GENERALIZED VOLTERRA AND WIENER NONLINEAR FILTERS FOR APPLICATIONS IN DIGITAL COMMUNICATIONS

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ABSTRACT

New developments in nonlinear dynamical systems are permitting Volterra and Wiener nonlinear functional series representations to be optimally approximated by artificial neural networks, which are capable of adaptation, learning and evolution by training with or without supervision.The framework based on Generalized Fock Space, enabling these developments is briefly reviewed. In particular, the realization of a best approximation to Wiener's Laguerre / Hermite representation in terms of a dynamical functional artificial neural network (D-FANN) is presented. Applications of this technology to emerging telecommunications systems such as wireless, digital subscriber lines, and cable TV are discussed.

1. INTRODUCTION

In the analysis and design of large-scale nonlinear dynamical systems, the two representations proposed by Vito Volterra[1] and Norbert Wiener [2] have played an especially prominent role.

Let f denote the input-output function of a large-scale nonlinear dynamical system. We may write

$$y(t) = f(x)(t) \tag{1}$$

where the input x and output y are real-valued functions on an interval I of the real line, and f is a functional which sends x to the value $y(t)$ of the output y at time t. Even though we limit the below discussion to signals, the result trivially generalizes to the case of images, x then being defined on a two-dimensional domain.

In the Volterra representation [1], f is expressed as an abstract (functional) power series in x of the form

$$y(t) = \sum_{k=0}^{\infty} \frac{1}{k!} \int_I \cdots \int_I h_k(t; t_1, \ldots, t_k) x(t_1)..x(t_k) \, dt_1 \ldots dt_k \tag{2}$$

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where the kernels satisfy appropriate conditions.

The Wiener representation [2] is of the form

$$y(t) = H(Lx(t)), \tag{3}$$

where L is an infinite linear differential dynamical system represented by a Laguerre network, and H is a Hermite expansion defined on the range of L (See Fig.1). Details are described in [2] and are omitted because their description would require too much space.

2. D-FANN BEST APPROXIMATION OF VOLTERRA AND WIENER SERIES

While the above two representations are powerful and especially relevant to emerging computer and information technologies, they have some important limitations. One is that they are computationally intensive. Another is that they are too difficult to implement. Also, in the way these implementations were originally developed, they do not lead to simpler representations which can be justified rigorously and which are capable of adaptation and learning.

In order to overcome these difficulties, the author and collaborators developed a broader-based rigorous framework for obtaining a best approximation $\tilde{f}$ of the input-output map f of a large scale nonlinear dynamical system [3]-[6]. This framework, briefly overviewed in the remaining part of this paper, combines features of the representations (2) and (3) and leads to optimal realizations of $\tilde{f}$ in the form of artificial neural networks[7]-[15]. These are called by the author OI (Optimal Interpolative), OS (Optimal Smoothing), OMNI (Optimal Multilayer Neural Interpolating), and OSMAN (Optimal Smoothing Multilayer Artificial Neural) networks. Two additional new generic classes of these networks[13] are called functional artificial neural networks (FANNs) and dynamical FANNs (D-FANNs). They are detailed elsewhere and hence will not be discussed in this write-up.

In the broader-based framework proposed by the author, f is assumed to belong to a Generalized Fock Space

(GFS) F of Volterra functional series. F constitutes a generalization of the conventional Fock Space, the state space of non-self interacting Boson fields in quantum-field theory. Specifically, F is a Reproducing Kernel Hilbert Space (RKHS) with a reproducing kernel K which can be expressed explicitly often in closed form. The best approximation $\tilde{f}$ of f is extracted from the training data by requiring that $\tilde{f}$ be the minimum norm element in F which satisfies the training input-output data constraints, i.e. $\tilde{f}$ is the unique solution of the problem

$$\min \|f\|_F \tag{4}$$

such that

$$f(x^i) = y^i, i = 1, \ldots, m \tag{5}$$

where $\|\cdot\|_F$ denotes the norm in F, and $(x^i, y^i), i = 1, \ldots, m$ are input-output training pairs. The solution $\tilde{f}$ to (4) and (5) satisfies a minimax error criterion and is of the form

