ADAPTIVE METHOD FOR 1-D SIGNAL PROCESSING BASED ON NONLINEAR FILTER BANK AND Z-PARAMETER

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ABSTRACT

An approach to synthesis of adaptive 1-D filters based on nonlinear filter bank and the use of Z-parameter is put forward. The nonlinearity of elementary filters ensures the predetermined robustness of adaptive procedure with respect to impulsive noise and outliers. In turn, a local adaptation principle enables to minimize the total output error being a sum of the residual fluctuation component and dynamic errors. The use of *Z*-parameter as a local activity indicator permits to “recognize ” the signal and noise properties for given fragment quite surely and, thus, to select a proper filter from the bank at disposal. Numerical simulation results confirming the efficiency of the proposed approach are presented.

1. INTRODUCTION

During recent years the filter banks attracted attention of many scientists and researchers. In fact, the filter banks formed themselves in a special area of knowledge and found various applications [1]. It is more easy to explain the essense of filter bank within linear approach [2] because of the well developed theory and the tools for linear filter design. However, for nonlinear filters a similar approach is also valid and useful [3].

A well known advantage of nonlinear filters, in particular, those ones based on order statistics is their ability to remove impulsive noise from data and, in general, to perform in appropriate manner in mixed noise environment [4]. Another excellent property of some nonlinear filters is that they are able to preserve discontinuities and abrupt changes of signals much better than their linear counterparts [4],[5]. In other words, the nonlinear filtering algorithms usually provide less dynamic errors in output signals in comparison to linear filters. One problem is that it is difficult to describe the dynamic properties of nonlinear filters analytically, especially taking into account that they depend upon noise characteristics [5].

Despite the aforementioned advantages of nonlinear filters no one of them is able to simultaneously satisfy a set of contradictory requirements: to provide the sufficient efficiency of fluctuative noise suppression, the reliable spike removal and minimal dynamic errors. Commonly, the higher the efficiency of noise suppression (provided by the scanning window size increasing), the worse dynamic propeties of the considered nonlinear filter and vise versa [6]. One way out is to apply an adaptive nonlinear smoother. A basic idea put behind the locally adaptive aproach is to process the noisy signal fragment by means of nonlinear filter suited in the best (optimal) manner for situation at hand. In other words, the local signal and noise properties and the priority of requirements of fragmentary data filtering should be taken into consideration. By minimizing the output local errors one obtains the minimization of the total error in MSE or MAE sense.

There already exist many locally-adaptive (data dependent) nonlinear 1-D filters. As examples, let us mention ones proposed by R. Bernstein [7], A. T. Fam et al [8], Ho Ming Lin [9], Runtao Ding and A. N. Venetsanopoulos [10], S. Siren, P. Kuosmanen and K. Egiazarian [11]. Some of them are heuristic ones [8],[9] and suited well for processing quite simple signals, the others [7],[10] require a priori knowledge of the noise component characteristics that is not always at disposal.

Our intention was to design an algorithm applicable in situations of a priori ambiguity of noise level and model. The only preliminary assumptions were the following:

1) the fluctuative (i. e., additive+multiplicative) noise has the zero mean and symmetrical probability density function (p.d.f.);
2) the noise samples are assumed to be i. i. d. random variables;
3) the impulsive noise is characterized by not very large probability of its occurrence.

The proposed locally-adaptive nonlinear filtering procedure is based on the use of a *Z*-parameter introduced in our papers [12],[13]. Besides, we give and apply quasioptimal rules for filter selection from the nonlinear filter bank. This bank is organized in such a manner that it contains the algorithms possessing the properties essentially different from each other. The operation of the proposed filters is explained and described below. Some numerical simulation results proving the efficiency of the considered approach are presented as well.

