COMBINATION OF TRANSFORM BASED DENOISING AND STACK FILTERING FOR NON-GAUSSIAN NOISE SUPPRESSION

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ABSTRACT

We consider the case of strongly non-Gaussian noise contamination. To handle this situation, we propose a combination of statistically optimized stack filters and transform based denoising. The purpose of using the stack filters is to reduce the noise level, remove outliers, while retaining important signal detail. The remaining noise at the output of this stage is closer to a Gaussian one. Consequently, transform based techniques become more effective in denoising. Numerical simulations confirming the effectiveness of the proposed method are presented.

1. INTRODUCTION

Multiresolution transforms, unlike traditional Fourier methods, are well suited for situations in which signals possess nonstationary behavior. Wavelet based techniques have been successfully applied in signal and image denoising applications, especially when the signal is embedded in Gaussian noise, which is evenly distributed over the wavelet coefficients [6]. Wavelet shrinkage removes Gaussian noise, but preserves sharp features of the signal. Unfortunately, thresholding of linear wavelet transforms does not work well in strongly non-Gaussian environments [7]. For example, a linear wavelet transform of i.i.d. Cauchy noise does not result in independent nor identically distributed wavelet coefficients [7]. Moreover, when the noise contains impulses, speckles, or is non-symmetrically distributed, wavelet methods do not produce satisfactory results.

In order to cope with heavy-tailed noise sources, several nonlinear multiresolution transform algorithms have been proposed [15], [7], [13] and have been mainly based on pyramidal decomposition structure. Only the case of zero-mean symmetric noise distributions (Gaussian, Laplacian, Cauchy, etc.) has been considered.

In order to handle non-Gaussian noise, much effort has gone into combining nonlinear filtering techniques with time-frequency domain analysis [12], [10], [5]. For example, in [10], the so-called smoother-cleaner wavelet decomposition was used to locate outliers and replace them with the local median. In [12], linear filters of wavelet decomposition were replaced with order statistic based Chameleon filters.

In the present work, we focus on non-Gaussian and specifically, non-symmetric noise distributions. Speckle noise is one such example and can be found in coherent imaging systems. Another example of a non-symmetric noise source is a mixture of two Gaussian densities. That is,

$$g(t) = p \cdot f(t; \mu_1; \sigma_1) + (1-p) \cdot f(t; \mu_2; \sigma_2) \quad (1)$$

where μ_1 and σ_1 are the mean and standard deviation of the first Gaussian density and μ_2 and σ_2 are the corresponding parameters for the second density. Such a model is often encountered when the noise originates from two separate sources.

We propose a two-stage processing approach to denoise signals embedded in non-Gaussian noise. In the first stage, we employ stack filters [16] which are statistically optimized for the given noise distribution. In the second stage, we apply multiresolution transform-based denoising, be it wavelet shrinkage or other similar techniques. We expect the stack filter to suppress outliers while retaining important detail information in the signal. Moreover, we expect the remaining noise at the output of the stack filter to be closer to a zero-mean Gaussian distribution than the noise at the input to the stack filter. Consequently, the purpose of multiresolution denoising applied after the first stage is to suppress the remaining noise.

In Section 2, we give a brief review of statistical stack filter optimization and discuss the details concerning the proposed method. Section 3 describes the second stage, which consists of transform based denoising. Finally, Section 4 contains numerical simulation results.

2. STACK FILTER OPTIMIZATION

Stack filters constitute an important class of nonlinear filters based on monotone Boolean functions [16]. Statistical properties of stack filters have been studied in terms of output distributions and moments for i.i.d. input signals [18],[1]. Consequently, it becomes possible to optimize stack filters in the mean square sense [2]. In other words, the knowledge of the distribution of the input, which is assumed to be i.i.d., allows us to find a stack filter which minimizes the output variance of the filter.

A window-width n stack filter is based on a monotone (positive) Boolean function $f : \{0,1\}^n \to \{0,1\}$. The well-known property of threshold decomposition [16] allows us to operate on the multilevel, rather than on the binary, domain. That is, since positive Boolean functions contain no negated literals in their minimal disjunctive normal forms, the operations of conjunction and disjunction, or equivalently, AND and OR, can be replaced by the MAX and MIN operations on the multi-level domain. For example, the Boolean function

$$f(x_1, x_2, x_3) = x_1 x_2 + x_2 x_3$$

(where $\cdot$ means conjunction and $+$ means disjunction) corresponds to

$$S_f(X_1, X_2, X_3) = \max\{\min\{X_1, X_2\}, \min\{X_2, X_3\}\}$$

where X_1, X_2, X_3 are real-valued variables. Suppose that the input variables of some stack filter $S_f(\cdot)$ are i.i.d. random variables with distribution

$$F(t) = \Pr\{X_i \le t\}$$

Then, it is well known [1] that the output

$$Y = S_f(X_1, \dots, X_n)$$

of the stack filter has output distribution

$$\Psi(t) = \sum_{i=0}^{n} A_i (1 - F(t))^i \cdot F(t)^{n-i} \qquad (2)$$

where

$$A_i = \left|\left\{x \in E^{n,i} : f(x) = 0\right\}\right|$$

The variance $\mu_2 = E\left\{(Y - E\{Y\})^2\right\}$ of the output Y of the stack filter can be written as [2]

$$\mu_2 = \sum_{i=0}^{n} A_i M(F, 2, n, i) - \left(\sum_{i=0}^{n} A_i M(F, 1, n, i)\right)^2 \qquad (3)$$

where

$$M(F, k, n, i) = \int_{-\infty}^{\infty} x^k \frac{d}{dx}\left((1 - F(t))^i \cdot F(t)^{n-i}\right)$$