$$y(t) = f(x)(t) = L_2 H(L_1 x(t)) \tag{6}$$

where L_1 and L_2 are linear operators and H is a zero-memory non-linear operator. This solution can be naturally realized as a two-layer artificial neural network. In this network, $H(L_1 \cdot)$ is implemented by the first (nonlinear) layer and L_2 by the second linear layer. By appropriate choice of the signal spaces in which x and y reside, these layers can be made analog or digital. If the input x is analog and hence the operator L_1 is analog and implemented by a linear differential dynamical system, we call the artificial neural network representing (6) a D-FANN (Dynamical Artificial Neural Network) [13]. Such networks recently proposed by the author permit intelligent processing of analog signals. In particular, if L_1 in a D-FANN is implemented as a Laguerre network, then the resultant D-FANN constitutes the best approximation to the Wiener representation (3) and is shown in Fig. 2. This figure shows that Wiener's Laguerre Network can be used as the filter bank for the initial stage of a D-FANN [13].

3. APPLICATIONS

Nonlinear filters based on the above generic structural models can be applied to the solution of a number of problems in digital communications. Within the bounds of the time allotted to this plenary lecture, the applications of these filters to one or more of the following problems will be discussed:

- Nonlinearities in QAM data transmission systems: Microwave radio and voice modems.
- Nonlinear satellite channel equalization.
- Distortion in high-density recording channels.
- Nonlinearities in echo cancellation.
- Nonlinear intermodulation distortion: CATV channels and FDM systems.
- Nonlinearities in optical fiber transmission: semiconductor laser diodes.
- Analog-to-digital conversion nonlinearities.
- Nonlinear filters in Phase Locked Loop techniques.

4. CONCLUSION

We presented a framework whereby functional artificial neural networks (FANNs) become a natural vehicle for implementation of Volterra and Wiener filters and the generalizations of these filters are described here. Applications to emerging digital communication systems will be discussed in the lecture.

5. REFERENCES

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Note: See Figures on next page.

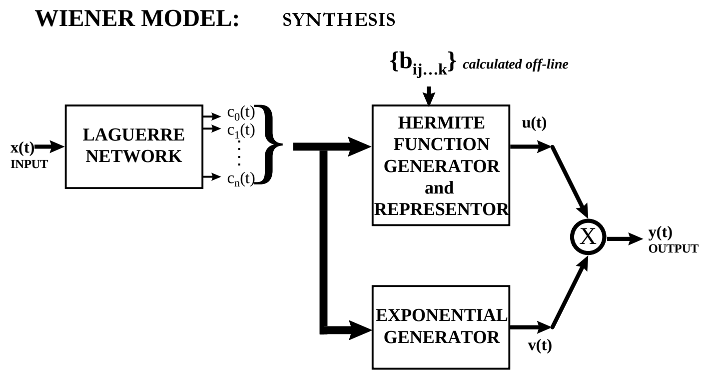

Figure 1: The Wiener Nonlinear Functional Series Model

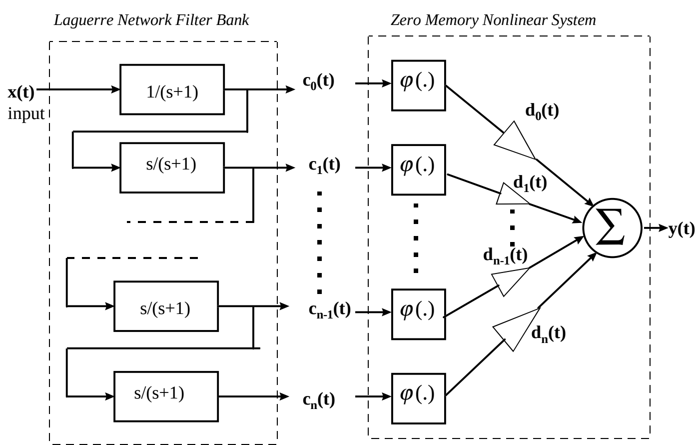

Figure 2: D-FANN Realization of the Best Approximation to the Wiener Model of Fig.1