SIGNAL ESTIMATION IN CHAOS USING NONLINEAR PREDICTION

Zhiwen Zhu

Communications Research Laboratory
McMaster University,
1240 Main Street W., Hamilton, Ontario
Canada, L8S 4K1
zhuz@sunburst.crl.mcmaster.ca

Henry Leung

Depart. of Electrical and Computer Engineering
University of Calgary
2500 University Drive N. W., Calgary, Albert
Canada, T2N 1N4
leungh@enel.ucalgary.ca

ABSTRACT

The problem of parameter estimation in chaotic noise is considered in this paper. Based on the inherently deterministic nature of a chaotic signal — the short term predictability, a novel estimation approach called minimum nonlinear prediction error (M) technique is proposed. The parameters of a signal can be accurately estimated by minimizing the nonlinear prediction error of the output of an inverse filter of the received signal. Monte Carlo simulations are carried out to demonstrate the efficiency of the MNPE approach. It is shown that not only could the chaotic approach provide an accurate estimation, but it is more effective than the conventional statistic approach in the sense that the chaotic estimation approach has a smaller mean squares error (MSE).

1. INTRODUCTION

Recent developments in nonlinear dynamics and chaos theory suggest that it may be possible to develop new and powerful alternative strategies for signal processing and communications [1]-[3]. A wide range of signal such as radar [1], speech [4], and indoor propagation [5] have been demonstrated to be chaotic rather than purely random. The development of new classes of signal models may naturally lead to new kinds of algorithms for processing such signals to obtain better performance that explicitly take into account their special structure.

The notions of using chaotic signals for signal processing applications have received increasing interest over the last few years [1],[6]-[9]. In some cases, it is the chaotic signal that is direct interest and is corrupted by random noise [2]. The problem becomes one of estimating a chaotic signal in noise. In other scenarios such as radar surveillance [1] and system identification in a chaotic modulation communications [9], the chaotic signal is a form of noise or other unwanted signal. In this case, we are interested in estimating an unknown signal in a background of chaotic noise. The former problem, which is basically the problem of parameter estimation of a nonlinear system in random noise, has been addressed by various researchers using various approaches including filtering [10], shadowing theory [11] and so on. The latter problem, which is to estimate parameters of a signal embedding in chaotic noise, is considered by Leung and Huang [8]. A minimum phase space volume (MPSV) approach is presented. The approach, which exploits that a chaotic signal has a finite volume in an embedded phase space, can provide an accurate estimation. However, the computational load of the phase space volume is heavy.

In this paper, we consider the problem of estimating parameters of a signal embedding in chaotic noise. A novel technique called minimum nonlinear prediction error (MNPE) is developed. The MNPE approach is based on the idea that a chaotic signal can be modelled by a deterministic function called nonlinear prediction function, and can be predicted in short term. To estimate the parameters, the received signal is first passed through an inverse filter, and then the parameters can be estimated by minimizing the nonlinear prediction error of the output signal of the inverse filter. It is shown that the parameters of the inverse filter will approach the correct parameters of the signal to be estimated as the nonlinear prediction error of the output signal goes to the minimum. Compared with the MPSV approach, the MNPE approach has low computational load. An autoregressive (AR) model driven by a chaotic noise is used in numerical simulations. The MNPE approach and conventional least squares (LS) approach are applied to estimate the coefficients of the AR model. We investigate the effects of the number of points for estimation, the difference between the approximated nonlinear prediction function and desired function, and noise in the received signal on the estimation performance of the two approaches. The results show that the MNPE approach obviously outperforms the LS approach in the sense that it has smaller mean squares estimation error.

2. THE MINIMUM NONLINEAR PREDICTION ERROR ESTIMATION TECHNIQUE

The estimation problem in this paper is formulated as follows:

$$x_n = s_n(\theta_0) + w_n, \tag{1}$$

where

- x_n received signal
- s_n signal of interest
- w_n chaotic noise
- θ_0 parameter vector to be estimated