Since we assume non-symmetric input noise distributions, our goal is to minimize μ_2 under the constraint of zero-mean output. This constraint can be expressed as

$$\sum_{i=0}^{n} A_i M(F, 1, n, i) = 0$$

and is easily incorporated into any optimization procedure. Moreover, it is important to have additional constraints on parameters A_i, since

$$0 \le A_i \le \binom{n}{i}, \; i = 1, \dots, n$$

So, as a result of the optimization, we obtain the parameter set $A = \{A_0, A_1, \dots, A_n\}$. It should be mentioned, however, that a set A of parameters does not uniquely define a stack filter. In other words, a number of stack filters can have the same parameters A, all having the same output variance and hence, all being statistically optimal. One approach to choosing one stack filter, which preserves details better than the rest of its statistically equivalent filters, is to use sample selection probabilities. This is discussed in [14].

3. TRANSFORM BASED DENOISING

Transforms have been used for a long time in noise-removal applications. Examples are linear and nonlinear filters based on FFT type structures, optimal Wiener and adaptive LMS type filtering in transform domain [3],[8]. Signal/image processing in transform domain rather than in spatial domain has certain advantages of incorporating a priori knowledge on signals into the design of processing algorithms and in terms of computational expenses. The transfer from spatial domain into the transform domain is especially useful if it is applied locally rather than globally. Local adaptive filters [17] work in the domain of an orthogonal transform in a moving window and nonlinearly modify the transform coefficients to obtain an estimate of the central pixel. Nonlinear filtering in wavelet transform domain was introduced in terms of wavelet de-noising by Donoho and Johnstone [6], and has been extended by several authors. In [4], translation invariant wavelet de-noising algorithms were introduced and tested on different signals. In [11], wavelet transform domain de-noising was combined with empirical Wiener filtering for better performance. In [9], local averaged transform domain de-noising was presented. The difference between this filter and the one in [17] is that nonlinear modifications of the transform coefficient within the window give us an estimate of the overall sub-image within the window and not only of the central pixel. Thus, it makes an overlap of estimates of pixels of neighboring windows; i.e. we obtain multiple estimates for the same pixel. It then averages all the above estimates to obtain the final estimate for this pixel.

Transform based denoising consists of the following three steps:

1. Computing spectral coefficients

$$\boldsymbol{\beta}(m) = \mathbf{H}Y(m)$$

   of the observed signal $\mathbf{Y}(m)$ (within the window in the case of local transform based denoising) over the chosen nonsingular transform $\mathbf{H}$.

2. Point-wise multiplication of the obtained transform coefficients with the filter coefficients $\{\eta_i(m)\}$: $\hat{\alpha}_i(m) = \eta_i(m)\beta_i(m)$ where $i = m, m+1, \ldots, m+M-1$.

3. Inverse transformation $\mathbf{H}^{-1}$ of the output signal spectral coefficients $\widehat{\boldsymbol{\alpha}}(m)$, $\hat{\mathbf{X}}(m) = \mathbf{H}^{-1}\widehat{\boldsymbol{\alpha}}(m)$.

In [8], an adaptive de-noising algorithm combining wavelet shrinkage and multi-base local averaged transform based filters has been developed. We apply this algorithm as our second stage of processing.

4. SIMULATION RESULTS

To test the effectiveness of our method, we selected a signal shown in Figure 1. As an additive noise source, we used a mixture of two Gaussian sources, as shown in (1). The parameters were chosen to be $p = 0.7$, $\mu_1 = 0$, $\sigma_1 = 0.3$, $\mu_2 = 1$ and $\sigma_2 = 0.1$. The contaminated signal is shown in Figure 2.

A stack filter was optimized using the method described in Section 2. Figures 3 and 4 show two histograms of the noise before and after application of the optimal stack filter. Three things can be seen from these figures: the variance of the noise is reduced, the outliers are essentially suppressed, and the resulting output distribution is closer to a Gaussian one. Thus, we expect that applying transform based denoising will result in significant reduction of noise level. Figure 5 shows the signal after applying a combined wavelet and local transform based denoising algorithm [8]. Finally, Figure 6 shows the histogram of the remaining noise after the entire filtering process.

