TIME-FREQUENCY DOMAIN SIGNAL FILTERING:

A NEXT STEP FORWARD

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ABSTRACT

State of the art of time-frequency signal representation methods and their use for signal and image processing are reviewed and the ways of their further development are discussed and illustrated by examples.

1. INTRODUCTION

Time-frequency domain signal representation has been known since 40-th as a powerful method for nonstationary signal analysis and processing ([1]). However, until recently its application was limited due to the lack of adequate computer power. Modern progress of digital computer and signal processing technology has opened new perspectives in signal processing in time-frequency domain. Recently proposed transform image coding methods, local adaptive linear filters ([2-9]) and nonlinear wavelet methods ([10-13]) represent a first advance in this direction.

Transform image coding methods are maybe the earliest example of application of time-frequency signal representation. In transform coding, image is divided into blocks whose transform representation is shrunk and quantized. This process can be regarded as sampling the time-transform representation of the signal followed by shrinkage and quantization.

The local adaptive filters work in a moving window in a domain of an orthogonal transform and, in each position of the window, nonlinearly modify the transform coefficients of the signal representation in order to obtain an estimate of the central sample of the window. In the simplest implementation for signal de-noising, the transform coefficients are compared with a threshold and those that do not exceed the threshold are discarded as noise.

In signal denoising by nonlinear subband (wavelet, lapped transform) methods such as wavelet shrinkage the signal is successively split into halves that are then expanded in orthogonal bases such as local trigonometric bases or wavelet packets, and, at each scale, transform coefficients are compared with a certain threshold. Those that do not exceed the threshold are discarded from signal reconstruction. Signal denoising by wavelet shrinkage have gained a considerable popularity in signal processing community.

None of these methods, however, uses the full potential of time(space)-frequency signal representation. In time (space)-frequency domain, they apply only point-wise operations such as, for instance, point-wise weighting, shrinkage or zonal quantization, with (practically) no regard to the transform representation for adjacent samples.

Time (space)– frequency signal (image) representation is a special case of what one can call time (space) – transform domain representation. In what follows, we'll be using the term "time-frequency representation" in this more general sense.

2. REDUNDANCY OF TIME-FREQUENCY DOMAIN SIGNAL REPRESENTATIONS

The existence of redundancy in time-frequency signal representation is associated with the fact that time-frequency signal representation contains SzW times more data then the signal, where SzW is size of the moving window used to generate the representation. Mathematically, one can treat this representation as a frame [20]. This redundancy exhibits itself in thee appearance of highly correlated and, to a certain degree, regular patterns in time-transform domain (Fig. 1). The regularities in time-frequency signal representation depend both on the type of the transform and on the correlations characteristic of particular signals. For instance, local DFT spectra $\{\alpha_r^k\}$ and $\{\alpha_r^{k+1}\}$ of a real valued signal $\{a_n, n=1,...,k,...\}$ in the moving window of SzW samples for adjacent (k)-th and $(k+1)$-th positions of the window are connected with the following relationships:

$$\alpha_r^{k+1} = \left(\alpha_r^k + \frac{a_{k-(N+1)/2} - a_{k+(N-1)/2}}{\sqrt{SzW}}\right)\exp\left(-i2\pi\frac{r}{SzW}\right) \quad (1)$$

$$\left|\alpha_r^{k+1}\right|^2 = \left|\alpha_r^k\right|^2 + 2\frac{a_{k-(N+1)/2} - a_{k+(N-1)/2}}{\sqrt{SzW}}\cos\left(2\pi\frac{r}{SzW}\right) \quad (2)$$

One can see from these formulas that sort of an auto-regressive model holds for local DFT signal spectra. Similar, though sometimes more involved recursive relationships can be derived for other transforms.

Due to the regularity, the patterns that signals form in their time-frequency domain representation are much easier to discriminate if they are treated as 2-D (4-D for space-transform domain) ones rather then point-wise as it is done now in above mentioned time-frequency domain signal processing methods. This may represent a substantial potential for improving time-frequency domain signal processing.

