ITERATIVE PROCEDURES FOR NONLINEAR FILTERING OF IMAGES

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ABSTRACT

The drawbacks and advantages of the well known local statistic Lee and Frost filter are discussed. To avoid the disadvantages of these filters an iterative procedure is proposed. The outputs of the local statistic Lee or Frost filters are processed by FIR median hybrid (FMH) filter as a second iteration. This provides better noise suppression and robustness with respect to spikes. Some quantitative analysis of the iterative processing algorithms is performed. Numerical simulation and real data processing results are presented.

1. INTRODUCTION

A typical situation for many imaging systems is that the obtained images are noisy ones and the noise is not always Gaussian and additive. Rather often the multiplicative noise is a dominant factor degrading image quality [1],[2]. Furthermore, its probability density function (p.d.f.) can be non-Gaussian. This happens for coherent imaging systems like synthetic aperture radar (SAR) or ultrasound medical imaging systems. If they use only one look for forming the image of the sensed object (terrain) then the p.d.f. of multiplicative noise is the Rayleigh or the negative exponential depending upon the fact whether the amplitude or intensity of spatial distribution is estimated [3]. Moreover, spikes can be present in real scene images and this is one more obstacle restricting the application of linear filters for image enhancement.

The requirements to the filtering algorithms to be applied in the considered situation are rather contradictory. It is desirable simultaneously to retain the mean level of homogeneous regions of the image, to get high efficiency in multiplicative noise suppression and in spike removal and to preserve edges and fine details well. Many known filters such as standard median, sigma, K nearest neighbor, center weighted median [4], etc. fail to perform well because of different reasons. In the case of spike absence an appropriate trade-off of properties is often provided by the local statistic Lee and Frost filters [5],[6],[7]. However, both of them are sensitive to impulsive noise. In our papers [8],[9] the iterative procedures based on application of the FMH filter to the outputs of local statistic Lee or Frost filters are proposed and analyzed. The application of the FMH filter [10] is intended on providing the robustness in respect to spikes. Due to this the iterative procedure provides an excellent combination of properties of the filters used.

The goal of this paper is to obtain quantitative estimations for the efficiency of the proposed filtering algorithms. Another purpose is to compare the performance of the considered iterative procedures depending upon the model of multiplicative noise. Two sizes of the scanning window and three types of multiplicative noise distributions are considered. The proposed procedure is tested for both simulated artificial and real scene images.

2. LOCAL STATISTIC LEE AND FROST FILTERS

Here we consider a generalized image/noise model typical for coherent one-look imaging systems and other images with prevailing influence of multiplicative noise. For this model

$$g(i,j) = \begin{cases} \mu(i,j)f(i,j), & \text{with probability } 1-P_a \\ A(i,j), & \text{with probability } P_a, \end{cases} \tag{1}$$

where $g(i,j)$ is the noisy image; $\mu(i,j)$ denotes the multiplicative noise (speckle) characterized by p.d.f. $\rho(\mu)$ with the mean $E\{\mu(i,j)\} = 1$ and the variance $Var\{\mu(i,j)\} = \sigma_\mu^2$; $f(i,j)$ defines the true image; P_a is the probability of impulsive noise; $A(i,j)$ denotes the amplitude of a spike for the ij-th pixel. The influence of additive noise is usually neglected because its variance is essentially less than the values $\sigma_\mu^2 f^2(i,j)$ for majority of image pixels. Depending upon the procedure of image formation the p.d.f. of multiplicative noise can be Rayleigh ($\sigma_\mu^2 = 0.273$) or one-side exponential ($\sigma_\mu^2 = 1$) for one-look imaging and rather close to Gaussian for many others [3], [7]. The distribution of the spike amplitude $A(i,j)$ can be different depending upon the reasons the spikes are originated [4].