5. CONCLUSION

We have considered the case of strongly non - Gaussian noise contamination. Additionally, we have allowed the probability density function of the input noise to be nonsymmetric. It is shown that the proposed procedure, which incorporates optimal stack filtering in the first stage and transform based denoising in the second stage, deals effectively with such noise models.

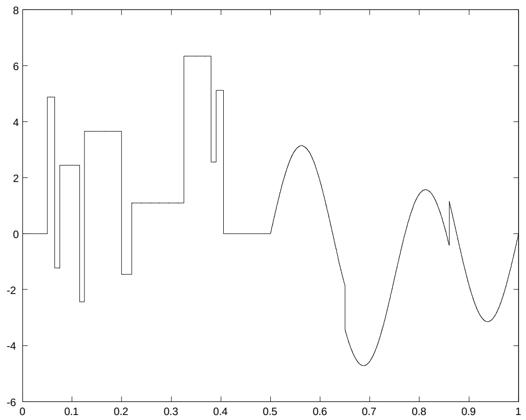

Figure 1: Test signal (concatenation of “Blocks” and “HeavySine”)

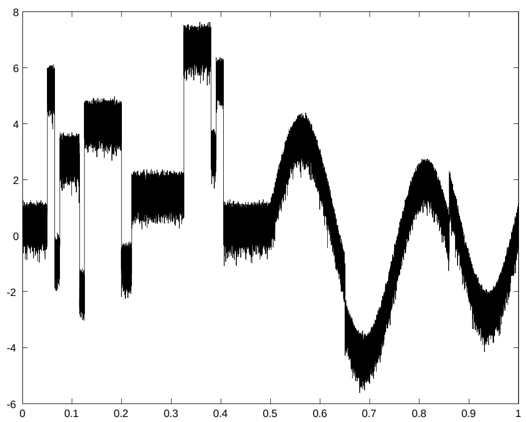

Figure 2: Noisy signal

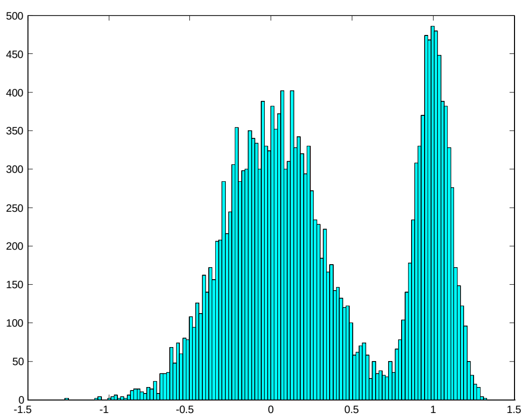

Figure 3: Histogram of input noise

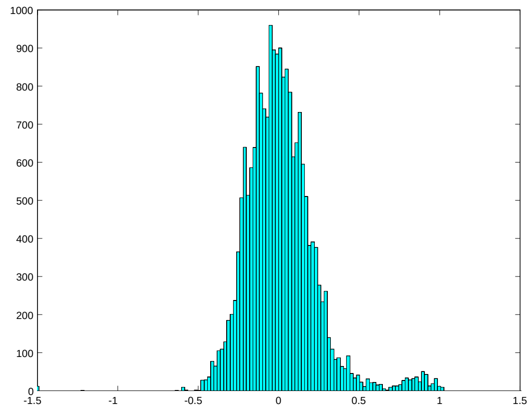

Figure 4: Histogram of the remaining noise at the output of the stack filter

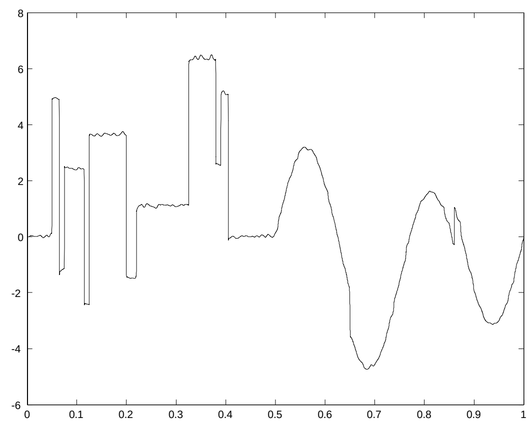

Figure 5: Resulting signal after applying stack filtering and transform based denoising

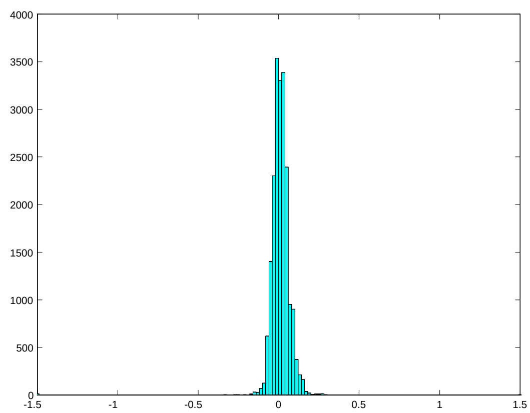

Figure 6: Histogram of the remaining noise after stack filtering and transform based denoising

6. REFERENCES

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