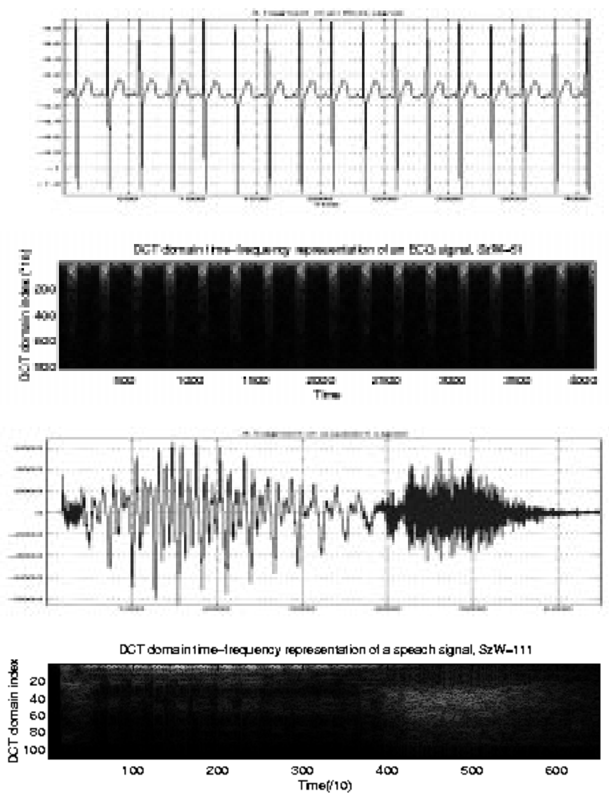

Fig. 1 Examples of ECG and speech signals and patterns they form in their time-frequency representations

At present, quite a number of image processing methods, linear and nonlinear, are known that are capable of efficient detection and restoration of patterns in noise clutter. The design and use of these methods for signal processing in time(space)-transform domain is simplified by the availability of formal models that describe signal correlations in time (space)- transform domain.

3. APPPLICATIONS OF TIME-FREQUENCY PROCEESSING

Important applications of time-frequency signal processing are:

1. Signal denoising, restoration and separation
2. Signal (image) coding
3. Signal interpolation
4. Signal recognition
5. Signal detection and target location
6. Signal and image watermarking and generating signals with a specified time-frequency pattern

In signal denoising and time-frequency separation, signal representation is segmented into different areas representing signal and noise or different signals. In signal restoration, signal time-frequency representation is modified to compensate degradations in system (signal and image blur, signal interruptions, etc.). In this process, correlations of patterns in time-frequency representation can be actively exploited to improve reliability of signal and noise separation.

In signal (image) transform coding methods, time-frequency domain shrinkage, quantization and signal restoration efficiency can be improved by making use of entire time-frequency domain rather then only of its samples. Recently proposed zero-tree wavelet coding method ([14]) represents an advance in this direction.

In signal interpolation for restoration of intermediate or lost signal samples, interpolation (linear or nonlinear) of interrupted patterns in signal time-frequency representation may improve signal interpolation quality.

In signal recognition, time-frequency signal representation is compared as a whole with corresponding template representations in order to improve discrimination capability of recognition of signals or their individual fragments.

In signal detection and target location, time-frequency representation processing is used to allow for signal (image) inhomogeneity and in this way to improve the detection discrimination capability ([9]).

In signal watermarking, a special marking pattern is introduced into the signal time-frequency representation to allow identificvation of the origin of the signal, and time-frequency representation redundancy can be used to improve detectability of water marks and more efficiently hide watermarks in the signal.

4. EXPERIMENTAL RESULTS

In what follows we demonstrate results of experiments to illustrate time-transform domain signal processing. One set of experiments demonstrates excavation of a signal buried in a very strong interference. The experiments were performed with a test ECG signal hidden in the mixture with a very strong frequency modulated interference. Separation of signal and interference was carried out in two steps. On the first step, squared module of the space-frequency representation of the mixture was smoothed by a box filter with the aperture appropriately adjusted to simplify subsequent detection of the interference component by thresholding. At the second pass, the interference signal separated on the first step was subtracted from the mixture, and the result was subjected to similar filtering with time-frequency domain smoothing window adjusted to suppress residuals of the interference left after its removal after the first pass. Fig.2 shows a test ECG signal. Fig.3 shows the mixture of the test signal and the interference. Fig.4 illustrates subsequent steps of processing time-frequency signal representation aimed at separating the interference from the mixture. Fig.5 demonstrates the separated interference after the first pass of the processing. The result of removal the interference from the mixture is shown in Fig.6. One can see that it contains some interference left after the separation This residual interference is filtered out at the second pass (Fig.7). The restored resulting signal is shown in Fig.8. One can see from these illustrations that appropriate time-frequency domain processing may allow signal restoration and its separation from interferences even when interferences are substantially stronger then the signal. Simple detection by time-frequency domain thresholding is not capable of such an efficient restoration.