2. CONSIDERED SIGNAL/NOISE MODEL

Let us consider the following signal/noise model of the sampled data sequence $\{U(t_i)\}, i=1,\dots,I$

$$U(t_i)=\begin{cases}\mu(t_i)S(t_i)+n_a(t_i), \text{with probability } 1-P_{imp} \\ S(t_i)+n_{imp}(t_i), \quad \text{with probability } P_{imp},\end{cases} \qquad (1)$$

where $S(t_i)$ is the true signal value of the i-th sample; μ denotes the multiplicative noise having the mean equal to one and the variance σ_μ^2 dependent upon signal amplitude $|S(t_i)|$, e. g., $\sigma_\mu^2(t_i)=k_0 S^2(t_i)$; n_a is the zero mean additive noise with the variance σ_a^2; n_{imp} defines the amplitude of impulsive noise occurring with the probability P_{imp} $\left(|n_{imp}|>3\sqrt{\sigma_a^2+\sigma_\mu^2(t_i)}\right)$. It is supposed that both additive and multiplicative noise have p.d.f.s symmetrical with respect to their means, k_0 is assumed to be not larger than 0.1.

We chose the following rather simple model of the test signal

$$S(t_i)=\text{arctg}(\gamma\, t_i), \qquad (2)$$

where γ is the parameter; $t_i=(i-i_0)\Delta t$ denotes the time coordinate, Δt define the sampling rate, i_0 is the parameter ($i_0 \approx I/2$). The values of γ, Δt, i_0, and I are chosen in such a manner that $|\gamma\, t_1|=\gamma\, t_I \approx 2\ldots3$. Due to this the test signal contains three types of fragment:

1) almost constant signal with values $|S(t_i)|\approx 1$, this is observed at the beginning and at the end of the signal (2);
2) fragments with large absolute values of the second derivative of signal component;
3) fragments with almost linear behavior (rapidly increasing signal) observed when $|\gamma\, t_I|<0.5$.

The reasons for selection of such a test signal are the following. The signal (2) does not contain discontinuities and this is important for derivations done below. The analysis of different fragments of the signal permits to evaluate filter properties from different points of view:

- to estimate the noise suppression efficiency for almost constant signal fragments;
- to estimate the dynamic errors (bias of filter output) and their influence on local and integral characteristics of filter performance;
- to study the behavior of filter outputs for rapidly changing signals (in the middle part of the signal (2)).

Moreover, on one hand, for the large values $\gamma\,\Delta t$, the test signal can be considered as a ramp (or smoothed) edge, on the other hand, the test signal is a so-called trajectory curve, i.e., the sequence of “noisy” estimates of angle coordinates of a moving target.

3. NONLINEAR FILTER BANK FORMATION

While selecting the nonlinear filters of the bank we took into account several aspects. First, these filters should have rather different properties. In other words, among them there should be the detail preserving filters (DPFs), the efficient noise suppressing filters (NSFs) as well as some filters with "intermediate" characteristics (ICFs), i. e., the filters combining not bad edge/detail preservation with rather well noise suppression ability. The next requirement is to provide an appropriate robustness with respect to spikes. In this sense the properties of nonlinear filters depend upon the scanning window size, the filter type, some parameters (as for α-trimmed or center weighted median (CWM) filters [4],[10]), and even upon the characteristics of impulsive noise. Finally, it is desirable to ensure rather high computational efficiency of the filters as well as of the adaptive procedure.

Taking the aformentioned requirements into consideration the following recommendations concerning selection of the filters of the bank seem to be reasonable. An appropriate DPF is a standard median filter with scanning window size $N=5$. The CWM and the FIR-hybriad median filters [4] are also quite good choice but they are less robust with respect to spikes. The other nonlinear filters with small scanning window sizes can be also used as the DPFs.

The acceptable NSFs are the Wilcoxon and Hodges-Lehman filters with large N [10] combining high efficiency of fluctuative noise reduction with perfect robustness with respect to spikes. However, their computation efficiency is not appropriate in many practical situations. That is why, instead of them as the NSFs it is possible to use the α-trimmed filters having $\alpha \approx 0.2\ldots0.25$ and $N=9\ldots13$.

