FUZZY VQ ALGORITHMS FOR COLOR QUANTIZATION

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ABSTRACT

Two new extensions of Fuzzy C-means (FCM) algorithm which minimize an objective function incorporating a validity index are proposed. These algorithms are applied to color quantization of images. In the first approach, we minimize an objective function including a term for partition index. This algorithm attempts to place the cluster centers such that the membership values of the pixels are maximized. In the second approach, we minimize an objective function including an inter-cluster separation term. The goal here is to move cluster centers apart from each other towards the convex hull of the color space, hence obtaining a color palette which is more suitable for dithering, an operation generally applied after the quantization of the images.

1. INTRODUCTION

Many image display and printing devices allow only a limited number of colors to be used. These colors constitute a palette, which typically contains 256 or fewer entries. Original images represent each color component with one byte, therefore they can contain up to 16 million different colors, which must then be mapped to the available colors in the palette. This process of selecting a suitable palette and mapping each pixel in the original image to an entry in the palette is called quantization.

The C-means vector quantization algorithm has been long applied to image palette design, where palette is generally referenced as codebook [1]. The C-means algorithm partitions a collection of n 3x1 vectors $\mathbf{x_j}$, j=1,...,n, into c clusters where c is the codebook size. The algorithm finds a cluster center in each group such that an objective function J is minimized. Euclidean distance is commonly chosen as the dissimilarity measure. This schema is also called hard quantization since each pixel is represented by only one codebook entry.

Fuzzy quantization is a generalization of hard quantization schema and the best known and most widely used fuzzy quantization technique is the Fuzzy C-means (FCM) algorithm developed by Dunn [2] and refined by Bezdek [3]. In the FCM algorithm, each data point belongs to a cluster with a degree specified by a membership grade between 0 and 1. Thus, the FCM algorithm partitions n vectors into c fuzzy groups. Summation of membership values is equal to unity:

$$\sum_{i=1}^{c} u_{ij} = 1 \quad \forall j = 1, ..., n \tag{1}$$

The objective function is defined as:

$$J = \sum_{j=1}^{n} \sum_{i=1}^{c} (u_{ij})^m \; d(\mathbf{x_j}, \mathbf{y_i}) \tag{2}$$

where m is the parameter of fuzziness and $\mathbf{y_i}$ denote the set of quantization colors. Although no formal method exists to define the optimal value of m, in the literature it is generally chosen around 2. Both hard quantization and FCM algorithm use an iterative procedure and produce locally optimal codebooks depending on initial code vector locations.

Cluster validity measures have been used to evaluate the quality of the clusters in quantitative and objective fashion. The quality of a clustering is indicated by a validity function which assigns a number to the output of a classifier to associate the data points to cluster centers. Examples of well known validity measures are the partition coefficient and classification entropy [3], proportion exponent [4], Dunn's index [2] and Davies-Bouldin index [5]. Validity indexes are generally used to determine the best choice of c to identify the structure in the data. In [6], it is also used for the re-clustering of a fixed number (c) of clusters through a split and merge approach to obtain a better codebook.

In this paper, we propose two new extensions of the FCM algorithm which minimize an objective function incorporating a validity index. In the first approach, we minimize an objective function including a term for partition index. This algorithm attempts to place the cluster centers such that the membership values of the pixels are maximized. Although the intended application domain is image quantization, it is applicable to any classification schema. In the second approach, we minimize an objective function including an inter-cluster separation term. The goal here is to move cluster centers apart from each other towards the convex hull of the color space, hence obtaining a color palette which is more suitable for dithering, an operation generally applied after the quantization of the images [7]. Although this algorithm is specifically designed for color image quantization, it may also be applied to general classification problems. In Section II, we introduce Fuzzy Quantization with Partition Index Maximization (PIM). In Section III, Fuzzy Quantization with Inter-cluster Separation (ICS) is introduced. The results of both approaches are in Section IV. In Section V, the current study is evaluated and future areas of research are pointed out.

