System Identification of Legged Locomotion via Harmonic Transfer Functions and Piecewise LTI Approximation

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1 Introduction

In this study our goal is providing a formal system identification framework applicable to a class of legged locomotion models [2] and robotic systems [3]. Our approach to analyzing legged locomotion dynamics is based on considering the locomotion as a hybrid nonlinear dynamical system with stable periodic orbit (limit-cycle). We introduce a formulation that addresses the input–output system identification problem in frequency domain for hybrid legged locomotion models. Specifically based on some assumptions regarding the hybrid dynamics of the legged systems, we approximate the hybrid dynamics around the limit-cycle as a linear-time periodic system (LTP). However, this first LTP approximation is infinite dimensional, making parametric identification challenging. In order to limit the parametric degree of freedom of this LTP system we further approximate the dynamics with a *piece-wise LTI* system model (still LTP) which has finite degrees of freedom.

We then describe the system identification problem in the frequency domain using harmonic transfer functions [5], infinite-dimensional operators that are the analogues of frequency response functions for LTI systems. Since we approach the problem with a grey-box model structure with finite parameters (Piece-wise LTI), it suffices to estimate a finite number harmonics (non-parametrically) to which we later fit parametric models.

2 Modeling Legged Locomotion as a Linear Time Periodic System

Our goal is to provide a system identification framework for a class of legged locomotion models using harmonic transfer functions. In order to directly use the time periodicity in LTP analysis, and to avoid the complications associated with estimating phase [1], we limit ourselves to "clock-driven" locomotion models. Fortunately, clock-driven controllers are widely used is legged robotic platforms [3].

The legged locomotion models can be modeled as hybrid dynamical systems and cannot be easily characterized with a single smooth dynamical flow. In the broadest sense a hybrid dynamical system is a set of smooth flows and discrete transformations, which are triggered by threshold functions.

We consider a hybrid dynamical system with two charts, i.e. $\mathcal{I} = \{0, 1\}$, similar to SLIP like legged models [2]. The approximate nonlinear dynamics of the clock-driven model we consider is in the form

$$\begin{aligned} \dot{\phi} &= 1 \\ \dot{q} &\approx \begin{cases} f_0(q, \phi, u)\ , & \text{if } \operatorname{mod}(t, T) \in [0, \hat{t}), \\ f_1(q, \phi, u)\ , & \text{if } \operatorname{mod}(t, T) \in [\hat{t}, T). \end{cases} \end{aligned} \tag{1}$$

Assuming that the system given above has a limit cycle $\bar{q}(t)$, whose period is T, linearization around $\bar{q}(t)$ yields

$$\dot{x}(t) = \begin{cases} A_0(t)x(t) + B_0(t)u(t), & \text{if } \operatorname{mod}(t, T) \in [0, \hat{t}), \\ A_1(t)x(t) + B_1(t)u(t), & \text{if } \operatorname{mod}(t, T) \in [\hat{t}, T), \end{cases}$$

which is a piece-wise smooth LTP system with the corresponding time periodic output equation in the form

$$y(t) = \begin{cases} C_0(t)x(t) + D_0(t)u(t), & \text{if } \operatorname{mod}(t, T) \in [0, \hat{t}), \\ C_1(t)x(t) + D_1(t)u(t), & \text{if } \operatorname{mod}(t, T) \in [\hat{t}, T). \end{cases}$$

Since system matrices $A_i(t), B_i(t), C_i(t), D_i(t)\ i \in \{0, 1\}$ are time-parametrized functions (i.e. infinite dimensional), parametric identification is challenging. At this point we hypothesize that for hybrid systems the variability within a chart is small compared to the change between charts, thus we approximate the LTP dynamics using a piecewise LTI (still LTP) approximation. The LTP equations of motion take the form

$$\dot{x}(t) \approx \begin{cases} A_0x(t) + B_0u(t), & \text{if } \operatorname{mod}(t, T) \in [0, \hat{t}), \\ A_1x(t) + B_1u(t), & \text{if } \operatorname{mod}(t, T) \in [\hat{t}, T), \end{cases}$$

$$y(t) \approx \begin{cases} C_0x(t) + D_0u(t), & \text{if } \operatorname{mod}(t, T) \in [0, \hat{t}), \\ C_1x(t) + D_1u(t), & \text{if } \operatorname{mod}(t, T) \in [\hat{t}, T). \end{cases}$$

Above formulation will be our framework for analyzing and identifying clock-driven legged locomotion models.

