ADAPTIVELY VARYING TRANSFORM SIZE SELECTION BY ICI RULE FOR TRANSFORM DOMAIN IMAGE DE-NOISING

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ABSTRACT

The local adaptive processing of signals and images in a transform domain within a sliding window suggests certain advantages in some signal and image de-noising applications due to incorporating an available a priori information about the signals and noises. However, an optimum transform size is also data dependent and generally is not known in advance. Performing the de-noising with the varying transform size suggests further improvements. The approach based on the intersection of confidence intervals (ICI) rule for a selection of the varying transform size is introduced.

1. INTRODUCTION

The discrete orthogonal transforms are widely used in the adaptive signal and image de-noising. In particular, it is known [11] that the discrete cosine transform ($\mathbf{DCT}$) is a good approximation of the optimal Karhunen-Loeve transform for highly correlated data. A transform size is an important parameter for a transform based adaptive filtering. Relatively homogeneous portions of an image have higher de-noising performance with increasing transform size, while nonhomogeneous transition parts need a smaller transform size for the better detail preservation. The transform with a spatially varying transform size is able to improve noise attenuation and the detail preservation and provide a better performance than it can be achieved by this transform with a fixed transform size.

The methods considered in this study are based on the following ideas. First, an image can be treated as consisting from more less homogeneous parts. Second, the location and size of these parts are not known in advance.

We use the sliding orthogonal transform with the spatially varying size. The ICI rule is proposed for the adaptive transform size selection. In principle, the ICI enables filters to become spatially adaptive over a wide range of the classes of signals in the sense that its quality is close to that which one could achieve if smoothness of the signals was known in advance [3],[6],[7].

2. LOCAL ADAPTIVE PROCESSING IN TRANSFORM DOMAIN

When spectra of a signal and a noise are separable in an invertible transform domain, a de-noising in the transform domain and performing the inverse transform are efficient methods of removing the noise [1], [2],[12].

General filtering algorithm consists of the following three steps:

1. Computing spectral coefficients
$$\mathbf{a} = \mathbf{T}\mathbf{b}$$
of the observed signal $\mathbf{b}$ within the window over the chosen orthogonal transform $\mathbf{T}$.

2. Modifying the spectral coefficients
$$a'_r = f(a_r, r)$$

3. Performing the inverse transform
$$\mathbf{b}' = \mathbf{T}^{-1}\mathbf{a}'$$

The signal and noise spectra are overlapping in the most of applications. Therefore, the higher levels of the noise attenuation is possible as a rule only at the cost of the lower level detail preservation [8],[9],[10],[12].

Generally, the hard or soft thresholding are conventional methods of the spectra modification [1],[2].

Performing the de-noising in a running window and combining (generally averaging) the results returned for the same sample improves the filter performance due to the improved detail preservation [9],[12]. We consider the values of the threshold in the spectrum modification and the window size as main adjustable parameters of our smoothing procedure and we select these parameters independently for every pixel of the image. In another words, we try to fit the best denoising procedure for the every pixel of the image.

While we assume that the used orthogonal transform as well as the spectrum thresholding are quite conventional the main efforts in this paper are produced in the direction of the threshold value and window size selection.

In our study we assume the hard thresholding in the conventional form

$$\begin{aligned} a_r' &= a_r \cdot 1(|a_r| \geq t), \\ 1(|x| &\geq t) = \begin{cases} 1, & |x| \geq t \\ 0, & |x| < t \end{cases}, \end{aligned}$$

where t is a value of the threshold and $1(|x| \geq t)$ is the indicator-function.

Thus, the main problem concerns a selection of the optimal varying locally adaptive thresholds t which are assumed to be different for the every pixel and window sizes also spatially adaptive.

3. THE ICI RULE

Let the image intensity model for the pixel (i,j) be of the form

$$x(i,j) = s(i,j) + n(i,j),$$

where $s(i,j)$ and $n(i,j)$ are a signal and noise respectively..

