray scale image $g(\vec{x})$. The local symmetry loci are depicted in Figure 2.

Detailed discussion of the edge strength function and the local symmetry axes is given in the references [9, 10].

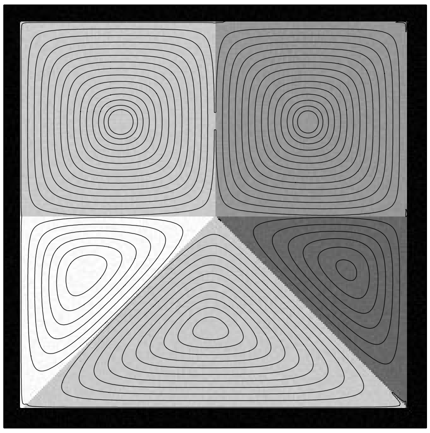

Figure 1: Level curves of $v(\vec{x})$

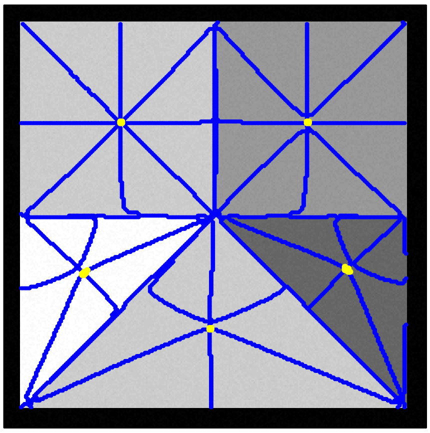

Figure 2: Local symmetry loci extracted from $v(\vec{x})$

3.1 Color Images

The idea presented above extends to color images. Let $g_i(\vec{x}), i = 1, 2, 3$ be the three color bands. The simplest way to compute a common edge strength function from 3 images is to consider the following functional:

$$\begin{aligned} E(u_i, v) &= \int_R (1-v(\vec{x}))^2 \left(\sum \alpha_i \|\nabla u_i(\vec{x})\|^2\right) d\vec{x} + \\ &\int_R \sum \beta_i (u_i(\vec{x}) - g_i(\vec{x}))^2 d\vec{x} + \\ &\int_R \frac{\rho}{2}\|\nabla v\|^2 + \frac{v^2}{2\rho} d\vec{x} \end{aligned}$$

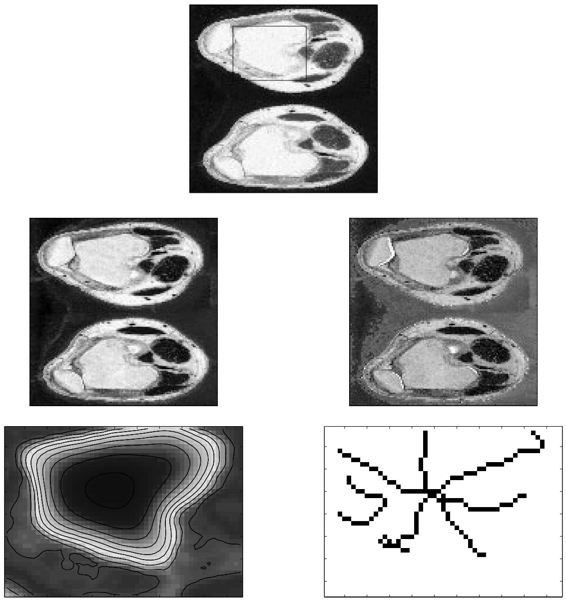

Figure 3: Color image. R, G and B components of the color image are shown in the first two rows. Level curves of v and the symmetry set in a selected area are shown in the bottom row.