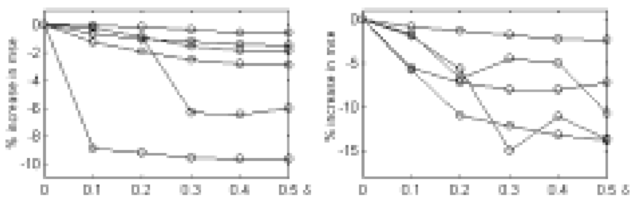

Figure 1: (a)Percentage decrease in mse for m=1.3, (b)Percentage decrease in mse for m=1.5

2. PARTITION INDEX MAXIMIZATION

Partition index is a measure of validity using $P_j = \sum_{i=1}^{c} (u_{ij})^m$ as a measure of how well the j^{th} data point has been classified. The closer a pixel is to a codebook entry, the closer P_j is to 1. If a pixel becomes an outlier to all cluster centers, the value of P_j approaches $1/c^{m-1}$, which is the minimum value it can have. Therefore, if we aim to minimize the fuzzy euclidian distance measure and maximize the membership values of the partitions, the objective function becomes

$$J(u,y) = \sum_{j=1}^{n}\sum_{i=1}^{c}(u_{ij})^m\, d(\mathbf{x_j},\mathbf{y_i}) - \alpha\sum_{j=1}^{n}\sum_{i=1}^{c}(u_{ij})^m \quad (3)$$

under the constraint of Equation 1. The parameter α controls the weight of the second term and will be further examined. Using the standard technique of Lagrange multipliers, the equation to be minimized is

$$J_j(u_j,\lambda) = \sum_{i=1}^{c}(u_{ij})^m d(\mathbf{x_j},\mathbf{y_i}) - \alpha\sum_{i=1}^{c}(u_{ij})^m - \lambda\Big(\sum_{i=1}^{c}u_{ij} - 1\Big) \quad (4)$$

where λ is the Lagrange multiplier. The minimization of $J_j(u_j,\lambda)$ proceeds as follows:

$$\frac{\partial J_j}{\partial u_{ij}} = m(u_{ij})^{m-1}\, d(\mathbf{x_j},\mathbf{y_i}) - \alpha\, m\,(u_{ij})^{m-1} - \lambda = 0 \quad (5)$$

Solving for u_{ij} yields:

$$u_{ij} = \left[\frac{\lambda}{m}\frac{1}{d(\mathbf{x_j},\mathbf{y_i}) - \alpha}\right]^{\frac{1}{m-1}} \quad (6)$$

from Equation 1:

$$\frac{\lambda}{m} = \frac{1}{\sum_{k=1}^{c}\frac{1}{d(\mathbf{x_j},\mathbf{y_k})-\alpha}} \quad (7)$$

and plugging Equation 7 in Equation 6 we obtain:

$$u_{ij} = \left[\frac{1}{\sum_{k=1}^{c}\frac{d(\mathbf{x_j},\mathbf{y_i})-\alpha}{d(\mathbf{x_j},\mathbf{y_k})-\alpha}}\right]^{\frac{1}{m-1}} \quad (8)$$

Note that if $\alpha = 0$, the proposed extension of the FCM reduces to the standard algorithm. Because the terms of the form $d(\mathbf{x_j},\mathbf{y_i}) - \alpha$ in Equation 8 may take 0 or negative values, resulting in undefined or negative membership values, we should define

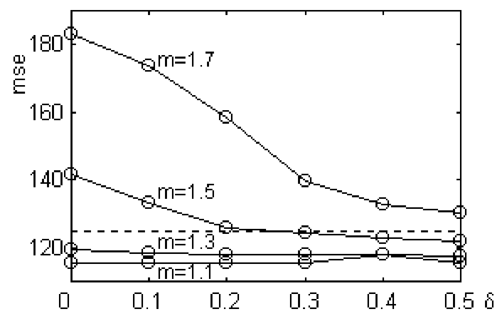

Figure 2: Mse vs δ for different m values for the image "peppers". Dotted line represents the mse obtained after k-means quantization.

$$u_{ij} = 1 \quad for\ d(\mathbf{x_j},\mathbf{y_i}) \le \alpha \quad (9)$$

The physical interpretation of Equation 9 is to define a hard region around quantization centers with radius α. Because P_j is independent of $\mathbf{y_i}$, the update equation is the same as in the FCM:

$$\mathbf{y_i} = \frac{\sum_{j=1}^{n}(u_{ij})^m x_j}{\sum_{j=1}^{n}(u_{ij})^m} \quad (10)$$