So, as appropriate ICFs one could use the α-trimmed, Wilcoxon and Hodges-Lehman filters with $N=7$ or 9. According to their properties they are in between the considered DPFs and NSFs. Certainly, someone could propose the other filters to be added to the filter bank according to his opinion or preferrances. But as it will be seen from further analysis the desirable effect of adaptive filtering is reached even with the use of such, rather small, filter bank.

4. PROPOSED ADAPTIVE PROCEDURE AND Z-PARAMETER PROPERTIES

Obviously, the dynamic error (the bias) Δ_{Di} of the i-th sample of the output signal and the variance of residual fluctuations σ_r^2 can be easily derived for the standard mean filter with scanning window size N used for processing the signal corrupted by zero mean fluctuative noise with locally constant variance σ_f^2. For example, one can derive the variance σ_r^2 of mean filter output for i.i.d. fluctuative noise as $\sigma_{mean}^2 \approx \sigma_f^2/N$.

In [14], it is shown that the variance of residual fluctuations of nonlinear filter outputs depends upon many factors. However, the following approximations of σ_r^2 can be used

$$\sigma_{wil}^2 \approx \sigma_{h-l}^2 \approx 1.2\sigma_f^2/N = 1.2\sigma_{mean}^2, \qquad (3)$$

$$\sigma_{\alpha-tr}^2 \approx 1.4\sigma_f^2/N = 1.4\sigma_{mean}^2, \qquad (4)$$

where σ_{wil}^2, σ_{h-l}^2, $\sigma_{\alpha-tr}^2$ are the variances of residual fluctuations for the Wilcoxon, Hodges-Lehman and α-trimmed (α=0.2) filters, respectively. The experiments have shown that these expressions were approximately valid not only for signal fragments with linear behavior but for other kinds of fragments as well. It is worth noting here that the standard median filter and some other DPFs lose their efficiency of noise supression for rapidly increasing/decreasing signals [14].

Now let us suppose that the signal component can be well approximated by the following expression

$$S_{i+k} = S(t_{i+k}) = S_i + S_i' k\,\Delta t + S_i'' k^2 \Delta t^2 , \qquad (5)$$

where $S_i = S(t_i)$; $S_i' = S'(t_i)$ and $S_i'' = S''(t_i)$ are the first and second derivatives of the signal component in the *i*-th sample, respectively. Then introducing notations $\Delta_{Si} = S_i' \Delta t$, $\Delta_{Si}^2 = S_i'' \Delta t^2$ one gets $S_{i+k} \approx S_i + \Delta_{Si}\, k + \Delta_{Si}^2\, k^2$.

Thus, the dynamic error (bias) Δ_{Di} of the mean filter can be derived as

$$\Delta_{Di}^{mean} = \left[\sum_{k=-\frac{N-1}{2}}^{\frac{N-1}{2}} \frac{S_i + \Delta_{Si}\, k + \Delta_{Si}^2\, k^2}{2k+1} \right] - S_i \approx 0.08 \Delta_{Si}^2\, N^2 , \qquad (6)$$

It can be shown that the term proportial to Δt^3 does not influence the dynamic error value Δ_{Di}^{mean}.

The investigation of dynamic errors of the considered nonlinear NSFs has resulted in the following approximations [15]

$$\Delta_D^{wil} \approx 0.9 \Delta_D^{mean}, \qquad (7a)$$

$$\Delta_D^{h-l} \approx 0.85 \Delta_D^{mean}, \qquad (7b)$$

$$\Delta_D^{\alpha-tr} \approx 0.6 \Delta_D^{mean}, \qquad (7c)$$

where Δ_D^{wil}, Δ_D^{h-l}, and $\Delta_D^{\alpha-tr}$ are the dynamic errors of the outputs of Wilcoxon, Hodges-Lehman, α-trimmed ($\alpha = 0.2$) filters with the same *N*, respectively.