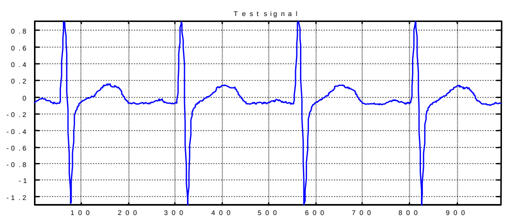

Fig. 2 A test signal

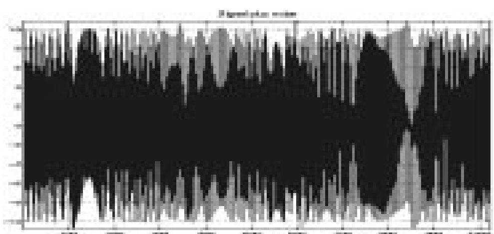

Fig. 3 Test signal camouflaged by a strong interference

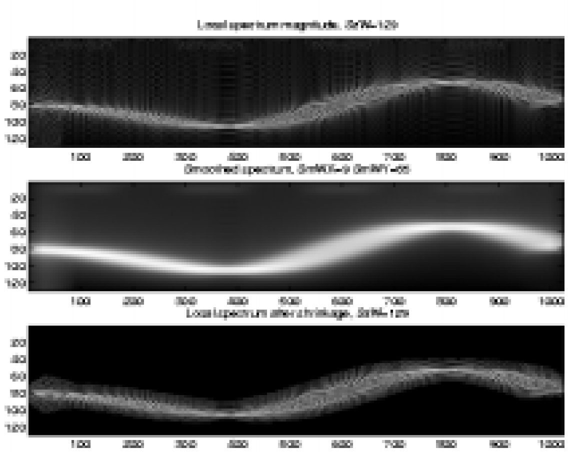

Fig. 4 From top to bottom: Magnitude of time-frequency (DCT domain shown in coordinates time versus spectrum magnitudes; window size is 129) representation of the mixure of signal and interference shown in Fig. 2; the same after smoothing by a box filter in the window 5 spectral components by 55 time components. Smoothing was aimed at time-frequency representation preparation for subsequent detection of the interference component by thresholding; time-frequency representation of the detected interference component.

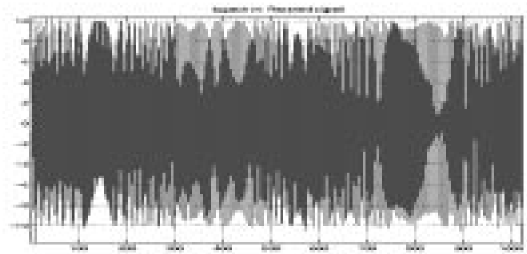

Fig. 5 Interference signal separated from the mixture

of Fig. 3

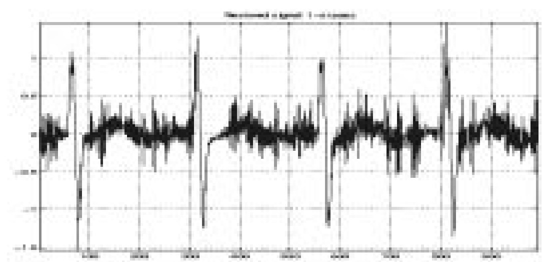

Fig. 6 Signal separated from the interference after the first pass.

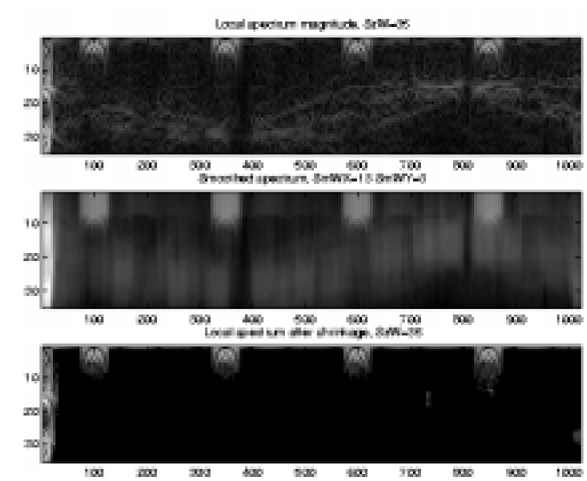

Fig. 7 Illustration of the second pass of processing of time-frequency domain of the separated signal of Fig. 5. From top to bottom: Magnitude of time-frequency (DCT domain shown in coordinates time versus spectrum magnitudes; window size is 35 samples) representation of the separated signal; the same after smoothing by a box filter in the window 13 spectral components by 3 time components. As in the first pass (Fig. 3), smoothing was aimed at time-frequency representation preparation for subsequent detection of the signal component by thresholding; time-frequency representation of the detected signal component.