3. INTER-CLUSTER SEPARATION MAXIMIZATION

In [6], the separation (s_i) of a fuzzy cluster (i) is defined as the sum of the distances from its cluster center (y_i) to the center of the other $(c-1)$ clusters

$$s_i = \sum_{t=1}^{c}\|\mathbf{y_i} - \mathbf{y_t}\|^2 \quad (11)$$

In [7], Modified Binary Tree Splitting (MBTS) algorithm is suggested to design a quantizer for color images in such a way that better results are obtained after dithering, an operation generally applied after the quantization of the images. Quantization of the images causes some visual artifacts such as false edges and color streaks. For this reason, quantization is generally followed by a dithering operation. The goal is to hide these defects and to achieve a more faithful reproduction of colors by using the averaging property of the human eye. In a dithering technique called error diffusion, this is achieved by spreading the quantization error to neighboring pixels. The idea in MBTS algorithm is to obtain a wider color space by displacing the pairs of quantization centers opposite to each other, therefore creating the illusion of more colors. For the same purpose, using s_i to give displacements to all quantization centers towards the convex hull of the image color space yields a larger volume of colors in the convex hull of the quantization colors. We incorporate s_i into the objective function J to minimize the fuzzy euclidian distance and maximize the inter-cluster separation. We define the objective function:

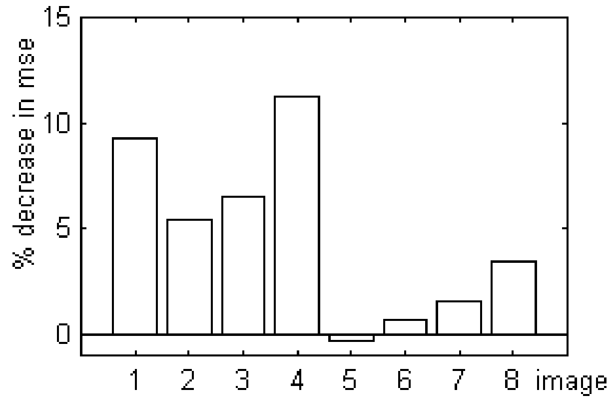

Figure 3: The percentage decrease in mse using HVS filter for 8 test images

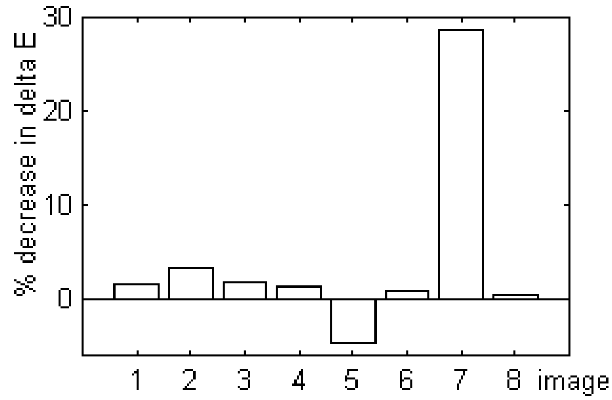

Figure 4: The percentage decrease in Δ E using HVS filter for 8 test images

$$J(u,y) = \frac{1}{n}\sum_{j=1}^{n}\sum_{i=1}^{c}(u_{ij})^m\, d(\mathbf{x_j},\mathbf{y_i}) - \frac{\gamma}{c}\sum_{t=1}^{c} d(\mathbf{y_i},\mathbf{y_t}) \quad (12)$$

Again, we minimize $J(u,y)$ using the Lagrange multipliers method. Because s_i is independent of u, the membership function is the same as in the FCM. The update function y_i is obtained as follows:

$$\frac{\partial J_i}{\partial y_i} = \frac{-2}{n}\sum_{j=1}^{n}(u_{ij})^m\,(\mathbf{x_j}-\mathbf{y_i}) - \frac{2\gamma}{c}\sum_{t=1}^{c}(\mathbf{y_i}-\mathbf{y_t}) = 0$$

$$-\frac{1}{n}\sum_{j=1}^{n}(u_{ij})^m\,\mathbf{x_j} + \frac{1}{n}\sum_{j=1}^{n}(u_{ij})^m\,\mathbf{y_i} - \gamma\mathbf{y_i} + \frac{\gamma}{c}\sum_{t=1}^{c}\mathbf{y_t} = 0$$

$$\mathbf{y_i} = \frac{\frac{1}{n}\sum_{j=1}^{n}(u_{ij})^m\,\mathbf{x_j} - \frac{\gamma}{c}\sum_{t=1}^{c}\mathbf{y_t}}{\frac{1}{n}\sum_{j=1}^{n}(u_{ij})^m - \gamma} \quad (13)$$