3 Harmonic Transfer Functions

A harmonic transfer function (HTF) is a linear operator which maps the coefficients of the input harmonics to the coefficients of the output harmonics in exponentially modulated periodic (EMP) signal space [5]. HTF is derived using the principle of harmonic balance and describes the input–output relationship between the harmonics of the input signal, and those of the output signal such that

$$\mathcal{Y} = \hat{G}\mathcal{U}, \tag{2}$$

where

$$\hat{G}(s) = \mathcal{C}[sI - (\mathcal{A} - \mathcal{N})]^{-1}\mathcal{B} + \mathcal{D}, \tag{3}$$

as long as its inverse exists. Here, the four-tuple $[\mathcal{A}, \mathcal{B}, \mathcal{C}, \mathcal{D}]$ represents the harmonic state space model and $\mathcal{N}$ represents the frequency modulation matrix. The details about the derivation of harmonic transfer functions can be found in [5].

4 System Identification of LTP Systems

In this section we describe the procedure to identify harmonic transfer functions of LTP systems explained in [4]. The HTF $\hat{G}$ of an LTP system can be written as below

$$\hat{G}(\omega) = (\Phi_{UU})^{-1}\Phi_{UY}, \tag{4}$$

where Φ_{UU} and Φ_{UY} represents power spectral density and cross-spectral density functions, respectively. $\hat{G}(\omega)$ is not a single transfer function as in LTI systems but each row of $\hat{G}(\omega)$ includes different transfer functions corresponding to different harmonics. The LTP system identification problem is underdetermined, since an input-output pair will not be sufficient to identify N different transfer functions.

The key idea for choosing the inputs is to consider the time of application of each input relative to the system period. In order to obtain a complete characterization of a periodic system behavior, the system must be excited at various times during its period. Therefore, we excite the system with N inputs by evenly separating their time of application in the system period.

5 Identification Results for a Piecewise LTI System

In this section we consider a piecewise LTI system model, where the system is damped for the first half of its period and lossless in the second half. Our equation is similar to the damped Mathieu equation but modified to make it applicable for legged locomotion models. The example system we are using is like a half-damped piecewise LTI system, which can be formulated as a LTP model.

$$\ddot{x}(t) + 2\zeta\omega_n s(t)\dot{x}(t) + \omega_n^2 x(t) = u(t), \tag{5}$$

where $\zeta = 0.3$ and $\omega_n = 2\pi$. The half-damped property is incorporated with $s(t)$ function, where $s(t) = 1$ when $Tn \leq t \leq Tn + T/2$ and $s(t) = 0$ otherwise for $n \in \mathbb{Z}$ and $T = 0.5s$.

Theoretical HTFs are computed by using (3) considering the system response up to 10^{th} harmonic. The results showed that there were no response in the positive harmonics and the response on the negative side was negligible after third harmonic. Therefore, we considered only the fundamental and three harmonics. The results for our identification studies can be seen in Fig. 1. The identified transfer functions are G_0, G_{-1}, G_{-2} and G_{-3}.

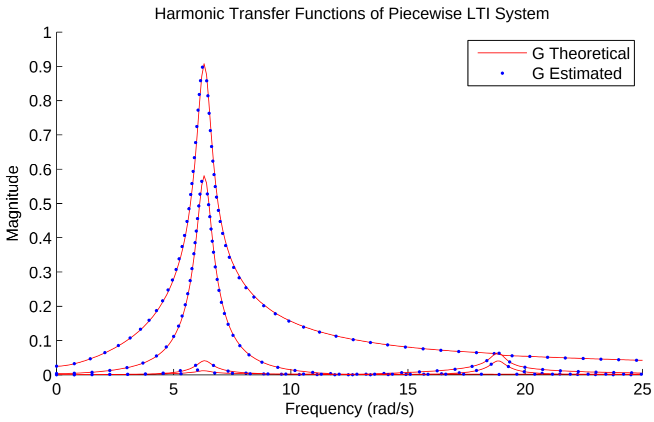

Figure 1: Four transfer functions of our piecewise LTI system obtained theoretically and empirically

Our results show that LTP system identification techniques are suitable for identification of piecewise LTI systems such as legged locomotion models when appropriate chirp inputs are used to activate periodic system. As a future work we plan to extend our method to identify the transfer functions of an actual legged locomotion model.

References

[1] S. Revzen and J. M. Guckenheimer. Estimating the phase of synchronized oscillators. *Phys Rev E*, 78(5 Pt 1):051907–051918, 2008.

[2] U. Saranli, O. Arslan, M. M. Ankaralı, and O. Morgül. Approximate analytic solutions to non-symmetric stance trajectories of the passive spring-loaded inverted pendulum with damping. *Nonlinear Dynamics*, 62:729–742, December 2010.

[3] U. Saranli, M. Buehler, and D. E. Koditschek. RHex: A simple and highly mobile robot. *Int J Robot Res*, 20(7):616–631, July 2001.

[4] A. Siddiqi. *Identification of the Harmonic Transfer Functions of a Helicopter Rotor.* M.sc., Massachusetts Institute of Technology, Dept. of Aeronautics and Astronautics, 2001.

[5] N. W. Wereley. *Analysis and Control of Linear Periodically Time Varying Systems.* Ph.d., Massachusetts Institute of Technology, Dept. of Aeronautics and Astronautics, 1991.