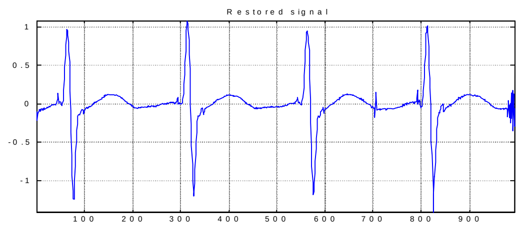

Fig. 8 Restored signal

The second set of experiments have been carried out to apply space-frequency filtering of images ([18,19]) as a post-processing stage to decompressed images that contain artifacts caused by compression.

Image compression algorithms used today are based mainly on block transform coding or subband decomposition ([14,15,16]). Discrete cosine transform (DCT) is the most commonly used transform that has many benefits. The main drawback of the DCT block coding are blocking artifacts ([17]). The borders between adjacent blocks become visible when the compression ratio is high. This visually disturbing phenomena is caused by the short non-overlapping basis functions of the DCT, hence the block borders become discontinuous after the quantization.

Subband and wavelet coding do not suffer from this kind of blocking artifacts since they do not operate on small discrete blocks. However, they may produce other artifacts. In the case of heavy quantization the long filters tend to cause visible ringing effects on high frequencies. Although this is not as visible as blocking, it can be disturbing if the amplitude of these oscillations is high enough. In Figures 9 and 10 are shown the piecewise constant image “Piecewise” and the corresponding decompressed image being compressed using “SPIHT” wavelet-based coder ([15]) with compression ratio CR=16, resulting in PSNR=29.5 dB.

Decompressed image in Fig. 10 has strong distortion (pseudo-Gibbs ripples near boundaries). In order to reduce this distortion one can use an adaptive denoising strategy based on a combination of wavelet and local denoising ([19]), which results in the following image shown in Figure 11. The method exploits local neighborhood correlations in local spatial-frequency domain by applying local transform-based (LTB) denoising ([18]) in order to update wavelet coefficients. Here, LTB filtered image is used as a “reference” image in an iterative adaptation scheme working in wavelet domain.

4. ACKNOWLEDGMENT

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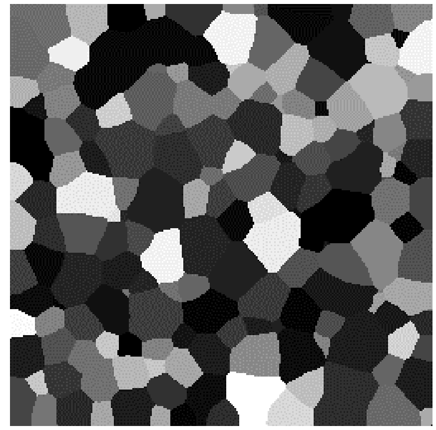

Fig. 9 Original “Piecewise” image

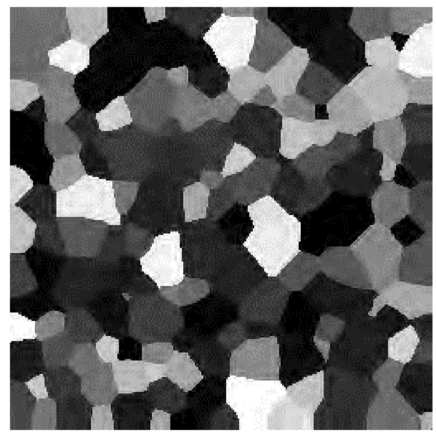

Fig. 10 Decompressed image “Piecewise”

(CR=1:16, PSNR=29.5)

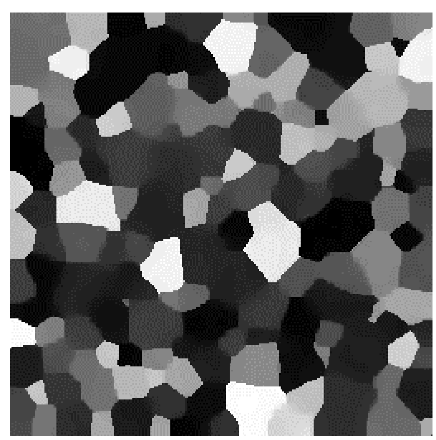

Fig. 11 Enhanced decompressed image “Piecewise”
(CR=1:16, PSNR=33.3)

5. REFERENCES

[1]. D. Gabor, *Theory of Communication*, J. Inst. Electr. Eng., 93,1946, pp. 429-457

[2]. L. Yaroslavsky, *Digital Signal Processing in Optics and Holography*, Radio I Svyaz', Moscow, 1987 (In Russian)