It is clear that $\gamma = 0$ corresponds to the standard FCM algorithm. Determination of the value of γ is investigated in Section 4.

4. RESULTS

4.1. Partition Index Maximization

We have implemented the PIM algorithm for color images. First, we needed to determine the value of α used in Equation 3 that controls the amount of contribution of partition index to fuzzy quantization. Equation 9 dictates that the value of α should not exceed the halfway between two cluster centers. Therefore, it is dependent to the distribution of the cluster centers. A straightforward approach is to set α to a fraction of the distance between the closest two quantization centers, that is $\alpha = \delta * min[d(\mathbf{y_i},\mathbf{y_j})]$, where $0 \le \delta < 0.5$. Hence, α is dynamically determined. In Figure 1, percentage change in mean squared error (mse) vs. δ for some test images is shown. $\delta = 0$ corresponds to classical fuzzy quantization and as δ increases, the hard region around the quantization centers grows. In Figure 1(a), the fuzziness parameter m is set to 1.3 and in In Figure 1(b) it is set to 1.5. It is seen that the PIM algorithm yields about 10% decrease in mse with respect to FCM for $m = 1.5$, while the decrease in mse is about 5% for $m = 1.3$. It is interesting to see that the percentage decrease in mse is greatly effected by the choice of m. That is, the fuzzier the quantizer, more improvement in mse is obtained as δ approaches to 0.5. The reason can be explained as follows: If a pixel $\mathbf{x_j}$ is in δ region of the quantization color $\mathbf{y_i}$, there is not much doubt about which quantization center it belongs to. Therefore, PIM algorithm assigns $\mathbf{x_j}$ to $\mathbf{y_i}$ in a hard manner, that is $u_{ij} = 1$. If $\mathbf{x_j}$ is outside of δ region of all quantization centers, its membership value is calculated by the fuzzy membership function defined in Equation 8. If m is small, the degree of fuzziness is already low and effect of introducing a hard region around quantization centers is also small. If m is larger, the degree of fuzziness is larger and the effect of the hard region defined by δ increases. Therefore, better codebooks are obtained by fuzzy quantization with respect to hard quantization, but even better codebooks are obtained by a semi-fuzzy quantization schema.

This result brings out the question of the effect of m and how δ contributes to the resultant mse for color image quantization. In Figure 2, the change in mse vs δ for different m parameters is shown for the image "peppers". It is seen that the value of m plays an important role for fuzzy quantization, and our experiments show that setting m to 1.3 produces satisfactory results for most images. However, introducing a hard region around quantization centers further reduces mse, more for relatively higher m values. Therefore, quantization process becomes less sensitive to the m parameter and lower mse values are obtained for any m value with respect to the FCM algorithm.

We also measured the error after applying the modulation transfer function (MTF) of human visual system (HVS) [8] [9] to resulting images. We transformed the color image to a gray level image by calculating the perceptual lightness at each pixel, filtered

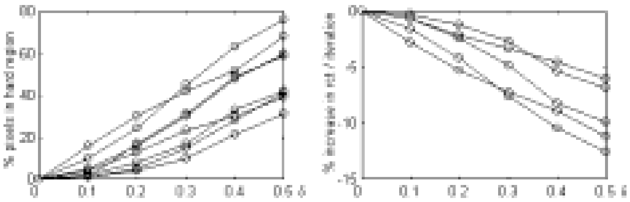

Figure 5: (a) The percentage of the pixels in the hard region vs δ for some test images, (b) The percentage increase in rct/iteration vs δ for some test images

with the HVS filter, and computed the mse afterwards as in [10].