Let $\hat{s}(i,j,h)$ be the estimate of $s(i,j)$ obtained by the transform with the window size h

Let us introduce a finite set of window sizes: $H = \{h_0 < h_2 < .... < h_M\}$, starting with a small h_0 and determine a sequence of the confidence intervals $D(h_k)$ of the biased estimates corresponding to the window size h_k

$$\begin{aligned} D(h_k) &= [U_k, L_k], \\ U_k &= \hat{s}(i,j,h_k) + \Gamma \cdot \sigma(i,j,h_k), \\ L_i &= \hat{s}(i,j,h_k) - \Gamma \cdot \sigma(i,j,h_k), \end{aligned} \tag{1}$$

where Γ is a threshold of the confidence interval, $\hat{s}(i,j,h_k)$ is the estimate of $s(i,j)$ using the window h_k and $\sigma(i,j,h_k)$ is the standard deviation of this estimate.

The ICI rule gives the adaptive window size as the following procedure:

Consider an intersection of the intervals $D(h_k)$, $1 \leq k \leq M$, with increasing h_k, and let m be the largest of those k for which the intervals $D(h_k)$, $1 \leq k \leq m$, have a point in common. This m defines the adaptive window size.

Thus, the adaptive window size is defined as the largest window size whose confidence interval of the corresponding estimate intersects with the confidence intervals of all smaller window sizes [4],[5].

4. ALGORITHM

Algorithm is composed of the following steps for zero mean white Gaussian noise $\mathbf{N}$:

- Estimate the standard deviation of the noise from the noisy observation $\mathbf{X} = \mathbf{S} + \mathbf{N}$:

  $$\begin{aligned} &\overline{\sigma}_n = \sqrt{2} median(abs(\mathbf{TX})) \\ &abs(\mathbf{a}) = \{|a_0|, |a_1|, ..., |a_f|\} \end{aligned}$$

  where $\mathbf{T}$ corresponds to the global transform having good signal compaction property. A local noise variance estimation may also be employed for locally varying noise [8].

- For each pixel location (i,j) in the observation

  1. Start with minimum window size h_0
  2. Form the local observation $\mathbf{o}_{i,j,k}$ by taking the elements inside the window having a size of h_k and located at (i,j).
  3. Obtain the $\mathbf{DCT}$ spectral coefficients of $\mathbf{o}_{i,j,k}$

     $$\mathbf{y}(i,j,h_k) = \mathbf{DCT}(\mathbf{o}_{i,j,k})$$

  4. Set the threshold t_k as a function of noise standard deviation and the window size. A simple linear model is used:

     $$t_k = (\gamma_c + \gamma_l h_k)\overline{\sigma}_n$$

     where γ_c, γ_l are empirically found constants. A size dependent threshold is preferred, because the more homogeneous the local portion the better performances are obtained both by larger window size and by higher threshold.
  5. Perform hard thresholding :

     $$\mathbf{y}'(i,j,h_k) = \mathbf{y}(i,j,h_k) \cdot 1(|\mathbf{y}(i,j,h_k)| > t_k)$$

     The spectral coefficient corresponding to the mean value of $\mathbf{o}_{i,j,k}$ is preserved.
  6. Perform the inverse transform of the thresholded spectral coefficients to obtain an estimate of the signal in the corresponding window

     $$\hat{\mathbf{s}}(i,j,h_k) = \mathbf{DCT}^{-1}(\mathbf{y}'(i,j,h_k))$$

  7. Set the confidence interval $D_{\hat{\mathbf{s}}(i,j,h_k)}$ for the pixel intensity estimate $\hat{s}(i,j,h_k)$ at the location (i,j) computed by using window size h_k

     $$\begin{aligned} D_{\hat{s}(i,j,h_k)} &= [U_{\hat{\mathbf{s}}(i,j,h_k)}, L_{\hat{\mathbf{s}}(i,j,h_k)}] \\ U_{\hat{s}(i,j,h_k)} &= \hat{s}(i,j,h_k) + \Gamma \overline{\sigma}_{e_{i,j,k}} \\ L_{\hat{s}(i,j,h_k)} &= \hat{s}(i,j,h_k) - \Gamma \overline{\sigma}_{e_{i,j,k}} \\ \overline{\sigma}_{e_{i,j,k}} &= \overline{\sigma}_n \frac{\#(\mathbf{y}'_{i,j,k})}{\#(\mathbf{y}_{i,j,k})}, \end{aligned}$$