The local statistic Lee and Frost filters are known to be among the best edge/detail preserving algorithms but they do not satisfy all the requirements mentioned above. In fact, both of them are locally adaptive soft-switching algorithms based on the use of local variance as the adaptation parameter. The output of the local statistic Lee filter [5] is expressed as

$$y(i,j) = \overline{g_{ij}} + \frac{\sigma_{ij}^2 - \overline{g_{ij}}^2 \sigma_\mu^2}{\left(\sigma_\mu^2 + 1\right) \sigma_{ij}^2} \left(g(i,j) - \overline{g_{ij}}\right), \qquad (2)$$

where $\overline{g_{ij}}$, and σ_{ij}^2 are the local mean and the local variance of the corrupted image around the ij-th pixel. For the Frost filter [6] the output image is obtained as

$$y(i,j) = \sum_{p,q} \theta a s_{ij}^2 \exp\left[-a s_{ij}^2 \left(|p-i| + |q-j|\right)\right] g(p,q), \qquad (3)$$

where $p = i - \frac{L-1}{2}, \ldots, i + \frac{L-1}{2}$; $q = j - \frac{M-1}{2}, \ldots, j + \frac{M-1}{2}$; $N = (2L+1) \times (2M+1)$ is the scanning window size; $a = \frac{4}{\sigma_\mu^2 \sqrt{N}}$; $s_{ij}^2 = \frac{\sigma_{ij}^2}{\overline{g_{ij}}^2}$; θ is a normalizing factor. The local statistic Lee filter can be implemented rather easily due to the application of recursive procedures of local mean and local variance calculation. The standard Frost filter is much more time consuming. However, in our paper [9] a hard-switching modification is proposed and it is shown that the properties of this modification are quite close to the standard Frost filter.

As it is seen from (2) and (3) for the application of both filters the preliminary knowledge of noise variance σ_μ^2 is required. But this parameter can be estimated, for instance, for some selected homogeneous region of the image. Sometimes it is a priori known because it depends upon the operation principles of the imaging system. That is why the considered filters can be applied to processing of images with different p.d.f.s of multiplicative noise. This is an important advantage of these filters.

Another useful property of both the local statistic Lee and Frost filters is that for homogeneous regions of the output image the p.d.f. of the residual fluctuation becomes close to Gaussian irrespective to the p.d.f. of speckle. This statement is valid if the spikes are absent in the scanning window. The phenomenon of the p.d.f transformation can be treated by the central limit theorem because both local statistic Lee and Frost filters perform similarly as the mean filter in homogeneous regions of image but with less efficiency of noise suppression. For the same reason the mean level is retained well enough. Thus, it is desirable to improve the noise suppression efficiency of both filters. However, spikes remain because both filters "consider" them like edges or fine details. This is the main drawback of the use of the local statistic Lee and Frost filters.

3. AN ITERATIVE PROCEDURE FOR IMAGE PROCESSING

Taking into account the aforementioned drawbacks of the local statistic Lee and Frost filters we have proposed [8],[9] to apply the FMH filter to their outputs. The corresponding iterative procedures are denoted as Lee+FMH and Frost+FMH, respectively. The most appropriate version of FIR median hybrid filter is 3LH+ (5x5 window size) [10] because it is able to preserve edges and fine details of almost arbitrary orientation. Besides, it possesses robust properties appropriate for many practical situations.

For homogeneous regions of the obtained preliminary output image the application of FMH filter (performing well for Gaussian and mixed Gaussian+ impulsive noise) results in spike elimination and additional noise reduction. As it is known, the degree of noise suppression for almost any filter with given scanning window size depends upon the correlation properties of noise. So we exploited the spatial correlation functions for the homogeneous regions of output images after the application of the local statistic Lee and Frost filters as well as of the mean filter (for comparison purposes). We assume here that for the initial test images the multiplicative noise was supposed to be spatially independent. The scanning window size for all filters was 7x7 pixels. The cross-sections of 2-D autocorrelation function are presented in Figure 1 for (a) Gaussian, (b) Rayleigh, and (c) negative exponential p.d.f., respectively. The vertical and horizontal cross-sections practically coincide.

It is clearly seen from Figure 1 that the width of the main lobe of the cross-correlation function is minimal if the local statistic Lee filter is applied. Then the Frost and, finally, the mean filter follows. This permits to expect that for the local statistic Lee filter the sequential application of FMH filter to its output should result in better additional noise suppression than if the Frost filter is used at the first stage.

