

# Adaptive Control of a One-Legged Hopping Robot Model

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**Keywords:** adaptive control, system identification, spring-mass hopper, legged locomotion

## Extended Abstract

Practical realization of model-based dynamic legged behaviors is substantially more challenging than statically stable behaviors due to their heavy dependence on second order system dynamics. This problem is further aggravated by the difficulty of accurately measuring or estimating dynamic parameters such as spring and damping constants for associated models and the fact that such parameters are prone to change in time due to heavy use and associated material fatigue. Fortunately, this issue is not confined to the control of legged locomotion and received considerable attention from the adaptive control community [1]. Motivated by work in this area, this study presents a new model-based adaptive control method for running with the well-known Spring-Loaded Inverted Pendulum (SLIP) model (see Fig. 1), emphasizing on-line estimation of unknown or miscalibrated dynamic system parameters.

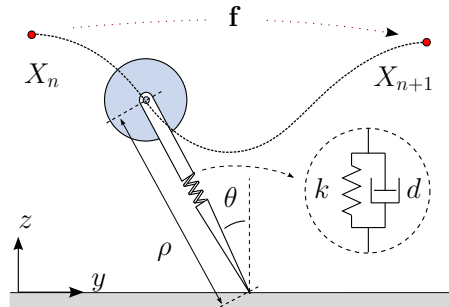


Figure 1: The Spring-Loaded Inverted Pendulum (SLIP) model. Dashed curve illustrates a single stride from one apex event to the next, defining the return map  $X_{n+1} = \mathbf{f}(X_n, u_n)$ .

In the presence of a sufficiently accurate model, gait control of the SLIP model can be achieved through a deadbeat strategy as described in [2]. Given a desired apex state  $X^*$ , inversion of the apex return map yields the controller  $u = f^{-1}(X^*, X_n)$ . Note, however, that the approximate return map and hence its inversion can only rely on possibly inaccurate parameter estimates for spring and damping constants.

The core of our adaptive control strategy relies on once-per-step corrections to these estimates based on the difference between predicted and measured apex states for each stride [3]. This corrective parameter adjustment is very similar to how estimation methods such as Kalman filters use innovation on sensory measurements to perform state updates.

Fig. 2 illustrates the block diagram for the adaptive parameter correction scheme we propose. Our method relies on the availability of an approximate return map  $\mathbf{g}$  that can predict the apex state outcome of a single stride. In this study, we consider two alternatives for this approximate predictor model. *Exact SLIP Model (ESM)*

predicts the outcome through numerical simulation of SLIP dynamics. In contrast, *Approximate Analytical Solution (AAS)* uses analytic differentiation of AAS derived in [2].

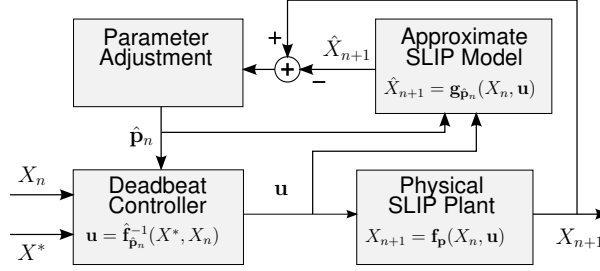


Figure 2: The proposed adaptive control strategy. Prediction errors of an approximate plant model  $g$  are used to dynamically adjust parameter estimates.

The first option is useful for accurate identification of the dynamic parameters of the system, whereas the second option will be useful in eliminating steady-state tracking errors for the gait-level control of SLIP running. Both of these goals can be defined as a function of steady-state behavior of the system. Consequently, we define three error measures where  $SSE_k$  and  $SSE_d$  capture system identification performance and  $SSE_a$  characterizes the tracking performance of the adaptive controller. In order to test our algorithm, we run a large number of simulations using different ranges of system parameters. Table I summarizes the average apex state tracking and parameter estimation errors and their standard deviations across all simulations.

It may be surprising that AAS predictor outperforms the ESM predictor for apex goal tracking. However, note that the deadbeat controller of [2] is based on the inversion of AAS. Naturally, when dynamic system parameters are adapted such that the predictions of these approximations are error-free, the resulting controller achieves zero tracking error in steady-state. In contrast, while the ESM predictor can accurately estimate the dynamic parameters, some remaining prediction errors still remain, leading to the small steady-state tracking errors as shown in Table I.

Table 1: Percentage apex tracking and parameter estimation errors

Error Measure:	$SSE_a$	$SSE_k$	$SSE_d$
Non-adaptive	$6.56 \pm 4.64$	$10 \pm 6.20$	$10 \pm 6.20$
AAS Adaptive	$0.002 \pm 0.001$	$2.34 \pm 1.45$	$5.53 \pm 2.81$
ESM Adaptive	$0.52 \pm 0.45$	$0.0008 \pm 0.0005$	$0.007 \pm 0.005$

In this study, we proposed a novel adaptive control algorithm to both support on-line identification of unknown dynamic parameters and improve steady-state tracking performance of previously proposed control algorithms for SLIP model. Our method used as a system identification tool addresses the practical difficulty of measuring possibly time-varying dynamic system parameters. In contrast, our method used as an adaptive controller allows effective elimination of steady-state tracking errors under different types of modeling errors for inverse dynamics controllers.

## References

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