where #(x) is a number of nonzero elements of the vector x, and the variance of the estimate $\overline{\sigma}_{e_{i,j,k}}$ is considered as the unattenuated noise components (considering the noise to be uniformly distributed in the spectral coefficients).

8. Compute the intersection of the confidence intervals

$$\begin{aligned} I_{\hat{s}(i,j,h_0)} &= D_{\hat{s}(i,j,h_0)}, \\ I_{\hat{s}(i,j,h_k)} &= D_{\hat{s}(i,j,h_k)} \cap I_{\hat{s}(i,j,h_{k-1})}, \\ k &= 1, 2, \ldots. \end{aligned}$$

9. Increase window size $h_k > h_{k-1}$ and repeat the and repeat from step 2 to 8.
10. Stop when the intersection $I_{\hat{s}(i,j,h_{m+1})}$ of the confidence intervals is empty. Then h_m is the selected window size of estimation for the pixel (i, j). The estimate is given by $\hat{s}(i, j, h_m)$.

- Pass to the next pixel.

As a result of the above calculations we obtain the estimate for the every pixel (i, j) and neighboring points in the adaptive window. As the windows are overlapping we have a varying number of estimates for the every pixel obtained in the different windows. The final estimate for each pixel is obtained by combining the multiple estimates available for the same pixel.

5. COMPARATIVE RESULTS

The algorithm was implemented in MATLAB and tested with the 'montage' image. The image having intensity values between 0 and 1 was corrupted by white Gaussian noise with standard deviation $\sigma = 0.1$. The proposed algorithm attenuated the root mean square error (RMSE) to 0.0287, while local adaptive de-noising with fixed size $\mathbf{DCT}$ attenuated RMSE to 0.0324 and Wiener filter of optimal size (5x5) attenuated to 0.0408. The maximum absolute error after suggested filtering was 0.265 and the mean absolute error (MAE) was 0.0193.

The comparative figures are given in Table 1. The original, corrupted and filtered by the proposed method images are plotted in Figures 1,2,3. The varying window sizes are plotted in Figure 4.

6. REFERENCES

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Figure 1: Original Image

Figure 2: Corrupted image

Table1: Comparative results.

Used Filter	RMSE	MAE
LAD with adaptive transform size	0.0287	0.0193
LAD with fixed transform size	0.0314	0.0204
Wiener Filter (5x5)	0.0408	0.0274
Wavelet Package Haar	0.0585	0.0444
Wavelet Package Sym 8	0.0464	0.0310
Wavelet PO Haar (4 levels)	0.0567	0.0411
Wavelet PO Sym 8 (4 levels)	0.0693	0.0485
Wavelet TI Haar (5 levels)	0.0317	0.0205
Wavelet TI Sym 8 (5 levels)	0.0365	0.0261

Figure 3: De-noised image

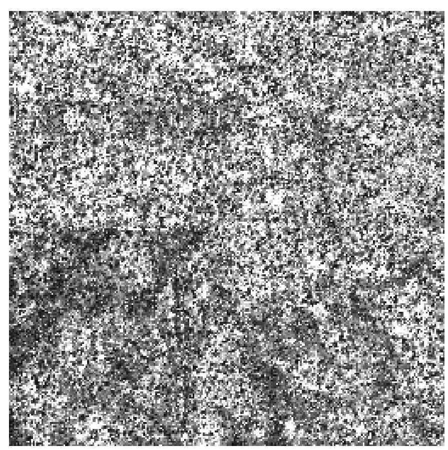

Figure 4: Distribution of window sizes: from windows containing a single pixel (darkest points) to the windows containing 64 and more pixels (brightest points)