The total error δ_i of the *i*-th sample can be determined as

$$\delta_i = \sigma_{ri}^2 + \left(\Delta_{Di}\right)^2 . \qquad (8)$$

Thus, taking expressions (3), (4), (6), (7) into account and assuming $\partial\delta / \partial N = 0$ one obtains the optimal window size $N_{\text{opt}\,i}$ of the filters for each sample under the criterion of minimal local total error δ_i. For the Wilcoxon, Hodges-Lehman and α-trimmed filters ($\alpha = 0.2$) the optimal window sizes are, respectively, derived as

$$N_{\text{opt}\,i}^{wil} \approx N_{\text{opt}\,i}^{h-l} = 2.4 \left(\sigma_{fi}^2 \Big/ \left(\Delta_{Si}^2\right)^2 \right)^{0.2} , \qquad (9)$$

$$N_{\text{opt}\,i}^{\alpha-tr} \approx 2.9 \left(\sigma_{fi}^2 \Big/ \left(\Delta_{Si}^2\right)^2 \right)^{0.2} . \qquad (10)$$

Therefore, in order to select optimal $N_{\text{opt}\,i}$ for processing the data one has to somehow estimate the local ratio $\sigma_{fi} / \Delta_{Si}^2$. For this purpose the Z-parameter can be used. It is defined as

$$Z_i = Z(t_i) = \frac{\sum_{m=-\frac{N_{p-1}}{2}}^{\frac{N_{p-1}}{2}} \left(U^{fp}(t_{i+m}) - U(t_{i+m}) \right)}{\sum_{m=-\frac{N_{p-1}}{2}}^{\frac{N_{p-1}}{2}} \left| U^{fp}(t_{i+m}) - U(t_{i+m}) \right|} , \qquad (11)$$

where $U^{fp}(t_i)$ is the output of preliminary filter having scanning window N_p and some “middle” properties. For this goal, the ICF from the nonlinear filter bank can be applied.

Both analytical and numerical analyses of the *Z*-parameter statistical properties resulted in the following approximation for the mean values

$$E\left[|Z_i|\right] \approx \beta_i \Big/ \left(0.8 + 0.1\beta_i + 0.3\beta_i^2\right), \qquad (12)$$

where $\beta_i = \left| \Delta_{Di} / \sigma_{fi} \right|$; the sign of Z_i coincides with the sign of Δ_{Di}. In fact, the *Z*-parameter values are within the limits $[-1;1]$.

Then if the value Z_i does not differ essentially from its mean value it becomes possible to derive β_i from Z_i. In order to estimate the ratio $\sigma_{fi}^2 \big/ \left(\Delta_{Si}^2\right)^2$ for the Wilcoxon and Hodges-Lehman filters one can use the expression $\sigma_{fi}^2 \big/ \left(\Delta_{Si}^2\right)^2 = 0.005 N_p^4 \big/ \beta_i^2$. For the α-trimmed filters, the formula $\sigma_{fi}^2 \big/ \left(\Delta_{Si}^2\right)^2 = 0.003 N_p^4 \big/ \beta_i^2$ is valid. Then, the optimal scanning window sizes of the considered nonlinear filters can be derived as

$$N_{\text{opt}\,i}^{wil} \approx N_{\text{opt}\,i}^{h-l} \approx 2.4 \left(0.005 N_p^4 \big/ \beta_i^2 \right)^{0.2} , \qquad (14)$$

$$N_{\text{opt}\,i}^{\alpha-tr} \approx 2.9 \left(0.003 N_p^4 \big/ \beta_i^2 \right)^{0.2} . \qquad (15)$$

This procedure may seem too complicated. However, the bank contains a limited number of filters. So, for N_p selected beforehand it is possible to calculate the mean values $E\left[|Z_i|\right]$ corresponding to each filter.