The results are presented in Figure 3 for 8 of the test images. It is seen that there is a perceptual improvement in 7 images. Although one of the images has a slight increase in mse after HVS filter is applied, it is less than 1%.

The color differences (ΔE) of the test images in $L^*a^*b^*$ coordinate system [11] is also calculated:

$$\Delta E = [(\Delta L^*)^2 + (\Delta a^*)^2 + (\Delta b^*)^2]^{1/2} \tag{14}$$

The results are seen in Figure 4 for 8 test images. In general, there is a decrease in color differences with respect to FCM quantization for most images, resulting in perceptually better quantized images.

Introducing a hard region also effects the speed of the quantization. The membership values of the pixels in the hard region do not need to be calculated. In Figure 5(a), the percentage of the pixels classified in a hard manner for some specific δ values are seen. If δ is chosen to be close to 0.5, about 40% of the pixels are classified in a hard manner on the average. This schema has an accelerating effect for most images. The increase in relative computation times (rct) per iteration vs δ is given in Figure 5(b). As δ approaches to 0.5, each iteration gets about 10% faster than it is in FCM. The percentage increase in rct with respect to FCM algorithm is calculated for test images. In general, the algorithm converges faster as hard region grows out. The percentage increase in rct for δ=0.49 is around 10-15% as shown in Figure 6.

We also implemented the PIM algorithm for 2 dimensional test data given in [6]. The data in Figure 7 consist of 2 clusters which are distinctly apart. Figure 7(a) shows the partition imposed by FCM with 2 clusters for $m = 1.3$. The FCM algorithm tries to partition the feature space into equal size clusters. The classification imposed by the PIM algorithm with $\delta = 0.4$ is seen in Figure 7(b). The PIM algorithm assigns higher membership values to nearby vectors and lower membership values to further vectors. This is seen in Figure 8, where the membership functions of both FCM and PIM algorithms are sketched in 2 dimensions. Therefore, better performance is obtained by PIM algorithm if the clusters are relatively compact.

4.2. Inter Cluster Separation

The ICS algorithm is tested with various color images. The true color test images are first quantized to 16 levels and dithered using ICS algorithm and Floyd-Steinberg error diffusion filter. The

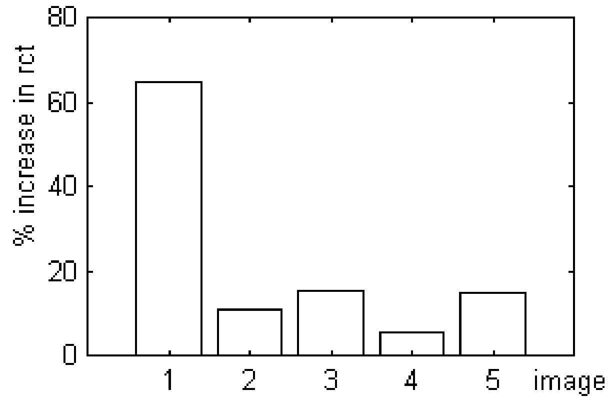

Figure 6: The percentage increase in rct for δ=0.49 for some test images

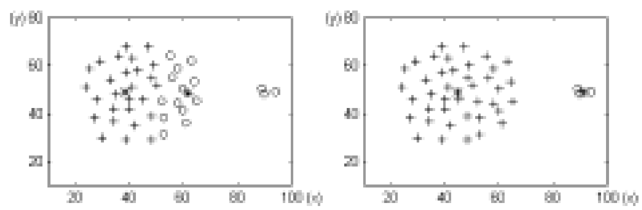

Figure 7: Two partitions obtained by (a)FCM (m=1.3) (b)PIM (m=1.3 $\delta = 0.4$)

goal is to obtain perceptually better images after dithering. Therefore, the error is measured after applying the modulation transfer function (MTF) of HVS to the resulting images.