As it is known, the FMH filter preserves edges and fine details well enough. Thus, we expect that its iterative application to the outputs of local statistic Lee or Frost filters should not result in additional undesirable

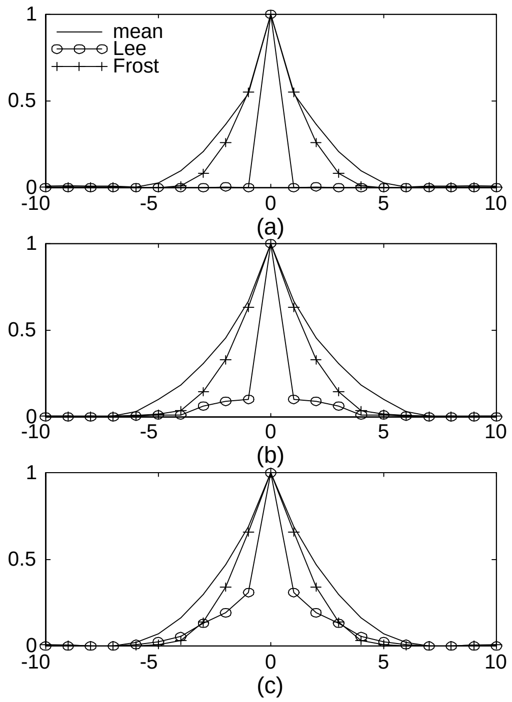

Figure 1: Horizontal cross-sections of the spatial correlation functions after filtering: (a) Gaussian; (b) Rayleigh; (c) exponential p.d.f.s of multiplicative noise.

smearing of edges and fine details. Therefore, all the main requirements to image processing algorithms are satisfied. One of the main advantages of the proposed iterative procedure of image processing is that it can be applied to data initially corrupted by different kinds of noise (i.e., with symmetrical and non-symmetrical p.d.f.s, when spikes are present or absent).

4. NUMERICAL SIMULATION RESULTS

The numerical simulations were performed for an artificial test image (it is shown in Figure 2) containing homogeneous regions, details with different contrasts and configurations, and large size objects having various contrasts with respect to background. The performance of the image filtering algorithms can be characterized by a set of characteristic figures evaluated quantitatively for different areas of the test image. As in [9] we calculated RMSE values δ_h, δ_e, and δ_d in the following way

$$\delta_h = \sqrt{\sum_{ij\in\Omega_h} \frac{(y(i,j)-f(i,j))^2}{L_h}}, \quad (4)$$

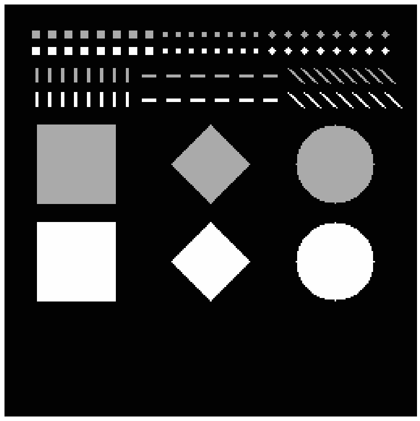

Figure 2: Test image model.

$$\delta_e = \sqrt{\sum_{ij\in\Omega_e} \frac{(y(i,j)-f(i,j))^2}{L_e}}, \quad (5)$$

$$\delta_d = \sqrt{\sum_{ij\in\Omega_d} \frac{(y(i,j)-f(i,j))^2}{L_d}}, \quad (6)$$

where Ω_h, Ω_e, and Ω_q are the sets of pixels belonging to homogeneous background (h), edges (e) and details (d) and their neighborhoods containing L_h, L_e, and L_d pixels, respectively. So these parameters describe the main properties of the considered filters. The numerical simulation results are given in Tables 1, 2, and 3 for three p.d.f.s of multiplicative noise (for Gaussian noise σ_μ^2 was equal to 0.03). They have been obtained for the artificial test image for the cases of spike absence ($P_a = 0.00$) and presence ($P_a = 0.01$). The "Salt-and-pepper" model of impulsive noise was used [4]. The data for initial images (denoted in the tables as "None") are presented in order to a) give the initial RMSE values δ_h, δ_e, and δ_d; b) show that the standard Lee and Frost filters are not robust; c) indicate the level of spike influence.