[3]. R. Y. Vitkus, L. Yaroslavsky, Recursive Algorithms for Local Adaptive Linear Filtration. *Mathematical Research*, Eds. L.P. Yaroslavskii, A. Rosenfeld and W. Wilhelmi, Berlin: Academy Verlag, pp. 34-39, 1987

[4]. L. P. Yaroslavsky, Linear and Rank Adaptive Filters for Image Processing, *Digital Image Processing and Computer Grasphics, Theory and Applications*, L. Dimitrov and E. Wenger, Eds., R. Oldenburg, Wien-Muenchen,1991, pp. 374-400

[5]. L.P. Yaroslavsky, Local Adaptive Image Restoration and Enhancement with the Use of DFT and DCT in a Running Window, *Wavelet Applications in Signal and Image Processing IV*, Eds.: M. A. Unser, A. Aldroubi, A.F. Laine, SPIE vol. 2825, pp. 2-11.

[6]. L.P. Yaroslavsky, Local Adaptive Filters for Image Restoration and Enhancement. *Int. Conf. on Analysis and Optimization of Systems. Images, Wavelets and PDE's, Paris, June 26-28, 1996*, Eds.: M.-O. Berger, R. Deriche, I. Herlin, J. Jaffre, J.-M. Morel, Springer Verlag, Lecture Notes in Control and Information Sciences, 219., 1996, pp. 31-39.

[7]. L. Yaroslavsky and M. Eden, *Fundamentals of Digital Optics*, Birkhauser, Boston, 1996.

[8]. L. Yaroslavsky, Nonlinear adaptive image processing in transform domain, *1977 IEEE Workshop on Nonlinear Signal and Image Processing, Sept. 7-11, 1997*, Grand Hotel, Mckinac Island, Michigan, USA

[9]. L. Yaroslavsky, Local Adaptive Filtering in Transform Domain for Image Restoration, Enhancement and Target Location. *Proceedings of SPIE, 6-th Imt. Workshop on Digital Image Processing and Computer Graphics (DIP-97). Applications in Humanities and Natural Sciences*, E. Wenger and L. I. Dimitrov, Eds., 20-22 Oct. 1997, Vienna, Austria, SPIE volume 3346, pp. 1- 17.

[10].D. L. Donoho, Nonlinear Wavelet Methods for Recovery of Signals, Densities, and Spectra from Indirect and Noisy Data, *Proc. Of Symposia in Applied Mathematics, American Mathematical Society*,1993, pp. 173-205

[11].D. L. Donoho, I. M. Johnstone, Ideal Spatial Adaptation by Wavelet Shrinkage, *Biometrica,* 81(3), pp. 425-455, 1994

[12].R.R. Coifman, D. L. Donoho, Translation- Invariant De-Noising, *Wavelet and Statistics*, Anestis Antoniadis, ed. Springer Verlag Lecture Notes, 1995

[13].D.L. Donoho, De-noising by soft thresholding, *IEEE Trans. Inf. Theory*, 41(3), 613-627, May 1995

[14]. J. M. Shapiro, "Embedded Image Coding Using Zerotrees of Wavelet Coefficients". *IEEE Trans. on Signal Processing*, vol. 41, no. 12, pp. 3445 - 3462, Dec. 1993.

[15].A. Said, and W. A. Pearlman, "A New Fast and Efficient Image Codec Based on Set Partitioning in Hierarchical Trees". *IEEE Trans. on Circuits and Systems for Video Technology*, vol. 6, no. 3, pp. 243 - 250, June 1996.

[16].Z. Xiong, O. G. Guleryuz, and M. T. Orchard, "A DCT-Based Embedded Image Coder". *IEEE Signal Processing Letters*, vol. 3, no. 11, pp. 289 - 290, Nov. 1996.

[17].Y.F. Hsu, and Y.C. Chen, "A new adaptive separable median filter for removing blocking effects". *IEEE Transactions on Consumer Electronics*, vol. 3, no. 39, pp. 510-513,1993.

[18].R. Oktem, L. Yarovslavsky, K. Egiazarian, "Signal and image denoising in transform domain and wavelet shrinkage: A comparative study", *Eusipco'98*, Signal Processing IX, Theories and Applications, Island of Rhodes, Greece, 8-11 Sept. 1998, v. 4, pp. 2269-2272.

[19].K. Egiazarian and J. Astola, "New algorithm for removing of mixed (white and impulsive) noise from images". Photonics West, *SPIE Proc.*, vol. 3646, San Jose, Jan. 1999.

[20].I.Daubechies, "The Wavelet Transform, Time-Frequency Localization and Signal Analysis", *IEEE Trans. Inform. Theory*, vol. 36, No. 5, pp. 961-1005, Sept. 1990.