Suppose the system which generates the chaotic noise is smooth, nonlinear function. According to the Takens embedding theorem [12], if a positive integer d is sufficiently larger than the attractor dimension, then an d-dimensional embedding will give in general a faithful representation of the attractor, and there is a nonlinear prediction function satisfying

$$w_n = f(\mathbf{w}_{n-1}), \tag{2}$$

where $\mathbf{w}_{n-1} = (w_{n-1}, w_{n-2}, \cdots, w_{n-d})$. Based on the prediction function, one step nonlinear prediction error (NPE) can then be defined as

$$P(w) = \sum_{n=d+1}^{\infty} (w_n - f(\mathbf{w}_{n-1}))^2. \tag{3}$$

To apply NPE to the estimation problem in (1), an inverse filter approach is employed here. More precisely, an inverse filter $u_n = x_n - s_n(\theta)$ for the received signal is constructed, and the signal model is substituted into the inverse filter to get $u_n = s_n(\theta_0) - s_n(\theta) + w_n$. The following theorem shows that by minimizing the nonlinear prediction error of u_n, the parameter vector θ of the inverse filter will converge to θ_0.

Theorem 1: If $s_n(\theta)$ is linear, then $\theta = \theta_0$ if and only if $P(u)$ is minimum.

Proof: For the only if part, if $\theta = \theta_0$, then $u_n = w_n$, and hence, $P(u) = P(w) = 0$. From the definition of prediction error, we know that $P(u) \geq 0$, therefore, $P(u) = 0$ is a minimum.

For the if part, assume that besides θ_0, there is another parameter vector θ_1 minimizing the nonlinear prediction error. Let $\hat{w}_n = x_n - s_n(\theta_1)$ and corresponding prediction error be $P(\hat{w})$. When $\theta = \theta_0$, $P(w) = 0$, so $P(\hat{w}) = 0$. That is

$$\hat{w}_n = f(\hat{\mathbf{w}}_{n-1}), \tag{4}$$

where $\hat{\mathbf{w}}_{n-1} = (\hat{w}_{n-1}, \hat{w}_{n-2}, \cdots, \hat{w}_{n-d})^{\mathrm{T}}$.

Subtracting (2) from (4) gets

$$\varepsilon_n = f(\mathbf{w}_{n-1} + \Upsilon_{n-1}) - f(\mathbf{w}_{n-1}), \tag{5}$$

where $\varepsilon_n = s_n(\theta_0) - s_n(\theta_1)$ and $\Upsilon_{n-1} = (\varepsilon_{n-1}, \varepsilon_{n-2}, \cdots, \varepsilon_{n-d})^{\mathrm{T}}$. While the right side of (5) is the nonlinear function of Υ_{n-1}, there only exists linear relationship between ε_n and Υ_{n-1} in the left side because the $s_n(\theta)$ is linear. Thus (5) can not hold except that $\varepsilon_{n-i} = 0, i = 0, 1, \cdots, d$, that is, $s_n(\theta_1) = s_n(\theta_0)$ for all n. In other words, $\theta_1 = \theta_0$ as the parameter vector in $s_n(\theta)$ is identifiable. Therefore, only when $\theta = \theta_0$, $P(u)$ is minimum. □

Theorem 1 tells us that the parameter vector can be accurately estimated by minimizing the nonlinear prediction error of the output of the inverse filter. For applications such as equalization of a chaotic communications system where the functional form of a chaotic signal is known as a prior, the exact nonlinear prediction function can be used. When the MNPE method is applied real signals such as speech deconvolution, the unknown mapping can be approximated using an approximation prediction function constructed by a universal function such as neural network and radial basis function. Let the approximation prediction function be $\hat{f}$.

Given the received signal $\{x_n, n = 1, 2, \cdots, N\}$, and the approximated prediction function $\hat{f}$, the parameter estimation using the MNPE technique can be summarized in the following steps:

1. Construct an inverse system $u_n = x_n - s_n(\theta)$;

2. Embed u_n into a d-dimensional phase space using the delay coordinate method and compute the NPE of the output of the inverse filter u_n

$$P(u) = \sum_{n=d+1}^{N} (u_n - \hat{f}(\mathbf{u}_{n-1}))^2, \qquad (6)$$

where $\mathbf{u}_{n-1} = (u_{n-1}, u_{n-2}, \cdots, u_{n-d})$; and

3. Minimize the NPE in (6) with respect to the parameter vector θ.

Note that the minimizing NPE in (6) is generally a nonlinear optimization problem. Here we use the random research technique to look for the global optimization solution. In particular, we use 2000 different points to initiate the optimization and take the best one as the solution.