The common edge strength function can be computed as the solution of the following 4 coupled diffusion equations.

$$\begin{aligned}
\frac{\partial u_1(\vec{x})}{\partial \tau_1} &= \frac{\alpha_1}{\beta_1}\left(\nabla \cdot (1-v)^2 \nabla u_1\right) - (u_1 - g_1) \\
\frac{\partial u_2(\vec{x})}{\partial \tau_2} &= \frac{\alpha_2}{\beta_2}\left(\nabla \cdot (1-v)^2 \nabla u_2\right) - (u_2 - g_2) \\
\frac{\partial u_3(\vec{x})}{\partial \tau_3} &= \frac{\alpha_3}{\beta_3}\left(\nabla \cdot (1-v)^2 \nabla u_3\right) - (u_3 - g_3) \\
\frac{\partial v(\vec{x})}{\partial \tau_v} &= \nabla^2 v - \frac{v}{\rho^2} + (1-v)\sum_{i=1}^{k} \frac{2\alpha_i}{\rho} ||\nabla u_i||^2
\end{aligned}$$

Notice that, each color band is nonlinearly smoothed away from boundaries and approximated by a piecewise smooth image that best represents the input image. At the same time the value of the distance function $v(\vec{x})$ is increased whenever any u has a large gradient. A better alternative for computing the edge strength function from color images can be found in [8]. Figure 3 depicts the level curves of $v(\vec{x})$ and the local symmetry axis computed from a color image.

4 Symmetry Axis: Texture Images

The preceding model makes the basic assumption that the true image can be represented by a piecewise constant function perturbed with random noise. The second term in Eq.(2) imposes that the image gradient should be small away from the discontinuity loci (i.e. edges). Many images, however, contain textures that are characterized by large but systematic texture gradients. We, now adopt the preceding symmetry extraction method to texture images in order to extract the local shape symmetries corresponding to the areas of homogeneous texture (Figure 4). Power responses of Gabor filters used as a measure of texture homogeneity. Given filter responses $P(\vec{x}, \sigma, \Theta)$ we compute its smooth approximation $u(\vec{x}, \sigma, \Theta)$ which varies slowly both in the spatial and the spectral domains. Large gradients in the power response $u(\vec{x}, \sigma, \Theta)$ are allowed only near the texture boundaries, i.e. where $v(\vec{x})$ is high.

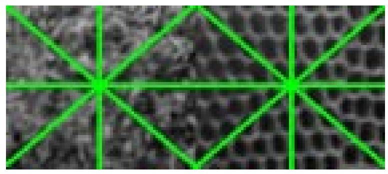

Figure 4: Sample image for illustration. Two identical axes of full symmetry corresponding to two square shaped image sections of uniform texture.

To be more specific, we consider the following functional:

$$\begin{aligned}
E = & \int_R \int_S \beta \left| u(\vec{x}, \sigma, \Theta) - P(\vec{x}, \sigma, \Theta) \right|^2 d\vec{x} d\sigma d\Theta + \\
& \int_R \int_S (1 - v(\vec{x}))^2 \alpha ||\nabla u||^2 d\vec{x} d\sigma d\Theta + \\
& \int_R \int_S \lambda_\Theta \frac{\partial u}{\partial \Theta}^2 + \lambda_\sigma \frac{\partial u}{\partial \sigma}^2 d\vec{x} d\sigma d\Theta + \\
& \int_R \left(\frac{\rho}{2} ||\nabla v||^2 + \frac{v(\vec{x})^2}{2\rho} \right) d\vec{x}
\end{aligned} \qquad (3)$$

Gradient descent equations for u and v are:

$$\begin{aligned}
\frac{\partial u(\vec{x}, \sigma, \Theta)}{\partial \tau} &= \beta(u(\vec{x}, \sigma, \Theta) - P(\vec{x}, \sigma, \Theta)) \\
&- \quad (1 - v(\vec{x}))^2 \alpha \nabla^2 u \\
&+ \quad \lambda_\Theta \frac{\partial^2 u}{\partial \Theta^2} + \lambda_\sigma \frac{\partial^2 u}{\partial \sigma^2}
\end{aligned} \qquad (4)$$

$$\frac{\partial v(\vec{x})}{\partial \tau} = (1 - v(\vec{x}))\alpha\|\nabla u\|^2 + \frac{v(\vec{x})}{2\rho} + \frac{\rho}{2}\nabla^2 v$$

The coupled equations (4) for gray scale texture images can be extended to color texture images by first decomposing the color image into three bands as discussed earlier for the piecewise constant gray images.

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