Then the adaptive procedure becomes rather simple and it consists in the following

$$U^{fa}(t_i) = \begin{cases} U_1(t_i), & if\ |Z_i| > Z_Q^{th} \\ U_2(t_i), & if\ |Z_i| \in \left]Z_{Q-1}^{th}; Z_Q^{th}\right] \\ \vdots & \vdots \\ U_{Q+1}(t_i), & if\ |Z_i| \le Z_1^{th}, \end{cases} \qquad (15)$$

where $Z_1^{th}, \ldots, Z_Q^{th}$ are the thresholds, Q is the number of thresholds; $U_1(t_i), \ldots, U_{Q+1}(t_i)$ are the outputs of the nonlinear filters sorted in order of improved noise supression efficiency and making worse their dynamic properties. In other words, the $U_1(t_i)$ corresponds to the best DPF and $U_{Q+1}(t_i)$ - to the best NSF. In many practical situations it is sufficient to use only three filters ($Q = 2$) - one serving as the DPF (for $|Z_i| > Z_2^{th}$), the other one performing preliminary processing and it coincides with the ICF (its output is assigned to adaptive filter output if $|Z_i| \in \left]Z_1^{th}; Z_2^{th}\right]$, and the latter acts as the NSF (for $|Z_i| \le Z_1^{th}$)

All the derivations done above are valid for the case of spike absense in the scanning window. If the *j*-th sample is corrupted by spike then the corresponding difference $\Delta U_j = \left| U^{fp}(t_j) - U(t_j) \right|$ is essentially larger than the other differences in (11) and, thus, the *Z*-parameter absolute values in the neighborhood of the *j*-the sample tend to one. According to (15) in this case the output of DPF is assigned to $U^{fa}(t_i)$. If the standard median filter is applied as the DPF, then the spike is removed properly.

5. NUMERICAL SIMULATION RESULTS

The numerical simulations were performed for different variances of additive noise and probabilities of spikes. Tables 1 and 2 present the MSE and MAE values for the cases of non-adaptive nonliner filter application and adaptive procedures based on (15). The MSE values are denoted as χ_5, χ_9, and χ_{13} for the DPF, preliminary filter (ICF) and NSF from the bank. The corresponding MAE values are denoted as κ_5, κ_9, and κ_{13}. The DPF, ICF and NSF have the scanning window sizes $N=5$, 9, and 13, respectively. The values χ_{ad} and κ_{ad} show the MSE and MAE for the proposed adaptive filters based on *Z*-parameter.

Table 1: MSE values

Filter bank	σ_a^2	P_{imp}	χ_5 $\times 10^{-3}$	χ_9 $\times 10^{-3}$	χ_{13} $\times 10^{-3}$	χ_{ad} $\times 10^{-3}$
Wil	0.001	0	0.26	0.37	1.04	0.22
Wil	0.003	0	0.70	0.62	1.23	0.51
Wil+M	0.001	0.01	0.55	0.49	1.19	0.53
Wil+M	0.003	0.01	1.30	0.80	1.40	0.97
H-L	0.001	0.	0.26	0.36	0.90	0.22
H-L	0.003	0	0.70	0.64	1.21	0.51
HL+M	0.001	0.01	0.57	0.50	1.04	0.49
HL+M	0.003	0.01	1.32	0.82	1.37	0.97
α*-tr*	0.001	0	0.26	0.20	0.27	0.19
α*-tr*	0.003	0	0.76	0.50	0.50	0.49
α*-tr+M*	0.001	0.01	0.54	0.32	0.47	0.40
α*-tr+M*	0.003	0.01	1.30	0.63	0.65	0.72