The numerical simulation results prove the assumptions mentioned above. It is seen that the sequential application of FMH filter provides robustness with respect to spikes (RMSE values radically decrease), edge/detail preservation properties do not become worse (δ_e and δ_d values become less or remain almost the same), and additional noise reduction in homogeneous regions is always observed and it is essential (δ_h reduces).

Another set of conclusions and recommendations resulting from the obtained data analysis are the following:

1. For Gaussian noise it is better to use the Frost filter at the preliminary stage. In this case the iterative

Table 1: The numerical simulation results for Gaussian p.d.f. of multiplicative noise.

Filter	Without 2nd iterat.			With 2nd iterat.		
	δ_h	δ_e	δ_d	δ_h	δ_e	δ_d
$P_a = 0.00$						
None	5.28	7.45	5.84	–	–	–
Frost 5x5	1.61	5.57	3.91	1.26	4.51	3.83
Lee 5x5	2.82	5.71	4.21	1.79	4.94	4.22
Frost 7x7	1.10	5.12	3.75	0.96	4.50	4.00
Lee 7x7	2.79	5.86	4.38	1.67	5.12	4.41
$P_a = 0.01$						
None	15.60	17.74	17.66	–	–	–
Frost 5x5	14.79	17.11	17.16	1.63	4.87	4.09
Lee 5x5	14.28	16.18	16.41	2.07	5.16	4.40
Frost 7x7	14.77	17.01	17.15	1.62	4.92	4.26
Lee 7x7	14.20	16.05	16.34	2.13	5.31	4.57

Table 2: The numerical simulation results for Rayleigh p.d.f. of multiplicative noise.

Filter	Without 2nd iterat.			With 2nd iterat.		
	δ_h	δ_e	δ_d	δ_h	δ_e	δ_d
$P_a = 0.00$						
None	10.46	28.29	17.8	–	–	–
Frost 5x5	3.08	15.98	12.9	2.44	13.76	11.68
Lee 5x5	2.93	15.77	11.8	2.28	14.84	12.64
Frost 7x7	2.07	14.45	12.0	1.84	13.70	11.77
Lee 7x7	2.43	16.66	12.5	1.80	15.84	13.42
$P_a = 0.01$						
None	18.39	31.99	24.6	–	–	–
Frost 5x5	15.70	20.59	21.4	2.90	13.95	11.68
Lee 5x5	11.49	18.30	16.9	2.69	14.82	12.71
Frost 7x7	14.97	17.68	19.9	2.87	13.85	11.98
Lee 7x7	11.19	18.87	17.2	2.48	15.71	13.40

Table 3: The numerical simulation results for one-side exponential p.d.f. of multiplicative noise.

Filter	Without 2nd iterat.			With 2nd iterat.		
	δ_h	δ_e	δ_d	δ_h	δ_e	δ_d
$P_a = 0.00$						
None	9.67	45.12	25.9	–	–	–
Frost 5x5	2.90	20.13	16.1	2.18	18.83	14.58
Lee 5x5	2.35	20.05	14.8	1.95	20.03	16.07
Frost 7x7	1.97	19.04	15.4	1.71	18.91	14.70
Lee 7x7	1.88	21.08	15.8	1.55	21.17	17.44
$P_a = 0.01$						
None	15.76	22.36	22.6	–	–	–
Frost 5x5	18.52	47.60	31.5	2.64	18.86	14.74
Lee 5x5	7.75	20.58	16.8	2.60	19.99	16.24
Frost 7x7	14.38	20.41	20.4	2.58	18.92	14.88
Lee 7x7	7.41	21.52	17.6	2.04	21.08	17.44

procedure provides better noise suppression efficiency than if the local statistic Lee filter is applied. The difference is especially obvious in the case of 7x7 scanning window of the filter applied at the first stage. Besides, the procedure Frost+FMH ensures better detail preservation than Lee+FMH.