A key issue is the determination of the parameter γ in Equation 13. It is the parameter controlling the degree of the deviation of quantization centers towards the convex hull of the color space of the image. The objective function in Equation 12 is normalized by $1/n$ to make γ independent of the image size. Therefore it should be a small number to perturb the quantization centers towards the convex hull. Setting $\gamma = 0$ means no perturbation, hence it corresponds to FCM. If γ is set to a large number, quantization centers may get too far away, even beyond the convex hull of the color space, thus clipping may be necessary and the resulting codebook will not represent the original image. Therefore, parameter γ will either be set to a small constant or it will be determined dynamically. In our experiments, we used both approaches to quantize the images to 16 colors. We saw that although $\gamma = 0.0005$ gave good results for most images, it needed some adjustment for better results for some of the images. Setting γ to some fixed small value in the beginning and slowly decreasing it resulted in a more robust schema for obtaining the desired enlarged palette. We used $\gamma = 0.001$ as the initial value and modified it in subsequent itera-

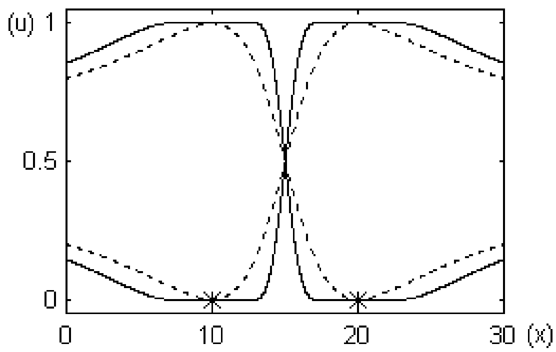

Figure 8: Membership functions (dotted line: FCM (m=1.5), solid line: PIM (m=1.5 $\delta = 0.3$))

tions according to

$$\gamma = \gamma \frac{\varepsilon_c}{\varepsilon_p} \tag{15}$$

where ε_c is the amount of error in current iteration and ε_p is the error in previous iteration.

In Figure 9, the results obtained for four different test images are presented. The vertical axis represents the percentage change in mse with respect to the quantized image with FCM algorithm ($m = 1.3$). The first set of data along the horizontal axis is the percentage increase in mse after ICS algorithm is applied for quantization. It is seen that the mse is generally higher than the FCM results. The second entry along the horizontal axis represents the change in mse after the images quantized with FCM algorithm are error diffused by Floyd-Steinberg filter. As expected, the error diffusion process introduces some improvement to the quantized images. The last set of data along the horizontal axis is the percentage decrease in mse after the image quantized with ICS algorithm is error diffused. It is seen that there is further improvement in the perceived error.

The examination of the resultant palettes for test images obtained from FCM and ICS algorithms show that the color space covered by the palette produced by the ICS algorithm is larger and the resultant mse after HVS filter is 15% smaller.

5. CONCLUSIONS

Two new extensions of FCM algorithm are introduced. The PIM algorithm is used to minimize an objective function including a term for partition index. The PIM algorithm sets a hard region around quantization centers and assigns higher membersip values to nearby vectors, lower membership values to further vectors. The resultant effect is lower mse and faster convergence for most images. The PIM algorithm may be viewed as a semi-fuzzy quantization technique. It may also be used for classification purposes.

The ICS algorithm is specifically developed to obtain better images after error diffusion, which is applied after quantization to remove side effects like false edges and color streaks. The goal is to enlarge the convex hull of the quantization colors to obtain the illusion of more colors after error diffusion. Although the mse

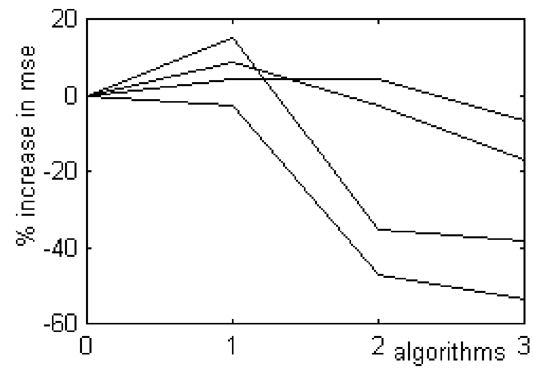

Figure 9: Percentage increase in mse using HVS approach: 0. Reference is the mse after fuzzy quantization(FCM) 1. Quantization with ICS algorithm 2. Error diffusion after FCM 3. Error diffusion after ICS

increased for most images after quantization, generally lower mse values are obtained after error diffusion compared to the use of FCM for quantization.

Our future research will include combining fuzzy quantization and error diffusion algorithms to obtain perceptually better images after error diffusion.

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