Table 2: MAE values

Filter bank	σ_a^2	P_{imp}	κ_5	κ_9	κ_{13}	κ_{ad}
Wil	0.001	0	0.043	0.057	0.092	0.044
Wil	0.003	0	0.070	0.067	0.101	0.065
Wil+M	0.001	0.01	0.080	0.069	0.103	0.079
Wil+M	0.003	0.01	0.117	0.082	0.111	0.106
H-L	0.001	0	0.043	0.058	0.093	0.045
H-L	0.003	0	0.071	0.072	0.109	0.066
HL+M	0.001	0.01	0.082	0.073	0.103	0.077
HL+M	0.003	0.01	0.117	0.085	0.115	0.107
α*-tr*	0.001	0	0.044	0.036	0.042	0.038
α*-tr*	0.003	0	0.072	0.057	0.054	0.057
α*-tr+M*	0.001	0.01	0.079	0.053	0.060	0.059
α*-tr+M*	0.003	0.01	0.116	0.070	0.070	0.081

We consider several variants of nonlinear filter bank. The first one contains only the Wilcoxon filters with $N=5$, 9, and 13 used as the DPF, ICF, and NSF, respectively. It is denoted as *Wil* in Tables 1 and 2. Similarly, the second variant contains Hodges-Lehman filters with the same *N* (*H-L*). The threshold values are $Z_1^{th}=0.15$ and $Z_2^{th}=0.35$ for the adaptive procedures based on the use of the Wilcoxon and Hodges-Lehman filters.

The third variant *(*α*-tr*) of filter bank includes three α-trimmed filters with $N=5$, 9, and 13. For the α-trimmed filter bank the thresholds are 0.2 and 0.4.

In case of spike absense the α-trimmed, Wilcoxon and Hodges-Lehman with $N=5$ are used as the DPFs. When the spikes are present the standard median filter ($N=5$) is used as DPF. This filter is added to aforementioned filter banks to provide better robustness of adaptive procedure because the other DPFs with $N=5$ have more poor robust properties than the standard median filter. The notations *Wil+M*, *H-L+M*, α*-tr+M* are used for the corresponding adaptive procedures.

As it can be seen from Table 1 for relatively small values of σ_a^2 ($\sigma_a^2=0.001$) and $P_{imp}=0$ the best results (minimal χ) among non-adaptive filters are provided by the DPFs with $N=5$. This happens because the influence of dynamic errors for small σ_a^2 is more essential than the influence of residual fluctuative noise. However, the adaptive procedure guarantees even smaller χ_{ad} value than the MSE value for any non-adaptive nonlinear filter of the filter bank. For larger σ_a^2 ($\sigma_a^2=0.003$) and $P_{imp}=0$ the best among the non-adaptive filters are those ones with $N=9$ (α-trimmed is the best one). In this case the adaptive filters also produce better results than the best among the considered non-adaptive filters.

When the spikes are present the adaptive procedures do not ensure the χ_{ad} less than the best non-adaptive filter. However, usually the adaptive procedures are superior in comparison with the two worst non-adaptive algorithms. The difference between χ_{ad} and $\min(\chi_5,\chi_9,\chi_{13})$ is not too large. That is why, in general, the adaptive filters provide the best results or they ensure the MSE values quite close to the best for rather wide variety of additive and impulsive noise characteristics.

The MAE criterion is often used in practice, especially for characterizing the filter properties in non-gaussian noise environments. The results of MAE value analysis for different non-adaptive filters and various adaptive procedures are in good agreement with the results obtained for MSE criterion. So, the proposed adaptive procedures are rather effective from different points of view.

One more conclusion is that the χ_{ad} and κ_{ad} values for the adaptive procedure based on α-trimmed filters are usually better than for the other ones. So taking this into account and comparing the computational efficiency of the component filters the last variant of adaptive procedure in the Tables seems to be preferrable for practical implementation.

6. CONCLUSIONS

It is shown that the adaptive procedure based on selection of nonlinear filter from the bank depending upon Z-parameter value has a theoretical background and can be optimized. The proposed adaptive procedure produces good quality of data processing in case if a priori unknown characteristics of mixed noise varying in rather wide limits.

7. REFERENCES

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