2. For Rayleigh and negative exponential distributions of speckle the iterative procedure Frost+FMH ensures a little bit better edge/detail preservation but poorer noise suppression efficiency in comparison with Lee+FMH.

3. In the cases of spike presence it is suggested to use the local statistic Lee filter at the first stage for Rayleigh and negative exponential p.d.f.s because this variant provides better robustness with respect to spikes. In the case of Gaussian noise and spike presence the situation is opposite.

4. Both iterative procedures perform well enough only in cases of rather low probability of spikes – not greater than $0.01 \ldots 0.02$. Even in this case the RMSE values rapidly increase with the growth of P_a. This fact is explained by relatively poor robustness features of FIR median hybrid filter applied at the second stage. In the cases of larger probabilities of impulsive noise more robust nonlinear filters, i.e., 3x3 standard median, can be used at the second stage. But the detail preservation properties of the corresponding iterative filtering algorithm can then become worse.

5. The noise suppression efficiency of both procedures approximates (in case of spike absence) the one of the mean filter with the same scanning window size – the difference in the variances of the remained fluctuations for the homogeneous regions of the image is not large for all considered distributions.

6. It is reasonable to use the Frost and the local statistic Lee filters having larger scanning window (7x7) than the FIR median hybrid (5x5) filter. In this case the detail/edge preservation properties of the iterative procedure remain practically the same as when 5x5 scanning windows are used at the first stage but the efficiency of multiplicative noise suppression is essentially improved.

The proposed iterative algorithms have been also tested for real radar images and the obtained results confirmed their efficiency. In order to illustrate this we present two figures. In Figure 3 the real SAR image is depicted. It was obtained by an airborne system of the Institute of Radiophysics and Electronics of Ukranian Academy of Science. The image after sequential application of the Frost filter and the FMH filter is presented in Figure 4. It is seen that the noise is essentially suppressed after processing.

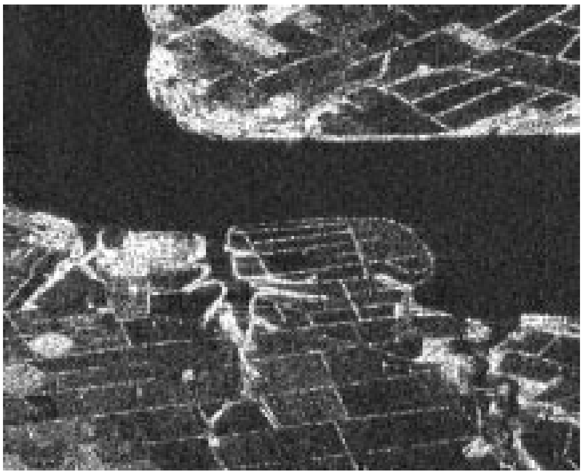

Figure 3: A SAR image.

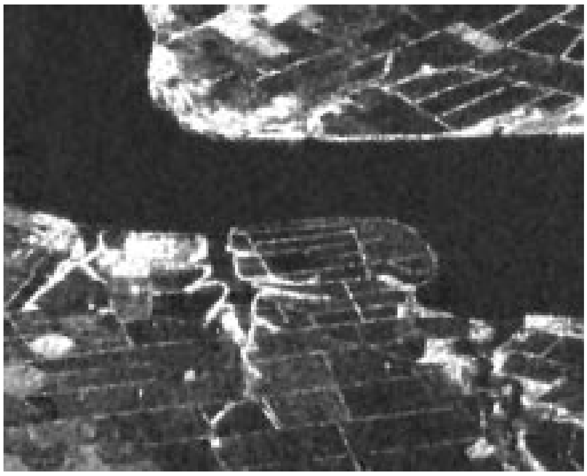

Figure 4: The SAR image of Figure 3 after iterative processing.

5. CONCLUSIONS

Two iterative techniques to process images corrupted by multiplicative noise and spikes are considered and compared quantitatively. It is shown that both of them are efficient when the probability of spikes is small (or they are absent). The selection between the two considered procedures depends upon the priority of requirements of the filtering algorithm as well as the multiplicative noise distribution. The iterative procedures have been successfully tested for real radar images.

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