3. COMPUTER SIMULATIONS

In this section, we evaluate the effectiveness of the MNPE approach by applying it to estimate the coefficients of an AR model:

$$\begin{aligned} x_n &= \sum_{i=1}^{p} a_i x_{n-i} + w_n \\ &= 0.195 x_{n-1} - 0.95 x_{n-2} + w_n. \end{aligned} \qquad (7)$$

The coefficients in (7) are chosen to have a stable AR model. The chaotic signal w_n is generated by the logistic map:

$$w_n = \lambda w_{n-1}(1 - w_{n-1}). \qquad (8)$$

When $\lambda = 4$, w_n is a white process.

Following the MNPE algorithm in Section 2, an inverse AR model $u_n = x_n - \hat{a}_1 x_{n-1} - \hat{a}_2 x_{n-2}$ is first constructed. The NPE of u_n is then computed and minimized by varying $\hat{a}_1$ and $\hat{a}_2$. Figure 1 plots the nonlinear prediction error P as an error function of the coefficients $\hat{a}_1$ and $\hat{a}_2$. As we can see, the error surface has a very sharp unimodal shape, and the correct parameter values are located right at the minimum of P (i.e., the maximum in Fig. 1, which displays $1/P$ for a clearer illustration).

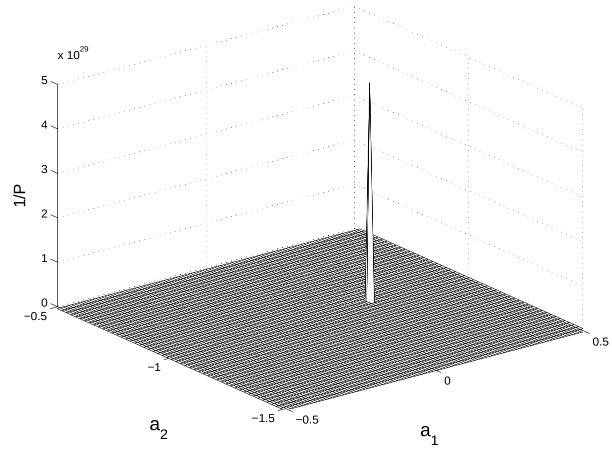

Figure 1: Prediction error versus the coefficients in the inverse filter.

Monte Carlo simulations are performed to investigate the effectiveness of the chaotic estimation approach. We used the mean squares error defined as

$$\text{MSE} = \frac{1}{T}\sum_{i=1}^{T}[(\hat{a}_1(i) - a_1)^2 + (\hat{a}_2(i) - a_2)^2], \qquad (9)$$

as a measurement of the estimation accuracy, where T is the number of trials.

First we consider the MSE of MNPE approach using different number of points for estimation and plot the results in Fig. 2. In this experiment, $d = 1$, $\hat{f}(x) = 4x(1-x)$, and $T = 500$. The estimation performance of LS approach is also given in Fig. 2. The MSE of the LS approach decreases with increasing number of points N. When 8 points are used in the estimation, the MSE of the LS approach is about -8dB, whereas for 250 points, its MSE is about -17dB. However, the MNPE approach remains the same MSE level (about -35dB) for all number of points. Compared the performance of the two approaches, the MNPE approach consistently outperforms the LS approach in all cases. The difference ranges from 18dB ($N \geq$250) to 27dB (N=8).

Next we investigate the effect of the difference between the approximated prediction function $\hat{f}$ and desired f on the performance of the MNPE approach. Here we fix $d = 1$, $N = 56$, $T = 500$, and vary a from 1.9 to 4 in the approximated prediction function $\hat{f}(x) = ax(1-x)$. Figure 3 presents the results. As expected, the MSE of the MNPE approach decreases with increasing the difference between a and $\lambda = 4$. The MSE is about -27dB when $a = 1.9$, and about -35dB when $a = 4$. The LS approach is independent of a, and has the same MSE (-13dB). Apparently, the MNPE approach has smaller MSE even if there exists a relatively large difference between the approximated prediction function and desired one.

We finally study the effects of noise in the received signal on the performance of two approaches. More precisely, the received signal x_n in the algorithm of Section 2 is replaced by $y_n = x_n + v_n$, where v_n is a Gaussian white noise process with zero mean. Its variance is varied to obtain different signal to noise ratios (SNR's). The simulation results in Fig. 4 show that the noise in the received signal has large effect on the performance of two approaches, especially the MNPE approach. When SNR$\geq$45dB, the MSE of the LS approach is about -13dB, and the MSE of the MNPE approach is about -35dB. The difference between them is 22dB. The difference decreases with increasing the variance of the Gaussian noise. When SNR=30dB, the difference becomes 10dB, and when SNR=10dB, the difference is almost zero. Thus the MNPE approach has better estimation performance than the LS approach for high SNR's, and the two approaches has the same performance for low SNR's.

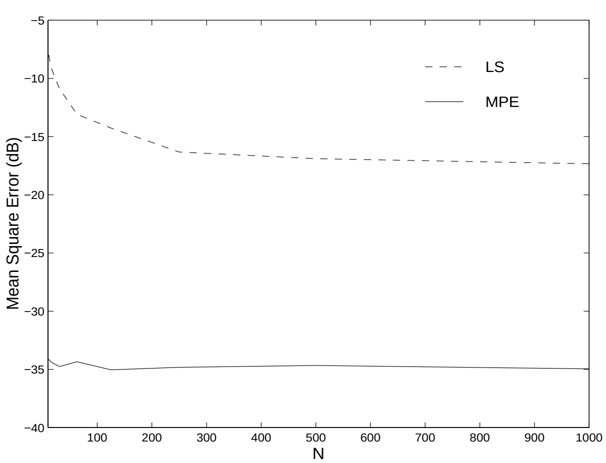

Figure 2: Comparison of the mean squares error of the MNPE and LS approach using different number of points.

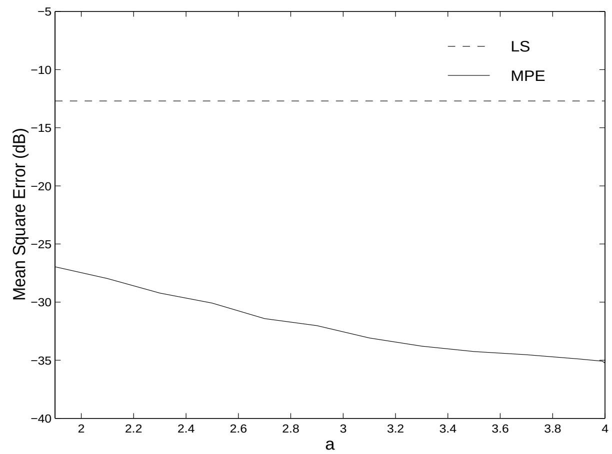

Figure 3: Comparison of the mean squares error of the MNPE and LS approach with different a in the approximated prediction function.

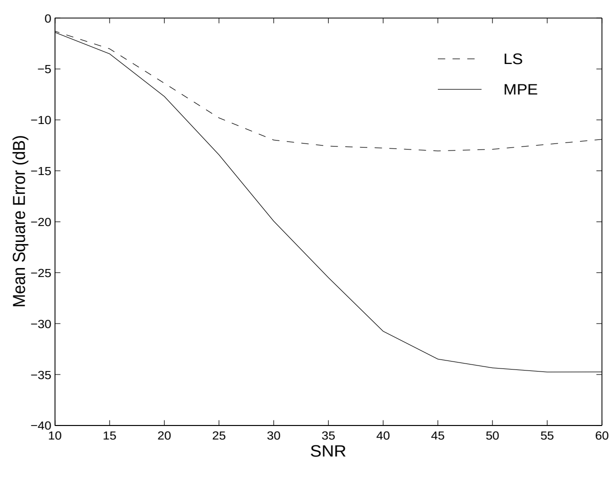

Figure 4: Comparison of the mean squares error of the MNPE and LS approach versus SNR when the received signal exists Gaussian noise.

4. CONCLUSIONS

We have proposed a novel approach called minimum prediction error (MNPE) for the parameter estimation in the chaotic noise. This approach exploits the inherently deterministic nature of a chaotic signal. More precisely, a chaotic signal can be predicted in the short term using a nonlinear prediction function. It is shown that the parameters can be accurately estimated by minimizing the nonlinear prediction error of the output of an inverse filter. Compared with the LS approach, the MNPE approach has smaller mean squares estimation error.

5. REFERENCES

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